DINVAR NUMBERS

DECINVARI
REFRACTI
~~INVARDEC~~1.WPD March 24, 2009

~~DIVAR #1~~ DINVAR ~ FRACTALS: FRACTAL NUMBERS

INVARIANT DECIMAL NUMBERS

REFRACT
DECINVAR ~~INVARDEC~~ NUMBERS ARE NUMBERS WHOSE DECIMAL PORTION REMAINS INVARIANT UNDER SQUARING AND OR INVERSION.

That is, if **D** is an ~~invardec~~ number, then $\mathbf{D}^2$ and / or $\mathbf{D}^{-1}$ is also an ~~invardec~~ DECINVAR number.

The most familiar ~~invardec~~ number is the golden ratio, $\Phi = (1 \pm \sqrt{5})/2$ REFRACT

$\Phi = 1.6180339887498948482045868343656.....$
$\Phi^{-1} = 0.6180339887498948482045868343656.....$
$\Phi^2 = 2.6180339887498948482045868343656$
But $\Phi^{-2} = 0.3819660112501051517954131656343 6$

Another example is K involving $3 \pm \sqrt{13}$ $K = \frac{\sqrt{13}-3}{2}$

$(\sqrt{13}-3)/2$
$(\sqrt{13}+3)/2$
$(\sqrt{13}\pm 1)/2$

$K = 0.3027756377319946465596106337352 5$
$K^{-1} = 3.3027756377319946465596106337352 5$
$K + 2 = 2.3027756377319946465596106337352 5$
$(K+2)^2 = 5.3027756377319946465596106337352 5$

$\frac{7+\sqrt{13}}{2} = 3 + \frac{(\sqrt{13}+1)}{2}$

Another example is $A = 5 - \sqrt{15}$

INFRACTI.WPD

$A = 1.1270166537925831148207346002176$
$A^2 = 1.2701665379258311482073460021 76$

But $A^{-1} = 0.8872983346207416885179265399782 4$
With $A^{-2} = 0.7872983346207416885179265399782 4$

$5+\sqrt{15} = 8.872983346$
$= 10A^{-1}$

$= (\frac{1}{A} + \sqrt{15})/10$

In this case a difference in the first place or two does not destroy the identity of the remaining decimal places.

Two sufficient conditions:
For a number $D = H + F\sqrt{K}$ to be invardec when squared, since $D^2 = H^2 + F^2 K + 2HF\sqrt{K}$
$2H$ must = 1 or 10^N where N is a positive integer

For a number $D = H + F\sqrt{K}$ to be invardec when inverted, since $D^{-1} = (H - F\sqrt{K})/(H^2 - F^2K)$
$F^2K - H^2$ must = 1 or 10^{-N} where N is a positive integer

INFRACT NUMBERS ARE NUMBERS WHOSE DECIMAL PORTION MAY BE CHANGED BY THE location of the decimal point but is otherwise invariant under squaring

When $\sqrt{a^2+4} = (1+4a)$

a	
1,1	5
3,3	13
5,7	29
7,13	53
9,21	85

$\frac{1}{x} + x^2$ same

	5−5	0.618034
8	13−13	.302773
16	29−29	.192582
24	53−53	.141054
32	85−85	

What non + integer

[... 1/x] are infracts?

[.. √x]

$[\frac{1}{x}]\ \sqrt{60} \to 0.127016$

$[\sqrt{r}]\ \sqrt{60} \to 7.8729$

$[\frac{1}{x}]\ \sqrt{6} \to$

R	2	5	10	17	26
√R	1	2	3	4	5
a	1	3	5	7	9
−a	0	0	0	0	0
	5	13	29	53	85
	1,1	3,3	5,7	7,13	

~~ROOTINV2.~~WPD
REFRACT2.WAD

April 5, 2009

Continued fraction inverted and decinvar

(Iterated root)^2 are decinvar

a = 1, a = 3 } decinvar

Modified Q inverse

CONTINUED FRACTION

R SAME AS R R^2 squared R

ITERATED ROOT

$\frac{1+2a+\sqrt{1+4a}}{2}$

a	$\sqrt{(a^2+4)}$	$[\sqrt{(a^2+4)}-a]/2$	$[\sqrt{(a^2+4)}+a]/2$	$[1+\sqrt{(1+4a)}]/2$	$a+[1+\sqrt{(1+4a)}]/2$	$\sqrt{(1+4a)}$
1	5	0.618033989	1.618033989	1.618033989	2.618033989	5
2	8	0.414213562 $\sqrt{2}-1$	2.414213562 $=\sqrt{2}+1$	2	4	9
3	13	0.302775638	3.302775638	2.302775638	5.302775638	13
4	20	0.236067977 $\sqrt{5}-2$	4.236067977 $\sqrt{5}+2$	2.561552813	6.561552813	17
5	29	0.192582404	5.192582404	2.791287847	7.791287847	21
6	40	0.162277660 $\sqrt{10}-3$	6.162277660 $\sqrt{10}+3$	3	9	25
7	53	0.140054945	7.140054945	3.192582404	10.192582404	29
8	68	0.123105626 $\sqrt{17}-4$	8.123105626 $\sqrt{17}+4$	3.372281323	11.372281323	33
9	85	0.109772229	9.109772229	3.541381265	12.541381265	37
10	104	0.099019514 $\sqrt{26}-5$	10.099019514 $\sqrt{26}+5$	3.701562119	13.701562119	41
11	125	0.090169944	11.090169944	3.854101966	14.854101966	45
12	148	0.082762530	12.082762530 $\sqrt{37}$	4	16	49
13	173	0.076473219	13.076473219	4.140054945	17.140054945	53
14	200	0.071067812	14.071067812 $\sqrt{50}$	4.274917218	18.274917218	57
15	229	0.066372975	15.066372975	4.405124838	19.405124838	61

def

1 $\sqrt{5}$ (2, 1)

2

3 $\sqrt{13}$ (3, 2)

4

5

6 $\sqrt{61}$ (6, 5)

7

8

5 (41, 4)

$5^3 =$ 118.066 413 342

A ~~118.000000367~~

118.000040367

17 293 0.058 621 384

17.058 621 384

$\frac{\sqrt{229}+15}{2}$ $\sqrt{65}$

5 fr n 7 IR

of 9 and 11 to 3 places

$\Delta = 4$

$(4.618033989)^{3/2} = 2.058171027$

~~ROOTINV2.WPD~~

REFRACT2.WPD

April 5, 2009

CONTINUED FRACTION Q — Q^{-1} INV or iterated division

$\sqrt{R}$ ITERATED ROOT — square root + R

a	(a^2+4)	$[\sqrt{(a^2+4)}-a]/2$	$[\sqrt{(a^2+4)}+a]/2$	$[1+\sqrt{(1+4a)}]/2$	$a+[1+\sqrt{(1+4a)}]/2$	$(1+4a)$
1	5	0.618033989	1.618033989	1.618033989	2.618033989	5
2	8	0.414213562	2.414213562	2	4	9
3	13	0.302775638	3.302775638	2.302775638	5.302775638	13
4	20	0.236067977	4.236067977	2.561552813	6.561552813	17
5	29	0.192582404	5.192582404	2.791287847	7.791287847	21
6	40	0.162277660	6.162277660	3	9	25
7	53	0.140054945	7.140054945	3.192582404	10.192582404	29
8	68	0.123105626	8.123105626	3.372281323	11.372281323	33
9	85	0.109772229	9.109772229	3.541381265	12.541381265	37
10	104	0.099019514	10.099019514	3.701562119	13.701562119	41
11	125	0.090169944	11.090169944	3.854101966	14.854101966	45
12	148	0.082762530	12.082762530	4	16	49
13	173	0.076473219	13.076473219	4.140054945	17.140054945	53
14	200	0.071067812	14.071067812	4.274917218	18.274917218	57
15	229	0.066372975	15.066372975	4.405124838	19.405124838	61

$\pm a \rightarrow \pm$

5 0 1 1 2
13 0 3 2 5
29 0 5 3 10
53 0 7 4 17
85 0 9 5 26

5
13
29
53
85
149
277

85

$A = m \cdot 8$

DINVAR ± → integer

ROOTINV1.WPS April 4, 2009

Iterated division

a	$\sqrt{(a^2+4)}$		$[\sqrt{(a^2+4)} - a]/2$ continued fraction	$[\sqrt{(a^2+4)} + a]/2$ inverse	iterated root	square
1	5	ϕ	0.618033989	1.618033989	1.618033989	2.618033989
2	8		0.414213562	2.414213562	2	4
3	13		0.302775638	3.302775638	2.302775638	5.302775638
4	20		0.236067977	4.236067977	2.561552813	6.561552813
5	29		0.192582404	5.192582404	2.791287847	7.791287847
6	40		0.16227766	6.16227766	3	9
7	53		0.140054945	7.140054945	3.192582404	10.192582404
8	68		0.123105626	8.123105626	3.372281323	11.372281323
9	85		0.109772229	9.109772229	3.541381265	12.541381265
10	104		0.099019514	10.099019514	3.701562119	13.701562119
11	125		0.090169944	11.090169944	3.854101966	14.854101966
12	148		0.0827625303	12.0827625303	4	16
13	173		0.076473219	13.076473219	4.140054945	17.140054945
14	200		0.071067812	14.071067812	4.274917218	18.274917218
15	229		0.066372975	15.066372975	4.405124838	19.405124838
16	260		0.0622577483	16.0622577483	4.531128874	20.531128874

