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NEBULAR DISTANCES DETERMINED FROM THEIR DIAMETERS

A. G. WILSON

I. Introduction

Some years ago Hubble and Tolman in a joint paper discussed the various methods which could be employed to determine the nature of the nebular red-shift. This paper considered how observable quantities could be applied to test various cosmological models. The observable parameters which were considered were 1) the total luminosities of individual nebulae, 2) the apparent size of the nebulae, 3) the spectra of the nebulae, and 4) the counts of the total number of nebulae to successive limiting magnitudes.

The procedure used was to determine the line element which characterized the cosmological model; then to derive from this line element the expected relations between the observable parameters. For example, Tolman constructed a homogeneous non-static model using the line element

$$ds^2 = -e^{g(t)}\left[\frac{dr^2}{1 - \frac{r^2}{R_o^2}} + r^2\, d\theta^2 + r^2 \sin^2\theta\, d\phi^2\right] + dt^2$$

Where R_o is a constant determined by the pressure and density of the model. Starting with this line element, it can be shown that the equation

$$\frac{l_b}{l_b'} = \frac{r'^2\,(1 + z')^2}{r^2\,(1 + z)^2} \qquad (1)$$

obtains between the apparent bolometric luminosities l_b and l_b' of two identical nebulae at distances r and r' and with spectral displacements z and z' where $z = \frac{\delta\lambda}{\lambda}$. (The distance r is the distance of the nebula at the time the observed light signal left the nebula.)

A second relation

$$\frac{\delta\theta}{\delta\theta'} = \frac{r'}{r}\,\frac{(1 + z)}{(1 + z')} \qquad (2)$$

can be shown to hold between the observed angular diameters $\delta\theta$ of two identical nebulae, the distances r, and the spectral displacements z.

Equations (1) and (2) are the only laws which provide for a direct determination of distance in the model. (Methods based on counts or properties of clusters contain further assumptions such as properties of a luminosity function.) Equation (1) is the form the familiar inverse square law takes in this particular model of Tolman's. The inverse square law, in one or another of its forms, (most familiar of which is $m - M = 5 \log r - 5$) has been the basic yardstick with which the universe has been measured.

Equation (2) is just as fundamental, but it has not been used in astronomy because diameters in general arenot observable quantities. In fact it is only when we come to consider the extragalactic nebulae that the use of equation (2) (or its analogue derived from some other line element) becomes possible.

This fact was realized by the pioneers in the extragalactic field and some interesting results concerning nebular diameters were obtained, such as Hubble's

$$m + 5 \log d = C$$

where C depends on the type of nebula and d is the diameter of the "main body" of the nebula.

But there are two serious difficulties inherent in the use of diameters as measures of distance. And these difficulties have precluded their use in the cosmological problem. First, and most fundamental, is the difficulty of establishing a suitable definition of diameter of a nebula. And by suitable definition we mean a diameter which could be used for the quantity $\delta\theta$ in equation (2).

If nebulae were balls, cans, or disks with sharply defined edges, there would be no difficulty. But nebulae are stellar aggregates whose density and luminosity vary throughout. In general the distribution of luminosity is such that there is a dense bright core or nucleus surrounded by a region in which the brightness gradually fades out. But neither the core nor the outer portion is sharply defined. The real spatial extent of a nebula is unknown. In fact, what is meant by real spatial extent is a paraphrase of the problem. A usable definition is: the distance from the center out to where the density falls to a specified value, such as one per cent of the nuclear density. For objects as nebulous as nebulae, the definition of diameter must take some such form. But actually we don't deal with the nebulae themselves but with such things as images on photographic plates and photocell readings. So, clearly, our definition of diameter must be an operational definition. And having agreed on an operational diameter, our problem is to find what modifications and corrections must be made to this operational diameter to give a quantity which can be used for $\delta\theta$ in equation (2).

What is really wanted in order to apply equation (2) is some linear dimension which can 1) be measured, and 2) be compared. And in order to be compared, our linear dimension must be measured on corresponding parts of similar galaxies. It is this requirement which brings us to the second fundamental difficulty in applying diameters to the cosmological problem, viz. the heterogeneity of the nebulae.

As is well known, nebuale differ in intrinsic form. A classification based on appearance divides them into ellipticals, spirals, and barred spirals. But a more refined classification shows that even those nebulae which appear similar may be quite different. For example, nebulae which appear globular (round) may actually be globular, or they may be flattened disks with their axes parallel to the line of sight. And in order to use equation (2) we must compare nebuale which are identical in every respect except as to their distance and velocity. So we must either determine by some method which nebulae are strictly comparable or resort to certain statistical devices which allow us to compare samples.

It is well to say at this point that this second difficulty is common to both equations (1) and (2) and is not peculiar to diameters. Distances determined from lunimosities are subject to the same sampling or identification difficulties.

The two foregoing fundamental problems afford a natural dichotomy in the analysis of the problem. So we shall divide the discussion into two parts: 1) The Definition Problem and 2) The Sampling (or Identification) Problem.

2. The Definition Problem.

In order to separate the two above problems, in the present section we shall treat a set of hypothetical standard nebulae all of whose intrinsic properties are identical, the differences in appearance thus being due to differences in velocity and distance. Our problem is to formulate an operational definition of diameter of such a standard nebula and then determine the corrections which must be employed to reduce the operational diameter to a quantity equivalent to $\delta\theta$ in equation (2).

In this procedure we may start at either end, i.e. start with a measured quantity and work toward a dimension of the nebula or start with a linear dimension of the nebula and work toward a measured quantity. Let us adopt the latter procedure and consider the process in two basic steps:

Step I: Conversion of the true spatial nebula at distance Δ with velocity v to the apparent projected light distribution received at our galaxy.

Step II: Conversion of the projected light pattern by the galactic and atmospheric absorption and the several recording instruments to the form of presentation on which the data is measured.

Step I. First we must define our standard nebula. In doing this, we must introduce several restrictions in order to allow a complete theoretical discussion. But to assure the applicability of the theory, the restrictions which are introduced must be of such nature that they define a standard nebula which is not unlike real nebulae, or better, which is like at least one type of real nebula. We can do the latter.

The standard nebula will be defined as a stellar aggregate possessing spherical symmetry. Let us suppose that it is composed of n different types of stars, each having a density distribution given by F_m (W), where W is the distance from the center. Each of the n types of stars will be characterized by a respective energy envelope $I_m(\Lambda)$.

Now Step I can be considered as composed of two sub-steps. A) The conversion of the spatial nebula to the projected luminosity pattern. It is in this projected luminosity pattern that our $\delta\theta$ of equation (2) must be defined, since equation (2) was based on the appearances of the projected nebulae, and B) The conversion of the real projected pattern to the apparent projected pattern due to the effects of Δ, the distance, and z, the spectral displacement.

A) If R is the projected radius in the plane of the sky and $P_m(R)$ is the projected density (stars per unit area), then von Zeipel's equation gives:

$$P_m(R) = \int_R^{\infty} \frac{W F_m(W)\, dW}{\sqrt{W^2 - R^2}}$$

The luminosity per unit area at a wave length Λ and at a distance R from the center due to stellar type m will be $I_m(\Lambda)\, P_m(R)$. The total luminosity at a distance R and at wave length Λ will be

$$I_T(\Lambda, R) = \sum_{m=1}^{n} I_m(\Lambda)\, P_m(R)$$

The function $I_T(\Lambda, R)$ is an axially symmetric function giving the distribution of radiant energy intensity per unit area as a function of Λ, the wave length in terms of an undilated scale at the nebula, and R, the linear distance from the center. But we may alternately consider $I_T^*(\lambda, \Theta)$ ~~as~~ which represent~~ing~~s the apparent distribution of energy as observed at some standard distance Δ_o. Where λ is the wave length measured on the observer's scale and Θ is the <u>angular</u> distance from the center. It is exactly this representation of the nebula which is employed in equation (2). So it is in terms of $I_T^*(\lambda, \Theta)$ that we must define $\delta\Theta$. (* used for observed quantities)

The function $I_T^*(\lambda, \Theta)$ will be an even function, i.e. $I_T^*(\lambda, \Theta) = I_T^*(\lambda, -\Theta)$, which has its maximum value at $\Theta = 0$ and is assymptotic to 0 as $\Theta \rightarrow \infty$.

Corresponding to the foregoing definition of diameter of the spatial nebula, viz., twice the distance from the center to a shell in which the density drops to some specified fraction of the central density, we can define $\delta\Theta$ as equal to $2\Theta_0$ where Θ_0 satisfies the equation

$$I^*_\tau(\lambda, \Theta_0) = \tau\, I^*_\tau(\lambda, 0) \quad , \qquad 0 < \tau < 1.$$

Whereas corresponding linear dimensions clearly are suited for use in equation (2) (or its analogues), it is not clear under what conditions ~~a~~ diameters defined in terms of a central intensity will constitute ~~constitute~~ corresponding linear dimensions. To investigate this question as well as the modifications of the projected luminosity pattern due to the velocity and distance factors in accordance with Step I, substep B), let us consider what we shall call <u>profiles</u>.