SOMATH01.WPG JUNE 28, 1997 #4

ROOTINV1.WPS April 3, 2009

a	continued fraction	inverse	iterated root	square
1 $(\sqrt{5}-1)/2 = \phi$	0.618033989	1.618033989	1.618033989	2.618033989
2 $\sqrt{2}-1$	0.414213562	2.414213562	2	4
3	0.302775638	3.302775638	2.302775638	5.302775638
4 $\sqrt{5}-2$	0.236067977	4.236067977	2.561552813	6.561552813
5	0.192582404	5.192582404	2.791287847	7.791287847
6 $\sqrt{10}-3$	0.16227766	6.16227766	3	9
7	0.140054945	7.140054945	3.192582404	10.192582404
8 $\sqrt{17}-4$	0.123105626	8.123105626	3.372281323	11.372281323
9	0.109772229	9.109772229	3.541381265	12.541381265
10 $\sqrt{26}-5$	0.099019514	10.099019514	3.701562119	13.701562119
11	0.090169944	11.090169944	3.854101966	14.854101966
12 $\sqrt{37}-6$	0.0827625303	12.0827625303	4	16
13	0.076473219	13.076473219	4.140054945	17.140054945
14 $\sqrt{50}-7$	0.071067812	14.071067812	4.274917218	18.274917218
15	0.066372975	15.066372975	4.405124838	19.405124838
16 $\sqrt{65}-8$	0.0622577483	16.0622577483	4.531128874	20.531128874

add $3 \pm \sqrt{41}$ 3.403124273 9.403124273

$a = 2$

new species

b = imaginary in $\sqrt{a+3}$

$$B+x = \frac{(1+2a)+\sqrt{1+4a}}{2}$$

$$A+x = \frac{1+\sqrt{1+4a}}{2}$$

$(A+x)^2 = B+x$

$B - A = a$

$x + A = \sqrt{B+x}$

$(B+x) = a + \sqrt{B+x}$

$(B+x) - a = \sqrt{B+x}$

SQ U BNN

$(B+x)^2 + 2a(B+x) + a^2 = (B+x)$

$(B+x)^2 + (2a-1)(B+x) + a^2 = 0$

$B+x = \dfrac{(1-2a) + \sqrt{(2a-1)^2 - 4a^2}}{2} = \dfrac{1-2a+\sqrt{4a^2-4a+1-4a^2}}{2}$

$B+x = \dfrac{(1-2a) \pm \sqrt{1-4a}}{2}$

$x =$

$(A+x)^2 = B+x$ $\quad$ $B+x = (A+x)^2$

$B - A = a$ $\quad$ $-B+A = -a$

$A+x = (A+x)^2 - a$

$(A+x)^2 - (A+x) - a = 0$

$(A+x) = \dfrac{+1 \pm \sqrt{1+4a}}{2}$

$(B+x) = \dfrac{1+2a \pm \sqrt{1+4a}}{2}$

$a^2 + 4 = 1 + 4a$

$a^2 - 4a + 3 = 0$

$a = \dfrac{4 \pm \sqrt{16-12}}{2} = \dfrac{4 \pm 2}{2}$, $a = 3$ or 1

∃ a's for which we do

$a + \sqrt{a^2+4} = 1 + \sqrt{1+4a}$ $\quad$ $= 2.643642941$

$a^2 + 4 = [(1-a) + \sqrt{1+4a}]^2 = (1-a)^2 - 2(1-a)\sqrt{\ } + 1 + 4a$

$a^2 + 4 = 1 - 2a + a^2 + 1 + 4a - 2(1-a)\sqrt{\ }$

$1 = -2a - (1-a)\sqrt{\ }$

$\dfrac{1+2a}{a-1} = \sqrt{\ }$

$\dfrac{4a^2+4a+1}{a^2-2a+1} = 1+4a$

$-3 \pm \sqrt{41}$ $\quad$ 3,403 124 273

9,403 124 237

$a = -\dfrac{3 \pm \sqrt{41}}{8}$

0,425 390 53

- 1, 175 390 53

Decimal Shift

FRACTNUM2.WPD May 9, 2008

$-5+\sqrt{35}$ = 0.916 079 783 099 616 042 567 328 291 561

$-5-\sqrt{35}$ = -10.916 079 783 099 616 042 567 328 291 562 −10

$(-5-\sqrt{35})^2$ = 11 9.16 079 783 099 616 042 567 328 291 562 +11

$(-5+\sqrt{35})^{-1}$ = 1.0 916 079 783 099 616 042 567 328 291 562 +10 any more like this?

$(-5-\sqrt{35})^{-1}$ = -0.0 916 079 783 099 616 042 567 328 291 562 −

$(-5+\sqrt{35})^{-2}$ = 1.1 916 079 783 099 616 042 567 328 291 562 +11

$(-5-\sqrt{35})^{-2}$ = 0.00 839 202 169 003 839 574 326 717 084 384

$(-5+\sqrt{35})^2$ = 0.839 202 169 003 839 574 326 717 084 384

$-5+\sqrt{35}$ $-5-\sqrt{35}$

1 1 neg
−1 −1 neg
−2 2
+2 −2

$V = \frac{13-\sqrt{117}}{2} = 1.0916\,73087$

$\frac{1}{V} = 0.916\,025\,747$

also 0.839...

$\frac{1}{V} = 0.916025$

$-5+\sqrt{35} = 0.916080$

$\Delta = 0.000055$

$\frac{1}{1} = 1.000060042$

$\frac{1}{\sqrt{10}}$

$\sqrt{10} = 3.162277\,66$

$\frac{1}{\sqrt{10}} = 0.316\,227\,766$

10.000
0.10000

cf $\frac{-13 \pm \sqrt{117}}{2}$ w $5 \pm \sqrt{35}$

PRODSUM1.mcd

$b := -13$ $c := 13$ $x^2 - 13x + 13 = 0$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 117$ $r = 10.8166538264$

$u := \frac{(-b+r)}{2}$ $v := \frac{(-b-r)}{2}$ $u + v = 13$

$v \times u = 13$

$u = 11.9083269$ $v = 1.0916730868$

$u^2 = 141.8082498715$ $v^2 = 1.1917501285$ $\sqrt{u} = 3.4508443768$ $\sqrt{v} = 1.0448316069$

$u^4 = 2.0109579732 \cdot 10$ $v^4 = 1.4202683687$ $\sqrt{\sqrt{u}} = 1.8576448468$ $\sqrt{\sqrt{v}} = 1.0221700479$

$u^{-1} = 0.0839748528$ $v^{-1} = 0.9160251472$ cf over page $0.839202 = (-5+\sqrt{35})^2$

$u^{-2} = 7.051775908 \cdot 10$ $v^{-2} = 0.8391020702$ $\sqrt{u^{-1}} = 0.2897841487$ $\sqrt{v^{-1}} = 0.9570920265$

$u^{-4} = 4.9727543457 \cdot 1$ $v^{-4} = 0.7040922843$ $\sqrt{\sqrt{u^{-1}}} = 0.5383160305$ $\sqrt{\sqrt{v^{-1}}} = 0.9783108026$

$v = \frac{-13 - \sqrt{169-52}}{2} = \frac{-13 - \sqrt{117}}{2} = -11.908$

$\frac{+13 - \sqrt{117}}{2} = 1.0916\,673087$

080

0.0000593

$\frac{1}{-5+\sqrt{35}} \approx 1.0916080$

169
52
117

117 = 3.39

$117 = 13^2 - 4.13$ = 13.9

DINVAR $A_{n+2} \to x^3$, $A_{n+1} \to x$, $A_n \to 1$

PISIGMA7.WPD December 17, 2005

NUMERICAL VALUES
FOR THE ROOTS OF $A_{n+2} = 10(A_{n+1} - A_n)$ → u ? how

$x^3 = 10x$ $x^2 - 10x + 10 = 0$

$u = 1.12701665379258311482073460021 76 = 10/v = u^2/10 + 1 = \sqrt{[10(u-1)]}$

$1/v = 0.11270166537925831148207346002177 = u/10$

$u^2 = 1.27016653792583114820734600217 6 = 10(u - 1)$

$u/v = 0.12701665379258311482073460021 76 = u - 1$

$1/v^2 = 0.01270166537925831148207346002176 = u^2/100 = (u - 1)/10$

$v = 8.87298334620741688517926539978 24 = 10/u = v^2/10 + 1 = \sqrt{[10(v-1)]}$

$1/u = 0.88729833462074168851792653997824 = v/10$

$v^2 = 78.72983346207416885179265399781 7 = 10(v - 1)$

$v/u = 7.8729833462074168851792653997824 = v - 1$

$1/u^2 = 0.7872983346207416885179265399782 4 = v^2/100 = (v - 1)/10$

$u^3 = 1.43149884133248033386611401958 4$

$u^2/v = 0.14314988413324803338661140195 84 = u^3/10$

$u/v^2 = 0.01431498841332480333866114019584 = u^3/100$

$1/v^3 = 0.00143149884133248033386611401958 41 = u^3/1000$

$v^3 = 698.56850115866751966613388598042$

$v^2/u = 69.8568501158667519666133885980 42 = v^3/10$

$v/u^2 = 6.985685011586675196661338859804 2 = v^3/100$

$1/u^3 = 0.69856850115866751966613388598043 = v^3/1000$

$v^2/2 = 39.364916731037084425896326998912 = 5(v - 1)$

$u^2/2 = 0.63508326896291557410367300108 8 = 5(u - 1)$

05-12-28

CREATIVITY

1° Mastery of Tools (Rules)
Words Music Crafts Forms Architecture, paint
Poetry Jewelry

2° Emulate many style

3° Create Symbols - semantic - semiotic ...