A profile is the intersection of the axially symmetric luminosity pattern with a plane passing through its center. Both the "solid" luminosity pattern and the profile may be represented by the function,

$$I^*_\tau(\lambda, \Theta)$$

In accordance with the cosmological model, we assume there exists some law connecting distance with red-shift, $z = h(\Delta)$. We do not know h, but will assume it to be uniform so that for every distance Δ, there will exist a unique value of z. (Present observational material out to about one-eighth the velocity of light indicates h to be a linear function.)

Now let $I^*_\tau(\lambda, \Theta)$ be the profile of the standard nebula located at some specified distance Δ_0. We shall call this the <u>reference nebula</u>. If we move this nebula to a distance Δ, and give it the corresponding value of z, its profile will assume a form $i^*_\tau(\lambda, \theta)$.

The parameters Θ and θ are corresponding angular diameters, i.e. angular distances from the center to corresponding points on the profiles. As the profile I^*_r is moved from the distance Δ_o to the distance Δ, (the spectral shift always maintaining the proper value defined by $z = h(\Delta)$). The abscissa Θ of each point on the profile will be modified according to equation (2).(or its analogue). The total bolometric luminosity, (or volume under the bolometric luminosity surface) will be modified according to equation (1).

Let

$$i^*_b(\theta) = \int_{\lambda=0}^{\infty} i^*_r(\lambda, \theta)\, d\lambda$$ (Bolometric profile)

then

$$l^*_b = 2\pi \int_{\theta=0}^{\infty} i^*_b(\theta)\, \theta\, d\theta$$

(Since the luminosity of a nebula is finite, the functions i^*_r and i^*_b must be of such a nature that these integrals exist.)

By Pappus theorem for volumes of solids of revolution we can write

$$l^*_b = 2\,\pi\,\overline{\theta}\,A$$

where $\overline{\theta}$ is the distance from the center to the center of gravity of the half-area A under the profile. But there exists a $\tilde{\theta}$, such that $A = i^*_b(0)\,\tilde{\theta}$ by the mean value theorem,

$$\therefore\ l^*_b = a\, i^*_b(0)\, \theta_{,}^2$$

where a is a constant and $\theta_,$ a determined angular dimension or abscissa. Similarly $L^*_b = a'\, I^*_b(0)\, \Theta_,^2$ where $\Theta_,$ and $\theta_,$ are corresponding abscissae, i.e. they are related by equation (2).

By means of equations (1) and (2) we may write:

$$\frac{\Delta_o^2\,(1+Z_o)^2}{\Delta^2\,(1+z)^2} = \frac{l^*_b}{L^*_b} = \frac{a\, i^*_b(0)\,\theta_,^2}{a'\, I^*_b(0)\,\Theta_,^2} = \frac{a\, i^*_b(0)\,\Delta_o^2\,(1+z)^2}{a'\, I^*_b(0)\,\Delta^2\,(1+Z_o)^2}$$

or
$$\frac{a\, i_b^{*}(0)}{a'\, I_b^{*}(0)} = \frac{(1 + Z_o)^4}{(1 + z)^4} \qquad (3)$$

But the constants a and a' are independent of z. In the case of no spectral displacements, the energy intensity per unit area is constant, $\therefore a = a'$.

Equations (2) and (3) give the variations of abscissae and the central ordinate of the bolometric profile with distance and velocity. Profiles have been expressed in terms of the angular distances Θ and θ from the center to corresponding points. Let Σ and σ be any two corresponding abscissae. If we introduce a variable x defined by

$$x = \frac{\Theta}{\Sigma} = \frac{\theta}{\sigma}$$

the profile, expressed in terms of x, is independent of equation (2). We defined a diameter of a nebula in terms of a parameter τ and the central intensity. If x_1 is the value of x, such that

$$I_b^{*}(x_1) = \tau_1 I_b^{*}(0)$$

then, clearly, every ordinate $I_b^{*}(x)$ is related to every ordinate $i_b^{*}(x)$ by equation (3). Thus, if we have bolometric profiles at our disposal, a diameter defined as above will transform by equation (2).

But we are faced with the difficulty of not having bolometric, but rather ~~p~~ limited-spectral-range profiles. And the usefulness of bolometric profiles depends on the property that equation (3) is (Since the total radiation is subject to number & energy effects, which do not depend when integrated) independent of x (or θ). In the case of photographic or other profiles, this is not always true, the ordinate transformation ~~being~~ may be a function of θ. In such a case, the foregoing diameter defined in terms of and central intensity would not transform according to equation (2). It will be necessary in such a case to resort to other operational devices to arrive at measurable parameters which transform according to equation (2).

Theory of Profiles based on Tolman's Line Element.

We wish to derive the transformations which govern luminosities and intensities in Tolman's homogeneous non-static model, which is characterized by the line element.

$$ds^2 = -e^{g(t)}\left[\frac{dr^2}{1-\frac{r^2}{R_0^2}} + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right] + dt^2$$

Geodesics in this universe are obtained by setting ds = 0. And by setting $d\theta$ and $d\phi = 0$, we get an expression for the radial velocity of light,

$$\frac{dr}{dt} = e^{-\frac{g(t)}{2}}\sqrt{1-\frac{r^2}{R_0^2}}$$

This differential equation may be solved by separation of variables.

$$\int_0^{\Delta} \frac{dr}{\sqrt{1-\frac{r^2}{R_0^2}}} = \int_{t_1}^{t_2} e^{-\frac{g(t)}{2}} dt$$

Where our coordinate system is centered at the radiating nebula. At time t_1, light leaves the nebula; it arrives at the observer at time t_2 after traversing a distance Δ.

Differentiating both members of this equation with respect to t_1, we have

$$0 = e^{-\frac{g_2}{2}}\frac{dt_2}{dt_1} - e^{-\frac{g_1}{2}}$$

or

$$\frac{dt_2}{dt_1} = e^{\frac{1}{2}(g_2 - g_1)}$$

If dt_1 represents a time interval at the nebula, dt_2 is the corresponding interval at the observer. For example, if dt_1 is the period between successive wave crests of a light pulse (photon) leaving the nebula, then

$$dt_1 = \frac{\lambda_1}{c} = \frac{1}{\nu_1}$$

and for the observed light, $dt_2 = \dfrac{\lambda_1(1+z)}{c} = \dfrac{1}{\nu_2}$

$$\therefore \frac{dt_2}{dt_1} = e^{\frac{1}{2}(g_2 - g_1)} = (1+z)$$

This change in wave length is the so-called energy effect. Let us next consider the rates of departure and arrival of the photons themselves. (The number effect). If dt_1 is the interval between successive departures of photons of wave length λ_1 from the nebula, $p_1 = \dfrac{1}{dt_{g1}}$ will be the rate of departure. The rate of arrival of these photons will be $p_2 = \dfrac{1}{dt_2}$ $\quad \therefore \dfrac{p_1}{p_2} = (1+z)$

But the photons which leave with wave lenth λ_1 arrive with wave length λ_2.

The observed monochromatic luminosity $l^*(\lambda_2)$ is defined as the total energy of photons of wave length λ_2 arriving from the nebula per unit time per unit area (at the receiver).

i.e. $$l^*(\lambda_2) = \frac{p_2(\lambda_2)}{4\pi e^{g_2}\Delta^2} \cdot \frac{hc}{\lambda_2}$$

but $p_2 = \dfrac{p_1(\lambda_1)}{1+z}$ and $\dfrac{hc}{\lambda_2} = \dfrac{hc}{\lambda_1(1+z)}$

$$\therefore l^*(\lambda_2) = \frac{hc}{4\pi e^{g_2}\Delta^2} \cdot \frac{p_1(\lambda_1)}{\lambda_1(1+z)^2} \quad \text{where} \quad \lambda_2 = \lambda_1(1+z)$$

where $p_1(\lambda_1)$ is the rate of departure of photons of wave length λ_1 and $\dfrac{hc}{\lambda_1}$ is the energy of each photon.

This may also be expressed in terms of the frequency, ν

$$l^*(\nu_2) = \frac{h\nu_1}{4\pi e^{g_2}\Delta^2} \cdot \frac{p_1(\nu_1)}{(1+z)^2} \quad \text{where} \quad \nu_2 = \frac{\nu_1}{1+z}$$

According as to whether we use frequency or wave length expressions, we can write

$$h\nu p_1(\nu) = E(\nu)(1+z) \quad \text{or} \quad \frac{hc\,p_1(\lambda)}{\lambda} = \frac{E(\lambda)}{1+z}$$

where E is the function representing the energy envelope of the nebula.

$$\therefore l^*(\lambda) = \frac{E\left(\frac{\lambda}{1+z}\right)}{4\pi e^{g_2}\Delta^2(1+z)^3}$$

The transformation law for monochromatic luminosities follows immediately

$$\frac{L^*(\lambda)}{l^*(\lambda)} = \frac{\Delta^2(1+z)^3}{\Delta_0^2(1+Z_0)^3} \cdot \frac{E\left(\frac{\lambda}{1+Z_0}\right)}{E\left(\frac{\lambda}{1+z}\right)} = \frac{L_b^*}{l_b^*}\,\frac{1+z}{1+Z_0}\,\frac{E\left(\frac{\lambda}{1+Z_0}\right)}{E\left(\frac{\lambda}{1+z}\right)} \quad (4)$$