4° Tools take charge

5° ~~Creator replaces the Creator~~

$u = \alpha\mu = 1.127074115$

$\frac{1}{u} = v = (\alpha\mu)^{-1} = 0.887253098$

$\frac{u}{v} = 1,270\,296\,061$

$\frac{1}{v^2} = 1,270\,296\,061$

$10 \times \frac{1}{v} = 11.2704115$

$u^2 = 1,270\,296\,061 = \frac{u}{v}$

1.127074115
1.270296061
.0000445

1,127074
1,127017
0,000057

PRODSUM

NUMBERS

PRODSUM2.WPD May 9, 2008

	√	+ = v	− = u	v^2	u^2	v^{-1}	u^{-1}	v^{-2}	u^{-2}
$x^2+14x-14$	√63	0.9372539	-14.93725	0.7716655	223.12155	1.0669467	-0.066946	1.1387528	0.0044818
$x^2+13x-13$	√221	0.9330344	-13.93303	0.8705531	194.12944	1.0717718	-0.071771	1.1486949	0.0051512
$x^2+12x-12$	√48	0.9282032	-12.92820	0.8615612	167.13843	1.0773502	-0.077350	1.1606836	0.0059831
$x^2+11x-11$	√165	0.9226163	-11.92261	0.8512208	142.14877	1.0838742	-0.083874	1.1747832	0.0070348
$x^2+10x-10$	√35	0.9160798	-10.91608	0.8392022	119.16079	1.0916079	-0.091608	1.1916079	0.0083920
x^2+9x-9	√117	0.9083269	-9.908326	0.8250577	98.174942	1.1009252	-0.100925	1.2120363	0.0101858
x^2+8x-8	√24	0.8989795	-8.898979	0.8081641	79.191835	1.1123724	-0.112372	1.2373724	0.0126275
x^2+7x-7	√77	0.8874822	-7.887482	0.7876246	62.212375	1.1267831	-0.126783	1.2696430	0.0160739
x^2+6x-6	√15	0.8729833	- 6.872983	0.762100	47.237895	1.1454972	-0.145497	1.3121364	0.0211694
x^2+5x-5	√45	0.8541019	-5.854102	0.7294902	34.270509	1.1708204	-0.170820	1.3708204	0.0291796
x^2+4x-4	√8	0.8284271	-4.828427	0.6862915	23.313708	1.2071067	-0.207106	1.4571067	0.0428932
x^2+3x-3	√21	0.7912878	-3.791287	0.6261365	14.373864	1.2637626	-0.263762	1.5970959	0.0695707
x^2+2x-2	√3	0.7320508	- 2.732051	0.5358984	7.4641016	1.3660254	-0.366025	1.8660254	0.1339746
x^2+x-1	√5	0.6180339	-1.618034	0.3819660	2.6180339	1.6180339	-0.618034	2.6180339	0.3819660
imaginary									
x^2-4x+4	√0	2	2	4	4	0.5	0.5	0.25	0.25
x^2-5x+5	√5	3.6180339	1.3819660	13.090166	1.9098301	0.2763932	0.7236068	0.0763932	0.5236068

	√	+ = v	− = u	v^2	u^2	v^{-1}	u^{-1}	v^{-2}	u^{-2}
x^2-6x+6	√3	4.7320508	1.2679492	22.392305	1.6076952	0.2113249	0.7886751	0.0446582	0.6220085
x^2-7x+7	√21	5.7912878	1.2087121	33.539014	1.4609850	0.1726731	0.8273268	0.0298160	0.6844696
x^2-8x+8	√8	6.8284271	1.1715728	46.627416	1.3725830	0.1464466	0.8535533	0.0214466	0.7285533
x^2-9x+9	√45	7.8541020	1.1458980	61.686917	1.3130823	0.1273220	0.8726779	0.0162108	0.7615669
x^2-10x+10	√15	8.8729833	1.1270167	78.729833	1.2701665	0.1127017	0.8872983	0.0127017	0.7872983
x^2-11x+11	√77	9.8874822	1.1125178	97.762304	1.2376958	0.1011379	0.8988620	0.0102288	0.8079529
x^2-12x+12	√24	10.898979	1.1010205	118.78775	1.2122461	0.0917517	0.9082482	0.0084183	0.8249149
x^2-13x+13	√117	11.908326	1.0916730	141.80824	1.1917501	0.0839748	0.9160254	0.0070517	0.8391020
x^2-14x+14	√35	12.916079	1.0839202	166.82512	1.1748830	0.0774228	0.9225771	0.0059943	0.8511485

PRODSUM2.WPD May 9, 2008 v^{-1}

	$\sqrt{}$	+ = v	− = u	v^2	u^2	v^{-1}	u^{-1}	v^{-2}	u^{-2}
x^2+14x-14	$\sqrt{}$63	0.9372539	-14.93725	0.7716655	223.12155	1.0669467	-0.066946	1.1387528	0.0044818
x^2+13x-13	$\sqrt{}$221	0.9330344	-13.93303	0.8705531	194.12944	1.0717718	-0.071771	1.1486949	0.0051512
x^2+12x-12	$\sqrt{}$48	0.9282032	-12.92820	0.8615612	167.13843	1.0773502	-0.077350	1.1606836	0.0059831
x^2+11x-11	$\sqrt{}$165	0.9226163	-11.92261	0.8512208	142.14877	1.0838742	-0.083874	1.1747832	0.0070348
x^2+10x-10	$\sqrt{}$35	0.9160798	-10.91608	0.8392022	119.16079	1.0916079	-0.091608	1.1916079	0.0083920
x^2+9x-9	$\sqrt{}$117	0.9083269	-9.908326	0.8250577	98.174942	1.1009252	-0.100925	1.2120363	0.0101858
x^2+8x-8	$\sqrt{}$24	0.8989795	-8.898979	0.8081641	79.191835	1.1123724	-0.112372	1.2373724	0.0126275
x^2+7x-7	$\sqrt{}$77	0.8874822	-7.887482	0.7876246	62.212375	1.1267831	-0.126783	1.2696430	0.0160739
x^2+6x-6	$\sqrt{}$15	0.8729833	- 6.872983	0.762100	47.237895	1.1454972	-0.145497	1.3121364	0.0211694
x^2+5x-5	$\sqrt{}$45	0.8541019	-5.854102	0.7294902	34.270509	1.1708204	-0.170820	1.3708204	0.0291796
x^2+4x-4	$\sqrt{}$8	0.8284271	-4.828427	0.6862915	23.313708	1.2071067	-0.207106	1.4571067	0.0428932
x^2+3x-3	$\sqrt{}$21	0.7912878	-3.791287	0.6261365	14.373864	1.2637626	-0.263762	1.5970959	0.0695707
x^2+2x-2	$\sqrt{}$3	0.7320508	- 2.732051	0.5358984	7.4641016	1.3660254	-0.366025	1.8660254	0.1339746
x^2+x-1	$\sqrt{}$5	0.6180339	-1.618034	0.3819660	2.6180339	1.6180339	-0.618034	2.6180339	0.3819660
imaginary									
x^2-4x+4	$\sqrt{}$0	2	2	4	4	0.5	0.5	0.25	0.25
x^2-5x+5	$\sqrt{}$5	3.6180339	1.3819660	13.090166	1.9098301	0.2763932	0.7236068	0.0763932	0.5236068

	$\sqrt{}$	+ = v	− = u	v^2	u^2	v^{-1}	u^{-1}	v^{-2}	u^{-2}
x^2-6x+6	$\sqrt{3}$ $\sqrt{12}$	4.7320508	1.2679492	22.392305	1.6076952	0.2113249	0.7886751	0.0446582	0.6220085
x^2-7x+7	$\sqrt{21}$	5.7912878	1.2087121	33.539014	1.4609850	0.1726731	0.8273268	0.0298160	0.6844696
x^2-8x+8	$\sqrt{8}$ $\sqrt{32}$	6.8284271	1.1715728	46.627416	1.3725830	0.1464466	0.8535533	0.0214466	0.7285533
x^2-9x+9	$\sqrt{45}$	7.8541020	1.1458980	61.686917	1.3130823	0.1273220	0.8726779	0.0162108	0.7615669
x^2-10x+10	$\sqrt{15}$ $\sqrt{60}$	8.8729833	1.1270167	78.729833	1.2701665	0.1127017	0.8872983	0.0127017	0.7872983
x^2-11x+11	$\sqrt{77}$	9.8874822	1.1125178	97.762304	1.2376958	0.1011379	0.8988620	0.0102288	0.8079529
x^2-12x+12	$\sqrt{24}$ $\sqrt{96}$	10.898979	1.1010205	118.78775	1.2122461	0.0917517	0.9082482	0.0084183	0.8249149
x^2-13x+13	$\sqrt{117}$	11.908326	1.0916730	141.80824	1.1917501	0.0839748	0.9160254	0.0070517	0.8391020
x^2-14x+14	$\sqrt{35}$ $\sqrt{140}$	12.916079	1.0839202	166.82512	1.1748830	0.0774228	0.9225771	0.0059943	0.8511485

THIS IS ROOTPROP3 x^2+bx+c=0

$c := 1,2..10$ $b := 1,2..10$

$$R_{b,c} := \sqrt{b^2 + 4 \cdot c}$$

$$P_{b,c} := \frac{(-b + R_{b,c})}{2} \qquad Q_{b,c} := \frac{(-b - R_{b,c})}{2}$$

c

b $P =$

	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0
1	1	1.30278	1.56155	1.79129	2	2.19258	2.37228	2.54138	2.70156
2	0.73205	1	1.23607	1.44949	1.64575	1.82843	2	2.16228	2.31662
3	0.56155	0.79129	1	1.19258	1.37228	1.54138	1.70156	1.8541	2
4	0.44949	0.64575	0.82843	1	1.16228	1.31662	1.4641	1.60555	1.74166
5	0.37228	0.54138	0.70156	0.8541	1	1.14005	1.27492	1.40512	1.53113
6	0.31662	0.4641	0.60555	0.74166	0.87298	1	1.12311	1.24264	1.3589
7	0.27492	0.40512	0.53113	0.65331	0.772	0.88748	1	1.10977	1.21699
8	0.24264	0.3589	0.47214	0.58258	0.69042	0.79583	0.89898	1	1.09902
9	0.21699	0.32183	0.42443	0.52494	0.62348	0.72015	0.81507	0.90833	1
10	0.19615	0.2915	0.38516	0.47723	0.56776	0.65685	0.74456	0.83095	0.91608

← products

$Q =$

	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0
1	-2.56155	-2.79129	-3	-3.19258	-3.37228	-3.54138	-3.70156
2	-3.23607	-3.44949	-3.64575	-3.82843	-4	-4.16228	-4.31662
3	-4	-4.19258	-4.37228	-4.54138	-4.70156	-4.8541	-5
4	-4.82843	-5	-5.16228	-5.31662	-5.4641	-5.60555	-5.74166
5	-5.70156	-5.8541	-6	-6.14005	-6.27492	-6.40512	-6.53113
6	-6.60555	-6.74166	-6.87298	-7	-7.12311	-7.24264	-7.3589
7	-7.53113	-7.65331	-7.772	-7.88748	-8	-8.10977	-8.21699
8	-8.47214	-8.58258	-8.69042	-8.79583	-8.89898	-9	-9.09902
9	-9.42443	-9.52494	-9.62348	-9.72015	-9.81507	-9.90833	-10
10	-10.38516	-10.47723	-10.56776	-10.65685	-10.74456	-10.83095	-10.91608

PRODSUM1.WPD October 6, 2005

PRODSUM PAIRS PART I

Prodsum pairs are pairs of numbers whose sum is equal to their product. There are two basic questions associated with prodsum numbers:

1) Given any real number x, what is its prodsum partner y, such that $x+y = x \cdot y$?

It immediately follows that if $x+y = x \cdot y$, then $y = x/(x-1)$ and $x = y/(y-1)$

Further: let $p = x+y = x \cdot y$. then $p = x^2/(x-1) = y^2/(y-1)$

2) Given any real number p, what are the prodsum numbers x and y such that

$$x + y = x \cdot y = p \ ?$$

From $p = x^2/(x-1)$, we have $x^2 - p \cdot x + p = 0$ note[1]

The two roots of this equation are $x = [p + \sqrt{(p^2-4p)}]/2$ and $y = [p-\sqrt{(p^2-4p)}]/2$

Some examples of prodsum pairs:

$x = \pi = 3.141592....$, $y = \pi / (\pi - 1) = 1.466942....$

$(3.141592) + (1.466942) = 4.608534$ and $(3.141592) \cdot (1.466942) = 4.608534$

$x = e = 2.718282....$, $y = e / (e - 1) = 1.581977....$

$(2.718282) + (1.581977) = 4.300259$ and $(2.718282) \cdot (1.581977) = 4.300259$

$x = \Phi = 1.618034...$, $y = \Phi / (\Phi - 1) = 2.618034....$

$(1.618034) + (2.618034) = 4.236068$ and $(1.618034) \cdot (2.618034) = 4.236068$

$x = 5 - \sqrt{15} = 1.127017....$, $y = 5 + \sqrt{15} = 8.872983....$

$(1.127017) + (8.872983) = 10$ and $(1.127017) \cdot (8.872983) = 10$

Properties of prodsum pairs:

$$x + y = x \cdot y = p$$

$$x = y/(y - 1), \quad y = x/(x - 1)$$

$$p = y^2/(y - 1) = x^2/(x - 1)$$

$$x = [p + \sqrt{(p^2 - 4p)}]/2, \qquad y = [p - \sqrt{(p^2 - 4p)}]/2$$

$$x^2 + y^2 = p^2 - 2p, \qquad x^2 - y^2 = p\sqrt{(p^2 - 4p)}$$

Many Prodsums are decimal diamonds

---

[1] This quadratic equation is the characteristic equation for the recursive equation $A_{n+2} = p (A_{n+1} - A_n)$, which has the explicit solution $A_n = (x^n - y^n)/(x - y)$

PRODSUM2.WPD October 5, 2005

PRODSUM PAIRS PART II

Prodsum pairs are pairs of numbers [x,y] whose sum is equal to their product:

$$x + y = x \cdot y.$$

If $p = x + y = x \cdot y$, then x and y in terms of p are given by the roots of the equation

$$x^2 - px + p = 0,$$

Namely, $x = [p + \sqrt{(p^2 - 4p)}]/2$ and $y = [p - \sqrt{(p^2 - 4p)}]/2$

TABLE 1. gives the values of the prodsum pairs corresponding to some integer values of p.

TABLE 1.

p	y	x	$y^2/2$	$x^2/2$	$\sqrt{}$
-6	-6.872983	0.872983	23.618947	0.381050	60
-5	-5.854102	0.854102	17.135255	0.364745	45
-4	-4.828427	0.828427	11.656853	0.343146	32
-3	-3.791288	0.791288	7.186932	0.313068	21
-2	-2.732051	0.732051	3.732051	0.267949	12
-1	-1.618034	0.618034	1.309017	0.190983.	5
+4	2	2	2	2	0
+5	1.381966	3.618034	0.954915	6.545085	5
+6	1.267949	4.732051	0.803847	11.196153	12
+7	1.208712	5.791288	0.730492	16.769508	21
+8	1.171573	6.828427	0.686291	23.313708	32
+9	1.145898	7.854101	0.656541	30.843459	45
+10	1.127017	8.872983	0.635083	39.364917	60

- The $\sqrt{}$ column gives the values of $\sqrt{(p^2 - 4p)}$
- The values of x and y are imaginary for p = 1, 2, and 3 and = 0 for p = 0.
- Note that for all p, ~~$x + y = x \cdot y = p$~~; $(x^2 + y^2)/2 = p \cdot (p-2)/2$; and ~~$x^2 \cdot y^2/4 = p^2/4$~~
- For p = +5, ~~$y = 3 - \Phi$~~ and ~~$x = 2 + \Phi$~~ where Φ is the golden ratio.
- When p = - 1, the values are those of the ~~Fibonacci numbers and the~~ golden ratio.