If $l_\alpha^* = \int_0^\infty \alpha(\lambda)\, l^*(\lambda)\, d\lambda$ a "limited-spectral-range" luminosity, we have:

$$\frac{L_\alpha^*}{l_\alpha^*} = \frac{\Delta^2(1+z)^3}{\Delta_0^2(1+Z_0)^3}\,\frac{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+Z_0}\right) d\lambda}{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+z}\right) d\lambda} = \frac{L_b^*}{l_b^*}\,\frac{(1+z)}{(1+Z_0)}\,\frac{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+Z_0}\right) d\lambda}{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+z}\right) d\lambda}$$

Hubble uses the notation K for the correction factor. Hence we write

$$K_\alpha(z, Z_0) = 2.5 \log \frac{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+Z_0}\right) d\lambda}{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+z}\right) d\lambda}$$

$$\therefore \quad \frac{L_\alpha^*}{l_\alpha^*} = \frac{L_b^*}{l_b^*}\,\frac{(1+z)}{(1+Z_0)}\,10^{0.4 K_\alpha(z, Z_0)} \quad (5)$$

(If we take $\alpha = 1$ equation (5) reduces to an identity.)

Surface brightness or unit intensity is defined as luminosity per unit area of the source. If $p_1(\lambda, r)\,dr$ is the total number of photons of wave length λ radiated per unit time in a ring from r to r + dr, then the luminosity of the ring will be

$$l^*(\lambda, \theta) = \frac{hc}{4\pi e^{g_2}\Delta^2(1+z)^2}\,\frac{p_1(\lambda', r)\,dr}{\lambda'} = \frac{e(\lambda', r)\,dr}{4\pi e^{g_2}\Delta^2(1+z)^3}$$

where $\frac{e(\lambda', r)}{1+z} = \frac{hc}{\lambda'}\, p_1(\lambda', r)$

and $\theta = \frac{r}{\Delta}(1+z)$ and $\lambda' = \frac{\lambda}{1+z}$

the brightness of the ring will be

$$i^*(\lambda, \theta) = \frac{l^*(\lambda, \theta)}{2\pi\theta\, d\theta} = \frac{e(\lambda', r)}{8\pi^2 e^{g_2}(1+z)^5\, r} \quad \text{the monochromatic intensity}$$

$$i_b^*(\theta) = \frac{\int_0^\infty e(\lambda', r)\, d\lambda}{8\pi^2 e^{g_2}(1+z)^5\, r} = \frac{e_b(r)}{8\pi^2 e^{g_2}(1+z)^4 r} \quad \text{the bolometric intensity}$$

$$i_\alpha^*(\theta) = \frac{\int_0^\infty \alpha(\lambda)\, e(\lambda', r)\, d\lambda}{8\pi^2 e^{g_2}(1+z)^5\, r} \quad \text{the "l. s. r." intensity}$$

Since $2\pi \int_0^\infty i^*(\lambda, \theta)\,\theta\, d\theta = l^*(\lambda)$

we have
$$\frac{\int_0^\infty e\left(\frac{\lambda}{1+z}, r\right) dr}{4\pi e^{g_2}\Delta^2(1+z)^3} = \frac{E\left(\frac{\lambda}{1+z}\right)}{4\pi e^{g_2}\Delta^2(1+z)^3}$$

or

$$E(\lambda) = \int_0^\infty e(\lambda, r)\, dr$$

Further:

$$\frac{I_\alpha^*(\Theta)}{i_\alpha^*(\theta)} = \frac{(1+z)^5}{(1+z_0)^5} \cdot \frac{\int_0^\infty \alpha(\lambda)\, e\left(\frac{\lambda}{1+z_0}, r\right) d\lambda}{\int_0^\infty \alpha(\lambda)\, e\left(\frac{\lambda}{1+z}, r\right) d\lambda} \tag{6}$$

(Equation (3) may be deduced by taking $\alpha \equiv 1$)

In the case of bolometric intensities, we have:

$$i_b^*(\theta_1) = \frac{e_b(r_1)}{8\pi^2 e^{g_0}(1+z)^4 r_1}$$

and

$$i_b^*(0) = \frac{[e_b'(r)]_0}{8\pi^2 e^{g_0}(1+z)^4}$$

$$\therefore \quad i_b^*(\theta_1) = \frac{e_b(r_1)}{r_1[e_b'(r)]_0}\, i_b^*(0) = \tau\, i_b^*(0)$$

Similarly

$$I_b^*(\Theta_1) = \frac{e_b(r_1)}{r_1[e_b'(r)]_0}\, I_b^*(0) = \tau\, I_b^*(0)$$

and θ_1 and Θ_1 satisfy equation (2) because they both correspond to r_1. Hence for bolometric intensities, diameters defined in terms of τ and the central intensity may be used in equation (2).

But in the case of i_α^*'s, we have

$$i_\alpha^*(\theta_1) = \frac{\int_0^\infty \alpha(\lambda)\, e\left(\frac{\lambda}{1+z}, r_1\right) d\lambda}{\int_0^\infty \alpha(\lambda)\, r_1 \left[e'\left(\frac{\lambda}{1+z}, r\right)\right]_0 d\lambda}\, i_\alpha^*(0) = \tau(1+z)\, i_\alpha^*(0)$$

Similarly

$$I_\alpha^*(\Theta_1) = \tau(1+z_0)\, I_\alpha^*(0)$$

θ_1 and Θ_1 both correspond to r_1, and therefore satisfy equation (2), but $\tau(1+z) \neq \tau(1+z_0)$ and the corresponding diameters consequently cannot be located by τ% of the central intensity, so some other operational diameter must be used for this case.

In the most general case, profiles can be used to derive the intensity and diameter transformation laws, if we can identify the untransformed and transformed position of the same point. This "reversed" approach would seem more desirable than deriving the transformation laws from a specified cosmological model, for account could be taken of evolutionary features as well as those considered above.

The great difficulty, and the one which vitiates this approach to the problem, is the identification of corresponding points. The structure of globular profiles is such that no such points can be definitely established. So even in the case derived from Tolman's model we have reached a "dead end." The points of inflection, which can be readily identified in transformed and untransformed profiles, suggest themselves as possibilities for applying equations (2) and (6). However, it can be seen that equation (6) does not necessarily transform an inflection point into an inflection point, which precludes use of diameters for determining distances in this case.

However, by making further assumptions as to the properties of our standard nebulae we may proceed. We shall assume that the color of the standard nebula is independent of the distance from the center. In making this assumption we are making our hypothetical nebulae like real globular nebulae, in which no appreciable color change has been measured.

Stebbins, using two sizes of diaphragms, has measured the colors of several near-by elliptical nebulae. There appears to be no color change with center distance in his results. Spectra of elliptical systems show that the dominant contribution is from dwarf stars of about class G. Further, the total range in color of the dwarf branch of population II is only about 0.8 magnitude. This is equivalent to

$$1 < \frac{I_r(\theta_i) - I_b(\theta_i)}{I_r(\theta_j) - I_b(\theta_j)} < 2 \quad \text{for all } i, j$$

Therefore, if we restrict our measurements to the nuclear portions of the nebula, we may safely assume that color variation is negligible on the basis of present observational evidence.

See also Holmberg's data

Then on the basis of no color change, we may write

$$\frac{I_T(\Lambda_2, r)}{I_T(\Lambda_1, r)} = \text{constant},$$

which implies that we may write,

$$I_T(\Lambda, r) = J_T(\Lambda)\, P_T(r)$$

And when the intensity function is separable, the problem is greatly simplified. (We may note here that color constancy implies either the same distribution for stars of different types, as is likely from the above discussion, or implies the same energy envelope for all the stars, which is highly unlikely.)

We an now rewite the foregoing equations in the separable form. Specifically,

take $e(\lambda, r) = e(\lambda)\cdot g(r)$

but $$\int_0^\infty e(\lambda, r)\, dr = e(\lambda)\int_0^\infty g(r)\, dr = E(\lambda)$$

$$\therefore\ e(\lambda, r) = E(\lambda)\frac{g(r)}{g} \qquad \text{where} \qquad g = \int_0^\infty g(r)\, dr$$

Hence $$i_\alpha^*(\theta) = \frac{g(r)}{r}\cdot\frac{\int_0^\infty \alpha(\lambda)\, E\left(\frac{\lambda}{1+z}\right) d\lambda}{8\pi^2 e\,\theta^2 (1+z)^5}$$

and $$I_\alpha^*(\theta) = \frac{g(r)}{r}\cdot\frac{\int_0^\infty \alpha(\lambda)\, E\left(\frac{\lambda}{1+Z_0}\right) d\lambda}{8\pi^2 e\,\theta^2 (1+Z_0)^5}$$

Thus, when we have separability, equation (6) becomes

$$\frac{I_\alpha^*(\theta)}{i_\alpha^*(\theta)} = \frac{(1+z)^5}{(1+Z_0)^5}\,\frac{\int_0^\infty \alpha(\lambda)\, E\left(\frac{\lambda}{1+Z_0}\right) d\lambda}{\int_0^\infty \alpha(\lambda)\, E\left(\frac{\lambda}{1+z}\right) d\lambda} = \frac{(1+z)^5}{(1+Z_0)^5}\, 10^{0.4\, K_\alpha(z, Z_0)} \qquad 7)$$

Dividing by equation (5), we obtain

$$\left(\frac{I_\alpha^*(\theta)}{i_\alpha^*(\theta)}\cdot\frac{l_\alpha^*}{L_\alpha^*}\right) = \frac{L_b^*}{l_b^*}\,\frac{(1+z)^4}{(1+Z_0)^4}$$

The left member of this equation contains only measurable quantities (i and I can be readily measured at the center), and both z and Z_0 are known; therefore we have the ratio of the bolometric luminosities. And from equation (2), the distances can be derived.