PRODSUM5.WPD PRODSUM TABLE October 6, 2007

p	x	x^2	$x^2/2$	y	y^2	$y^2/2$	√
-8	0.898979	0.808163	0.404082	-8.898979	79.191827	39.595914	96
-7	0.887482	0.787624	0.393812	-7.887482	62.212372	31.106186	77
-6	0.872983	0.762099	0.381050	-6.872983	47.237895	23.618947	60
-5	0.854102	0.729490	0.364745	-5.854101	34.270499	17.135255	45
-4	0.828427	0.686292	0.343146	-4.828427	23.313707	11.656853	32
-3	0.791288	0.626137	0.313068	-3.791288	14.373865	7.186932	21
-2	0.732051	0.535898	0.267949	-2.732051	7.464103	3.732051	12
-1	0.618034	0.381966	0.190983.	-1.618034	2.618034	1.309017	5
+4	2	4	2	2	4	2	0
+5	3.618034	13.090170	6.545085	1.381966	1.909830	0.954915	5
+6	4.732051	22.392306	11.196153	1.267949	1.607695	0.803847	12
+7	5.791288	33.539016	16.769508	1.208712	1.460985	0.730492	21
+8	6.828427	46.627415	23.313708	1.171573	1.372583	0.686291	32
+9	7.854101	61.686902	30.843459	1.145898	1.313082	0.656541	45
+10	8.872983	78.729834	39.364917	1.127017	1.270167	0.635083	60
+11	9.887482	97.762300	48.881150	1.112518	1.237696	0.618848	77
+12	10.898979	118.787743	59.393871	1.101021	1.212247	0.606124	96

PRODSUM5.WPD PRODSUM TABLE October 6, 2007

p	x	x^2	$x^2/2$	y	y^2	$y^2/2$	√
-8	0.898979	0.808163	0.404082	-8.898979	79.191827	39.595914	96
-7	0.887482	0.787624	0.393812	-7.887482	62.212372	31.106186	77
-6	0.872983	0.762099	0.381050	-6.872983	47.237895	23.618947	60
-5	0.854102	0.729490	0.364745	-5.854101	34.270499	17.135255	45
-4	0.828427	0.686292	0.343146	-4.828427	23.313707	11.656853	32
-3	0.791288	0.626137	0.313068	-3.791288	14.373865	7.186932	21
-2	0.732051	0.535898	0.267949	-2.732051	7.464103	3.732051	12
-1	0.618034	0.381966	0.190983.	-1.618034	2.618034	1.309017	5
+4	2	4	2	2	4	2	0
+5	3.618034	13.090170	6.545085	1.381966	1.909830	0.954915	5
+6	4.732051	22.392306	11.196153	1.267949	1.607695	0.803847	12
+7	5.791288	33.539016	16.769508	1.208712	1.460985	0.730492	21
+8	6.828427	46.627415	23.313708	1.171573	1.372583	0.686291	32
+9	7.854101	61.686902	30.843459	1.145898	1.313082	0.656541	45
+10	8.872983	78.729834	39.364917	1.127017	1.270167	0.635083	60
+11	9.887482	97.762300	48.881150	1.112518	1.237696	0.618848	77
+12	10.898979	118.787743	59.393871	1.101021	1.212247	0.606124	96

PRODNUMBt2.WPD March 26, 2009

5	$x = 3.618033989$	$x^2 = 13.090169944$	$\frac{1}{x} = 0.276393202$	$y = 1.381966011$	$y^2 = 1.909830056$	$\frac{1}{y} = 0.723606798$	5
6	$x = 4.732050808$ $\sqrt{3}+3$	$x^2 = 22.392304845$	$\frac{1}{x} = 0.211324865$	$y = 1.267949192$	$y^2 = 1.607695155$	$\frac{1}{y} = 0.788675135$	12
7	$x = 5.791287847$	$x^2 = 33.539014932$	$\frac{1}{x} = 0.172673165$	$y = 1.208712153$	$y^2 = 1.460985068$	$\frac{1}{y} = 0.827326835$	21
8	$x = 6.828427125$ $\sqrt{8} = 2.828427125$ $\sqrt{8}+4$	$x^2 = 46.627416998$	$\frac{1}{x} = 0.146446609$	$y = 1.171572875$	$y^2 = 1.372583002$	$\frac{1}{y} = 0.853553391$	32
9	$x = 7.854101966$	$x^2 = 61.686917696$	$\frac{1}{x} = 0.127322004$	$y = 1.145898034$	$y^2 = 1.313082304$	$\frac{1}{y} = 0.872677996$	45
10	$x = 8.872983346$	$x^2 = 78.729833462$	$\frac{1}{x} = 0.112701665$	$y = 1.127016654$	$y^2 = 1.270166538$	$\frac{1}{y} = 0.887298335$	60

PRODNUMBt2.WPD March 26, 2009

5	$x = 3.618033989$	$x^2 = 13.090169944$	$\frac{1}{x} = 0.276393202$	$y = 1.381966011$	$y^2 = 1.909830056$	$\frac{1}{y} = 0.723606798$	5
6	$x = 4.732050808$ $\sqrt{3}+3$	$x^2 = 22.392304845$	$\frac{1}{x} = 0.211324865$	$y = 1.267949192$	$y^2 = 1.607695155$	$\frac{1}{y} = 0.788675135$	12
7	$x = 5.791287847$	$x^2 = 33.539014932$	$\frac{1}{x} = 0.172673165$	$y = 1.208712153$	$y^2 = 1.460985068$	$\frac{1}{y} = 0.827326835$	21
8	$x = 6.828427125$ $\sqrt{8} = 2.828427125$ $\sqrt{8}+4$	$x^2 = 46.627416998$	$\frac{1}{x} = 0.146446609$	$y = 1.171572875$	$y^2 = 1.372583002$	$\frac{1}{y} = 0.853553391$	32
9	$x = 7.854101966$	$x^2 = 61.686917696$	$\frac{1}{x} = 0.127322004$	$y = 1.145898034$	$y^2 = 1.313082304$	$\frac{1}{y} = 0.872677996$	45
10	$x = 8.872983346$	$x^2 = 78.729833462$	$\frac{1}{x} = 0.112701665$	$y = 1.127016654$	$y^2 = 1.270166538$	$\frac{1}{y} = 0.887298335$	60

PRODNUMBt1.WPD

P

−6	$x = 0.872983346$	$x^2 = 0.762099923$	$\frac{1}{x} = 1.145497224$	$y = -6.872983346$	$y^2 = 47.237900077$	$\frac{1}{y} = -0.145497224$	60
−5	$x = 0.854101966$	$x^2 = 0.729490169$	$\frac{1}{x} = 1.170820393$	$y = -5.854101966$	$y^2 = 34.270509831$	$\frac{1}{y} = -0.170820393$	45
−4	$x = 0.828427125$	$x^2 = 0.686291501$	$\frac{1}{x} = 1.207106781$	$y = -4.828427125$	$y^2 = 23.313708499$	$\frac{1}{y} = -0.207106781$	32
	$\sqrt{8} \sim 2.828427125$						
−3	$x = 0.791287847$	$x^2 = 0.626136458$	$\frac{1}{x} = 1.263762616$	$y = -3.791287847$	$y^2 = 14.373863542$	$\frac{1}{y} = -0.263762616$	21
	cf $\sqrt{5}$...						
−2	$x = 0.732050808$	$x^2 = 0.535898385$	$\frac{1}{x} = 1.366025404$	$y = -2.732050808$	$y^2 = 7.464101615$	$\frac{1}{y} = -0.366025404$	12
	$\sqrt{3} \sim 1.732050808$						
−1	$x = 0.618033989$	$x^2 = 0.381966011$	$\frac{1}{x} = 1.618033989$	$y = -1.618033989$	$y^2 = 2.618033989$	$\frac{1}{y} = -0.618033989$	5

PRODNUMBt3.WPD March 26, 2009

11	x = 9.887482194	x^2 = 97.762304131	$\frac{1}{x}$ = 0.101137982	y = 1.112517806	y^2 = 1.237695869	$\frac{1}{y}$ = 0.898862018	77
12	x = 10.898979486	x^2 = 118.787753827	$\frac{1}{x}$ = 0.09175171	y = 1.101020514	y^2 = 1.212246173	$\frac{1}{y}$ = 0.90824829	96
15	x = 13.922616289	x^2 = 193.83924434	$\frac{1}{x}$ = 0.071825581	y = 1.077383711	y^2 = 1.16075566	$\frac{1}{y}$ = 0.928174419	165
4	x = 2	x^2 = 4	$\frac{1}{x}$ = 0.5	y = 2	y^2 = 4	$\frac{1}{y}$ = 0.5	0
−10	x = 0.916079783	x^2 = 0.839202169	$\frac{1}{x}$ = 1.091607978	y = - 10.916079783	y^2 = 119.160797831	$\frac{1}{y}$ = - 0.091607978	140

refract
−10
+29
−6

PRODSUM CALCULATIONS PRODSUM6.MCD

$p := -10$

$$x := \frac{\left(p + \sqrt{p^2 - 4 \cdot p}\right)}{2} \qquad y := \frac{\left(p - \sqrt{p^2 - 4 \cdot p}\right)}{2}$$

$x = 0.916079783$ $x^2 = 0.839202169$ $\frac{1}{x} = 1.091607978$ $y = -10.916079783$ $y^2 = 119.160797831$ $\frac{1}{y} = -0.091607978$

General Recursives
$A_{n+2} = -aA_{n+1} - bA_n$ $x^2 + ax + b = 0$
Prodsum $a = -p, b = +p$
Special Case $x^2 - px + p = 0$
$A_{n+2} = pA_{n+1} - pA_n$

PRODSUM1.WPD October 6, 2005

PRODSUM PAIRS PART I

Prodsum pairs are pairs of numbers whose sum is equal to their product. There are two basic questions associated with prodsum numbers:

1) Given any real number x, what is its prodsum partner y, such that $x+y = x \cdot y$?
   It immediately follows that if $x+y = x \cdot y$, then $y = x/(x-1)$ and $x = y/(y-1)$
   Further: let $p = x+y = x \cdot y$. then $p = x^2/(x-1) = y^2/(y-1)$

2) Given any real number p, what are the prodsum numbers x and y such that
   $x + y = x \cdot y = p$?
   From $p = x^2/(x-1)$, we have $x^2 - p \cdot x + p = 0$ note[1]
   The two roots of this equation are $x = [p + \sqrt{(p^2-4p)}]/2$ and $y = [p-\sqrt{(p^2-4p)}]/2$

Some examples of prodsum pairs:

$x = \pi = 3.141592....,$ $y = \pi / (\pi - 1) = 1.466942....$
$(3.141592) + (1.466942) = 4.608534$ and $(3.141592) \cdot (1.466942) = 4.608534$

$x = e = 2.718282....,$ $y = e / (e - 1) = 1.581977....$
$(2.718282) + (1.581977) = 4.300259$ and $(2.718282) \cdot (1.581977) = 4.300259$

$x = \Phi = 1.618034...,$ $y = \Phi / (\Phi - 1) = 2.618034....$
$(1.618034) + (2.618034) = 4.236068$ and $(1.618034) \cdot (2.618034) = 4.236068$

$x = 5 - \sqrt{15} = 1.127017....,$ $y = 5 + \sqrt{15} = 8.872983....$
$(1.127017) + (8.872983) = 10$ and $(1.127017) \cdot (8.872983) = 10$

Properties of prodsum pairs:

$x + y = x \cdot y = p$

$x = y/(y - 1),$ $y = x/(x - 1)$

$p = y^2/(y - 1) = x^2/(x - 1)$

$x = [p + \sqrt{(p^2 - 4p)}]/2,$ $y = [p - \sqrt{(p^2 - 4p)}]/2$

$x^2 + y^2 = p^2 - 2p,$ $x^2 - y^2 = p\sqrt{(p^2 - 4p)}$

---

[1] This quadratic equation is the characteristic equation for the recursive equation $A_{n+2} = p(A_{n+1} - A_n)$, which has the explicit solution $A_n = (x^n - y^n)/(x - y)$

PRODSUM2.WPD October 5, 2005

PRODSUM PAIRS PART II

Prodsum pairs are pairs of numbers [x,y] whose sum is equal to their product:

$$x + y = x \cdot y.$$

If $p = x + y = x \cdot y$, then x and y in terms of p are given by the roots of the equation

$$x^2 - px + p = 0,$$

Namely, $x = [p + \sqrt{(p^2 - 4p)}]/2$ and $y = [p - \sqrt{(p^2 - 4p)}]/2$

TABLE 1. gives the values of the prodsum pairs corresponding to some integer values of p.

TABLE 1.

p	y	x	$y^2/2$	$x^2/2$	$\sqrt{}$
-6	-6.872983	0.872983	23.618947	0.381050	60
-5	-5.854102	0.854102	17.135255	0.364745	45
-4	-4.828427	0.828427	11.656853	0.343146	32
-3	-3.791288	0.791288	7.186932	0.313068	21
-2	-2.732051	0.732051	3.732051	0.267949	12
-1	-1.618034	0.618034	1.309017	0.190983.	5
+4	2	2	2	2	0
+5	1.381966	3.618034	0.954915	6.545085	5
+6	1.267949	4.732051	0.803847	11.196153	12
+7	1.208712	5.791288	0.730492	16.769508	21
+8	1.171573	6.828427	0.686291	23.313708	32
+9	1.145898	7.854101	0.656541	30.843459	45
+10	1.127017	8.872983	0.635083	39.364917	60

- The $\sqrt{}$ column gives the values of $\sqrt{(p^2 - 4p)}$
- The values of x and y are imaginary for p = 1, 2, and 3 and = 0 for p = 0.
- Note that for all p, $x + y = x \cdot y = p$; $(x^2 + y^2)/2 = p \cdot (p-2)/2$; and $x^2 \cdot y^2/4 = p^2/4$
- For p = +5, ~~x~~ $= 3 - \Phi$ and ~~y~~ $= 2 + \Phi$ where Φ is the golden ratio.
- When p = - 1, the values are those of the ~~Fibonacci numbers and the~~ golden ratio.

$y = -\Phi$, $x = +\varphi$

$p = \pi \Rightarrow ?$

PRODSUM1.mcd x^2 +b x + c = 0

$b := 1$ $\quad$ $c := -1$

$r := \sqrt{b^2 - 4 \cdot c}$ $\quad$ $r^2 = 5$ $\quad$ $r = 2.2360679775$

$u := \frac{(-b + r)}{2}$ $\quad$ $v := \frac{(-b - r)}{2}$ $\quad$ $\frac{r^2}{4} = 1.25$

$u = 0.6180339887$ $\quad$ $v = -1.6180339887$

$u^2 = 0.3819660113$ $\quad$ $v^2 = 2.6180339887$

$u^4 = 0.1458980338$ $\quad$ $v^4 = 6.8541019662$

$u^{-1} = 1.6180339887$ $\quad$ $v^{-1} = -0.6180339887$

$u^{-2} = 2.6180339887$ $\quad$ $v^{-2} = 0.3819660113$

$u^{-4} = 6.8541019662$ $\quad$ $v^{-4} = 0.1458980338$

$\sqrt{u} = 0.7861513778$ $\quad$ $\sqrt{v} = 1.2720196495i$

$\sqrt{\sqrt{u}} = 0.8866517793$ $\quad$ $\sqrt{\sqrt{v}} = 0.7975022412 + 0.7975022412i$

$\sqrt{u^{-1}} = 1.2720196495$ $\quad$ $\sqrt{v^{-1}} = 0.7861513778i$

$\sqrt{\sqrt{u^{-1}}} = 1.1278384856$ $\quad$ $\sqrt{\sqrt{v^{-1}}} = 0.6269574857 + 0.6269574857i$

PRODSUM1.mcd x^2 +b x + c = 0

$b := 1$ $c := -1$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 5$ $r = 2.2360679775$

$u := \frac{(-b + r)}{2}$ $v := \frac{(-b - r)}{2}$ $\frac{r^2}{4} = 1.25$

$u = 0.618034$ $v = -1.6180339887$

$u^2 = 0.3819660113$ $v^2 = 2.6180339887$

$u^4 = 0.1458980338$ $v^4 = 6.8541019662$

$u^{-1} = 1.6180339887$ $v^{-1} = -0.6180339887$

$u^{-2} = 2.6180339887$ $v^{-2} = 0.3819660113$

$u^{-4} = 6.8541019662$ $v^{-4} = 0.1458980338$

$\sqrt{u} = 0.7861513778$ $\sqrt{v} = 1.2720196495i$

$\sqrt{\sqrt{u}} = 0.8866517793$ $\sqrt{\sqrt{v}} = 0.7975022412 + 0.7975022412i$

$\sqrt{u^{-1}} = 1.2720196495$ $\sqrt{v^{-1}} = 0.7861513778i$

$\sqrt{\sqrt{u^{-1}}} = 1.1278384856$ $\sqrt{\sqrt{v^{-1}}} = 0.6269574857 + 0.6269574857i$

PRODSUM1.mcd x^2 +b x + c = 0

$b := 10$ $c := -10$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 140$ $r = 11.8321595662$

$u := \frac{(-b + r)}{2}$ $v := \frac{(-b - r)}{2}$ $\frac{r^2}{4} = 35$

$u = 0.9160798$ $v = -10.9160797831$

$u^2 = 0.839202169$ $v^2 = 119.160797831$

$u^4 = 0.7042602805$ $v^4 = 1.419929574 \cdot 10^4$

$u^{-1} = 1.0916079783$ $v^{-1} = -0.0916079783$

$u^{-2} = 1.1916079783$ $v^{-2} = 8.39202169 \cdot 10^{-3}$

$u^{-4} = 1.419929574$ $v^{-4} = 7.0426028046 \cdot 10^{-5}$

$\sqrt{u} = 0.9571205687$ $\sqrt{v} = 3.3039491193i$

$\sqrt{\sqrt{u}} = 0.97832539$ $\sqrt{\sqrt{v}} = 1.2852916244 + 1.2852916244i$

$\sqrt{u^{-1}} = 1.044800449$ $\sqrt{v^{-1}} = 0.3026680993i$

$\sqrt{\sqrt{u^{-1}}} = 1.0221548068$ $\sqrt{\sqrt{v^{-1}}} = 0.3890167729 + 0.3890167729i$