In the case of separability:

$$i_\alpha^*(\theta_1) = \frac{g(r_1)}{r_1} \frac{\int_0^\infty \alpha(\lambda)\, E(\frac{\lambda}{1+z})\, d\lambda}{8\pi^2 e g_2 (1+z)^5} = \frac{g(r_1)}{r_1} S(1+z)$$

and $i_\alpha^*(0) = [g'(r)]_0\, S(1+z)$

hence $$i_\alpha^*(\theta_1) = \frac{g(r_1)}{r_1} \frac{1}{[g'(r)]_0}\, i_\alpha^*(0) = \tau\, i_\alpha^*(0)$$

And since τ depends only on r, this type of diameter may be used in equation (2).

A few final remarks on the theory of profile transformation are in order here. The color excess found by Stebbins and Whitford suggests that some alterations in present concepts are in order. First, space may not be transparent as previously supposed. There may exist an intergalactic absorbing medium. If this absorbing agency is isotropic, diameters can still be of use for the determination of distance.

Say absorption is a function of wave length and distance. $k = k(\Delta, \lambda)$

Now $\frac{E(\lambda)\, g(r)}{g(1+z)}$ will be the total number of photons of wave length λ leaving the nebula per unit time from a ring r to r + dr.

The observed monochromatic intensity in this ring with no absorption will be: $$i^*(\lambda,\theta) = \frac{E(\lambda/1+z)\, g(r)}{g r\, 8\pi^2 e g_2 (1+z)^5}$$

with absorption: (use bar) $$\bar{i}^*(\lambda,\theta) = i^*(\lambda,\theta)\, e^{\int_0^\Delta k(\tilde{\lambda},\Delta)\, d\Delta}$$

and $$\bar{i}_\alpha^*(\theta) = \frac{g(r)}{g r} \frac{\int_0^\infty \alpha(\lambda)\, E(\frac{\lambda}{1+z})\, e^{\int_0^\Delta k(\tilde{\lambda},\Delta)\, d\Delta}\, d\lambda}{8\pi^2 e g_2 (1+z)^5}$$

$\tilde{\lambda}$ is the effective wave length presented to the absorbing medium and varies with the per cent of the path traversed. It follows that the ordinate transformation is independent of r and τ diameters may be used in equation (2).

Another possible interpretation for the Whitford excess reddening is nebular evolution. Evolution of course may be either in intensity or in radius or both. In the 2×10^8 years since light left the Bootes cluster where an excess of about 0.3 magnitude is observed, a star such as the sun will have depleted its mass by about one part in ten thousand. The total change in the mass of the nebula must be of this order: 1 part in 10^4 and certainly not more than 1 part in 10^3. We may therefore assume little change in distribution. The evolution is predominantly in luminosity rather than in size.

We can generalize the emission function so that E (λ) is E (λ, t). Our only restriction being that changes are such that color is independent of r. In the evolutionary case, the law of ordinate transformation becomes:

$$\frac{i_\alpha^*(\theta_1)}{I_\alpha^*(\theta_1)} = \frac{(1+z_0)^5}{(1+z)^5} \frac{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+z}, t\right) d\lambda}{\int_0^\infty \alpha(\lambda) E\left(\frac{\lambda}{1+z_0}, T\right) d\lambda} = \frac{(1+z_0)^5}{(1+z)^5} 10^{0.4 K_\alpha(z, z_0, t, T)} \tag{8}$$

Since this is independent of r, τ diameters may be used in equation (2) to give true distances.

In the case of evolution, luminosities will be transformed by the law:

$$\frac{l_\alpha^*}{L_\alpha^*} = \frac{\Delta_0^2 (1+z_0)^3}{\Delta^2 (1+z)^3} 10^{0.4 K_\alpha(z, z_0, t, T)} \tag{9}$$

Equation (8), therefore, offers a method of obtaining the evolutionary K term which can be used in (9) for obtaining distances from luminosities. We may then write a general equation for derivation of distance, which is valid with nebular evolution and/or intergalactic absorption.

$$\Delta = \Delta_0 \frac{(1+z)}{(1+z_0)} \sqrt{\frac{i^*_\alpha(0)}{I^*_\alpha(0)} \cdot \frac{L^*_\alpha}{l^*_\alpha}} \qquad (10)$$

Note that in (10) all quantities except Δ and Δ_0 are observables

Area under profile may be used instead of luminosity

$\Delta = \Delta_0 \frac{(1+z)}{(1+z_0)} \frac{i^*_\alpha(0) \cdot A}{I^*_\alpha(0) \cdot a}$

Step II. We now come to step II, the conversion of the theoretical nebular profile by local absorptions and various instrumental effects to the form which we measure.