PRODSUM1.mcd x^2 +b x + c = 0

$b := 10$ $c := -10$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 140$ $r = 11.8321595662$

$u := \frac{(-b + r)}{2}$ $v := \frac{(-b - r)}{2}$ $\frac{r^2}{4} = 35$

$u = 0.9160797831$ $v = -10.9160797831$

$u^2 = 0.839202169$ $v^2 = 119.160797831$

$u^4 = 0.7042602805$ $v^4 = 1.419929574 \cdot 10^4$

$u^{-1} = 1.0916079783$ $v^{-1} = -0.0916079783$

$u^{-2} = 1.1916079783$ $v^{-2} = 8.39202169 \cdot 10^{-3}$

$u^{-4} = 1.419929574$ $v^{-4} = 7.0426028046 \cdot 10^{-5}$

$\sqrt{u} = 0.9571205687$ $\sqrt{v} = 3.3039491193i$

$\sqrt{\sqrt{u}} = 0.97832539$ $\sqrt{\sqrt{v}} = 1.2852916244 + 1.2852916244i$

$\sqrt{u^{-1}} = 1.044800449$ $\sqrt{v^{-1}} = 0.3026680993i$

$\sqrt{\sqrt{u^{-1}}} = 1.0221548068$ $\sqrt{\sqrt{v^{-1}}} = 0.3890167729 + 0.3890167729i$

PRODSUM1.mcd x^2 +b x + c = 0

$b := -10$ $c := 10$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 60$ $r = 7.7459666924$

$u := \frac{(-b + r)}{2}$ $v := \frac{(-b - r)}{2}$ $\frac{r^2}{4} = 15$

$u = 8.8729833$ $v = 1.1270166538$

$u^2 = 78.7298334621$ $v^2 = 1.2701665379$

$u^4 = 6.198386677 \cdot 10^3$ $v^4 = 1.6133230341$

$u^{-1} = 0.1127016654$ $v^{-1} = 0.8872983346$

$u^{-2} = 0.0127016654$ $v^{-2} = 0.7872983346$

$u^{-4} = 1.6133230341 \cdot 10^{-4}$ $v^{-4} = 0.6198386677$

$\sqrt{u} = 2.9787553351$ $\sqrt{v} = 1.0616104058$

$\sqrt{\sqrt{u}} = 1.725907105$ $\sqrt{\sqrt{v}} = 1.0303447995$

$\sqrt{u^{-1}} = 0.335710687$ $\sqrt{v^{-1}} = 0.9419651451$

$\sqrt{\sqrt{u^{-1}}} = 0.5794054599$ $\sqrt{\sqrt{v^{-1}}} = 0.9705488886$

PRODSUM1.mcd x^2 +b x + c = 0

$b := -10$ $c := 10$

$r := \sqrt{b^2 - 4 \cdot c}$ $r^2 = 60$ $r = 7.7459666924$

$u := \frac{(-b + r)}{2}$ $v := \frac{(-b - r)}{2}$ $\frac{r^2}{4} = 15$

$u = 8.8729833462$ $v = 1.1270166538$

$u^2 = 78.7298334621$ $v^2 = 1.2701665379$

$u^4 = 6.198386677 \cdot 10^3$ $v^4 = 1.6133230341$

$u^{-1} = 0.1127016654$ $v^{-1} = 0.8872983346$

$u^{-2} = 0.0127016654$ $v^{-2} = 0.7872983346$

$u^{-4} = 1.6133230341 \cdot 10^{-4}$ $v^{-4} = 0.6198386677$

$\sqrt{u} = 2.9787553351$ $\sqrt{v} = 1.0616104058$

$\sqrt{\sqrt{u}} = 1.725907105$ $\sqrt{\sqrt{v}} = 1.0303447995$

$\sqrt{u^{-1}} = 0.335710687$ $\sqrt{v^{-1}} = 0.9419651451$

$\sqrt{\sqrt{u^{-1}}} = 0.5794054599$ $\sqrt{\sqrt{v^{-1}}} = 0.9705488886$

$u = 5 - \sqrt{15} = 1.127\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$u^2 = 10(u-1) = 1.27\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$v^{-1} = u/10 = 0.1\ 127\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$v^{-2} = = 0.0127\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$u^{-1} = v/10 = 0.8\ 872\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$v = 5 + \sqrt{15} = 8.872\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$v^2 = 10(v-1) = 7\ 8.72\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$u^{-2} = = 0.7872\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$u + v = 10 \qquad u \cdot v = 10$

$v^2/2 = 39.364\ 916\ 731\ 037\ 084\ 425\ 896\ 327$

$\sqrt{(v^2/2)} = 6.274\ 146\ 693\ 458\ 568\ 189\ 289\ 070$

$\Phi = (1 + \sqrt{5})/2 = 1.618\ 033\ 988\ 749\ 894\ 848\ 204\ 586\ 834$

$\Phi^{1/2} = 1.272\ 019\ 649\ 514\ 068\ 964\ 252\ 422\ 461\ 737\ 537$ cf u^2

$\Phi^{1/4} = 1.127\ 838\ 485\ 561\ 682\ 260\ 264\ 835\ 483$ cf u

$\Phi^{-1/2} = 0.78\ 615\ 137\ 775\ 742\ 328\ 606\ 955\ 858\ 584\ 296$ cf $v^2/100$

$\Phi^{-1/4} = 0.88\ 665\ 177\ 931\ 216\ 225\ 923\ 351\ 658\ 315\ 073$ cf $v/10$

$\Phi^{-1/2}/2 = 0.39\ 307\ 568\ 887\ 871\ 164\ 303\ 477\ 929\ 292$

$\sqrt{(5\Phi^{-1/2})} = 6.269\ 574857\ 027\ 481\ 437\ 125\ 870\ 978\ 544$

$\Phi^{1/4} + 10\ \Phi^{-1/4} = 9.994\ 356\ 278\ 683\ 304\ 852\ 600\ 001$

$\sqrt{5} = 2.2360679 \div 2 = 1.1180339$

$\sqrt{15} = 3.87298333 \quad 5 - \sqrt{15} = 1.127017$

$\sqrt{35} = 5.9160797$

spans

618034	381966	10
127017	872983	10
916080	839202	10

$\sqrt{1} \cdot \sqrt{5} = \sqrt{5}$ like in 8

$\sqrt{3} \cdot \sqrt{5} = \sqrt{15}$ like in 6, 2

~~$x^2+x-1=\sqrt{5}$~~ ~~$x^2-6x+5=\sqrt{5}$~~ ~~like in 2, 2~~

$x^2+x-1 \to \sqrt{5}$	(6)	.618034	(2)	.381966	
$x^2-5x+5 \to \sqrt{5}$	(1)	.618034	(1)	.381966	
	(2)	.236068	(2)	.763932	
$x^2+6x-6 \to \sqrt{15}$	(2)	.872983	(2)	.145497	
$x^2-10x+10 \to \sqrt{15}$	(4)	.127017	(4)	.872983	
$x^2+10x-10 \to \sqrt{35}$	(6)	.916079	(2)	.839202	
$x^2-14x+14 \to \sqrt{35}$	(1)	.916079	(1)	.839202	
$x^2+5x-5 \to \sqrt{45}$	(2)	.854102	(3)	.708204	
$x^2-9x+9 \to \sqrt{45}$	(4)	.854102			

exception to rule

TOTAL POSSIBLE 16

$\sqrt{5}$	(7)	618	(3)	381	(2)	236	(2) 763
$\sqrt{15}$	(6)	872	(2)	127	(2)	145	
$\sqrt{35}$	(9)	916	(3)	839			
$\sqrt{45}$	(3)	854	(3)	708			

14 999 999

10 999

10 9999

6

$b + \bar{b} = 16$

$\Delta c = 20$

60 $\sqrt{15}$	
196 − 136	$x^2+14x+34$
144 − 84	$x^2+12x+21$
100 − 40	$x^2+10x+10$ [pos]
64 − 4	x^2+8x+1
36 + 24	x^2+6x-6 [pos]
16 + 44	$x^2+4x-11$
4 + 56	$x^2+2x-14$

140 $\sqrt{35}$	
196 − 56	$x^2+14x+14$
144 − 4	$x^2+12x+1$
100 + 40	$x^2+10x-10$ [neg]
64 + 76	$x^2+8x-19$
36 + 104	$x^2+6x-26$
16 + 124	$x^2+4x-31$
4 + 136	$x^2+2x-34$

$\Delta c = 30$ 20 $\sqrt{5}$ $\div 2 \to \varphi$'s

196 − 176	$x^2+14x+44$
144 − 124	$x^2+12x+31$
100 − 80	$x^2+10x+20$
64 − 44	$x^2+8x+11$
36 − 16	x^2+6x+4
16 + 4	x^2+4x-1
4 + 16	x^2+2x-4

.11 8034

The "Genetic" Sequence

	span similarity
$\sqrt{1} \cdot \sqrt{5} = \sqrt{5}$	6, 2
$\sqrt{3} \sqrt{5} = \sqrt{15}$	4, 4
$\sqrt{5} \sqrt{5} = 5$	$x^2+8x-9=0$ 9, −1 or 1, −9 not products
$\sqrt{7} \sqrt{5} = \sqrt{35}$	6, 2
$\sqrt{9} \sqrt{5} = \sqrt{45}$	3, 2
$\sqrt{11} \sqrt{5} = \sqrt{55}$	

Those with same $\sqrt{n5}$ have at least 2 common spans

$u = 5 - \sqrt{15} = 1.127\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$u^2 = 10(u-1) = 01.27\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$v^{-1} = u/10 = 0.1\ 127\ 016\ 653\ 792\ 583\ 114\ 820\ 734\ 600$

$u^{-1} = v/10 = 0.8\ 872\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$v = 5 + \sqrt{15} = 8.872\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$v^2 = 10(v-1) = 7\ 8.72\ 983\ 346\ 207\ 416\ 885\ 179\ 265\ 400$

$u + v = 10$ $\quad u + 10u^{-1} = 10$

$u \cdot v = 10$

$v^2/2 = 39.364\ 916\ 731\ 037\ 084\ 425\ 896\ 327$

$\Phi = (1+\sqrt{5})/2 = 1.618\ 033\ 988\ 749\ 894\ 848\ 204\ 586\ 834$

$\Phi^{1/2} = 1.272\ 019\ 649\ 514\ 068\ 964\ 252\ 422\ 461\ 737\ 537$ cf u^2

$\Phi^{1/4} = 1.127\ 838\ 485\ 561\ 682\ 260\ 264\ 835\ 483$ cf u

$\Phi^{-1/2} = 0.78\ 615\ 137\ 775\ 742\ 328\ 606\ 955\ 858\ 584\ 296$ cf $v^2/100$

$\Phi^{-1/4} = 0.88\ 665\ 177\ 931\ 216\ 225\ 923\ 351\ 658\ 315\ 073$ cf u^{-2}

$\Phi^{-1/2}/2 = 0.39\ 307\ 568\ 887\ 871\ 164\ 303\ 477\ 929\ 292$

$\Phi^{1/4} + 10\,\Phi^{-1/4} = 9.994\ 356\ 278\ 683\ 304\ 852\ 600\ 001$ cf $u + 10u^{-1} = 10$

$(\alpha\mu) + 10(\alpha\mu)^{-i\alpha} = 9.999606$

PRODSUM NOTES

09-06-06

~~A~~ $\pm \sqrt{B}$ roots prodsums

Roots $A \pm \sqrt{B}$ will be prodsum if

$B = A(A-2)$ where $A \geq +2$ [rational or irrational]

$(A+\sqrt{B}) + (A-\sqrt{B}) = 2A = (A+\sqrt{B})(A-\sqrt{B}) = A^2 - B$

Product = Sum $= A^2 - A^2 + 2A$

e.g.

$A=5$	$5 \pm \sqrt{15}$	10
$A=4$	$4 \pm \sqrt{8}$	8
$A=3$	$3 \pm \sqrt{3}$	6
$A=2$	$B=0$	4
$A=6$	$\sqrt{24}$	
7	$\sqrt{35}$	
8	$\sqrt{48}$	
9	$\sqrt{63}$	
10	$\sqrt{80}$	

3 8 15 25 35 48

2 5 7 9 11 13

Are these Prodsums also Divans?

Negative A

same $B = A(A-2)$

$-5 \pm \sqrt{35}$

$-4 \pm \sqrt{24}$

$-3 \pm \sqrt{15}$

$-2 \pm \sqrt{8}$

$-6 \pm \sqrt{48}$

$-7 \pm \sqrt{63}$

-5 -7

4	2	$\pm\sqrt{0}$	0	0
ΠΣ	A	$=A(A-2)$	A	ΠΣ
6	+3	$\pm\sqrt{3}$	−1	−2
8	+4	$\pm\sqrt{8}$	−2	−4
10	+5	$\pm\sqrt{15}$	−3	−6
12	+6	$\pm\sqrt{24}$	−4	−8
14	+7	$\pm\sqrt{35}$	−5	−10
16	+8	$\pm\sqrt{48}$	−6	−12
18	+9	$\pm\sqrt{63}$	−7	−14
20	+10	$\pm\sqrt{80}$	−8	−16
22	+11	$\pm\sqrt{99}$	−9	−18
24	+12	$\pm\sqrt{120}$	−10	−20

A	ΠΣ
$-\frac{1}{2}$	→ 3/2

Excluded

~~A = 0 to 2~~

$A > 0$ to $< +2$

$0 < A < +2$

EXCLUDED

Quadratic equation for prodsums

$ax^2 + bx + c = 0$

root $-\frac{b}{2} \pm \sqrt{(\frac{b}{2})^2 - ac}$

" "

A $\sqrt{A^2 - ac}$

$A^2 - 2A$

$\therefore ac = 2A = -b$

$\boxed{ac = -b}$

INFORMATION

Another aspect of this has been pointed out by Tony Rothman. Only those systems obeying Maxwell-Boltzman statistics are subject to the second law of thermodynamics. Systems obeying other statistics seem to be immune. Maxwell-Boltzman goes with analog, Einstein-Bose and Fermi-Dirac reside in other modes. On the one hand, digital codes may readily be restored, similar in ways to holograms, while the analog, preserved from decay by continual amplification, is always subject to information loss.

But there is also energy/information stored in the relationship, in the link. This energy/information is both static or stored and flowing. Over time the e/i in a relationship can become very rich, like a savings account of large magnitude. When the link is broken, the e/i begins to flow. For one party it can be like a spending spree, very euphoric [the euphoria comes both from the e/i released from the broken link and the flow of e/i into the new configuration.] For the other party the flow is draining the energy from the link, lost and diminished. There is no access to the e/i redeposited in the new bank account. In the case of death when we are drained does this mean that the e/i has been available to the departed one (cf ancient burial of e/i in tombs with kings, etc.) and if we have not lost significant e/i does this mean that there is little for the departed one?

Information is always at boundaries, whether these are boundaries in time, space, or level of abstraction.

Gregory Bateson

9. INFORMATION: THOUGHTS WITHOUT A THINKER

Degree of Surprise-- Shannon
Negentropy-- Szilard
Bits and Bytes--
Useful Data-- F(t,x,y,z,person)
Minimum Length of Description

We begin with a set of experiences, say those that are permitted by our biological structure. Soon some of these are emphasized (usually those with a large repetition rate) which results in the negation of others. This is like a rut in the side of a hill. The future flow of water will choose these existing ruts and develop them into gorges. Which is to say that whatever is selected operates through the Principle of Plenitude, confirming itself and blocking other choices. or as the Law of Hardening puts it, whenever information concerning a particular area is extracted this precludes information being extracted from other areas. That is, SELECTION CREATES INHIBITORS, which is to say that selection destroys access to that which is not selected. This process results in an ever narrowing and increasingly static

PRODSUMS

Roots $A+\sqrt{B}$, $A-\sqrt{B}$

PROD A^2-B^2

SUM $2A$

$\therefore A^2-B^2=2A$

$A^2-2A-B^2=0$

$A=\frac{2\pm\sqrt{4+4B^2}}{2}=1\pm\sqrt{1+B^2}$

$\therefore B^2=(A-1)^2-1 \therefore B=\sqrt{A(A-2)}$

Equation

Table

	A	± B	A	EQUATION
	−10	$\sqrt{120}$	+12	
	−9	$\sqrt{99}$	+11	
	−8	$\sqrt{80}$	+10	
$x^2+14x-14=0$	−7	$\sqrt{63}$	+9	
	−6	$\sqrt{48}$	+6	
$x^2+10x-10=0$	−5	$\sqrt{35}$	+7	$x^2-14x+14=0$
	−4	$\sqrt{24}$	+6	
$x^2+6x-6=0$	−3	$\sqrt{15}$	+5	$x^2-10x+10=0$ $5\pm\sqrt{15}$ 6 D pen^
$x^2+4x-4=0$	−2	$\sqrt{8}$	+4	
	−1	$\sqrt{3}$	+3	$x^2-6x+6=0$
	0	$\sqrt{0}$	+2	
	+1	$\sqrt{-1}$	+1	
	+2	$\sqrt{0}$	0	

$x^2+ax-a=0 \rightarrow$ prodsum $\quad \Sigma=\Pi=-a$

$x^2-ax-a=0 \rightarrow$ not prodsum

$x^2+ax+a=0 \rightarrow$ not prodsum

$x^2-ax+a=0 \rightarrow$ Prodsum $\quad \Sigma=\Pi=a$