A careful analysis of this aspect of the problem has been made by De Vaucouleurs (Ann. d'Astrophisique t. 11, no. 4, 1948 pp 247-265). Most of the technical problems involved in microphotometric photometry are common to ordinary photographic photometry. We may list:

1. Galactic absorption. Account must be taken of the latitude effect either by the cosecant law or some refinement. If the ordinate transformation is independent of r, it follows that

$$l_u = a\, i_u\, \theta_1^2$$
$$l_c = a\, i_c\, \theta_1^2$$

or $\frac{l_u}{i_u} = \frac{l_c}{i_c}$ where u designates uncorrected, and c designates corrected values.

2. Atmospheric absorption. This can be taken into account by the function $\alpha(\lambda)$ used above, except for differential effects. Zenith distance corrections also follow the rule

$$\frac{l_u}{i_u} = \frac{l_c}{i_c}$$

3. Seeing. As in ordinary photometry, seeing can be a source of error. Comparison of tracings made of the same object with various seeings do not show appreciable difference except in the more stellar-like images. However, as in all careful photometric work, plates to be compared must be taken under similar seeing conditions.

4. Sky background. This can be a rather serious source of error. Since certain critical measurements, such as $I^*_\alpha(0)$ depend on the background level, variations can vitiate the results. All the usual precautions must be taken.

There are also instrumental errors -

5. Diffraction, focus, guiding and instrumental selectivity. Vaucouleurs has studied these parameters in his paper. Again all the usual precautions are in order.

Photographic plates and processing.

6. The problems and sources of error here are the same as in ordinary photometry. (Again, see Vaucouleur loc. cit.) There is one factor here which is of a more serious nature than in ordinary photographic photometry, and that is turbidity or irradiation. Because of the spreading of the image, the photographic profile may depart considerably from the nebular profile. Turbidity effects have been studied by several authors, especially Ross and Mees. It is found that there is negligible spreading of the image until the density reaches saturation, (Mees, Theory of the Photographic Process, p. 888), after which the size increases according to the well-known Greenwich formula. Fine-grain emulsions show much less turbidity. So for reasons of resolution as well as turbidity, fine-grain emulsions should be used with exposures short enough to prevent the center of the image reaching saturation.

Calibration.

7. Conversion of densities to intensities is the same problem encountered in ordinary photographic photometry. H-$\mathfrak{D}$ curves for the same developing and exposure conditions must be had. There is still the uncertainty introduced by the differences of the spectral characteristics of the sources. Though exposure time may be duplicated in laboratory calibrations, the light source is not comparable to the nebulae. Holmberg has obtained very satisfactory results by calibrating his nebular tracings against

extrafocal tracings of stars. At present this promises to be the most satisfactory method. However allowance for highly reddened distant sources must be made.

Measurement.

8. The most likely source of error is in the establishment of the background level. In addition to the usual precautions, e.g. taking ample background on both sides of the image, there is an additional complication arising in clusters. Tracings made of clusters on 48-inch Schmidt plates show that the background at the center of the cluster does not fall to what may be called the mean sky background. This will not show on an individual tracing, so wide-angle tracings must be used to determine the mean sky level to correct for any local effects.

Resolution.

9. Another source of error is the instrumental effect of the microphotometer. Because of the finite size of the scanning aperature, the tracing will not provide a true profile. This problem is an old one in spectroscopy where, given, the instrumental characteristics and the observed profile (tracing), the true profile is desired. Whereas the spectroscopic problem is a two-dimensional problem, the reduction of nebular tracings is three-dimensional. There is a slit-length effect as well as a slit-width effect. Both Holmberg and Vaucouleurs mention this problem but both are justified in neglecting it in their work which is the photometry of large near-by nebulae. However, when we wish to work with distant nebulae, the area of the scanning spot becomes a sizable fraction of the total area of the nebula and this instrumental error can no longer be omitted.

One method of getting an accurate profile would be to count grains, as has been done in spectrophotometry. But even this method is not so simple when we are dealing with the three-dimensional case. It is best, first, to get an idea of the nature of the error from a theoretical study. We essentially have an integral equation to solve of the form:

$$H(x,y) = \int_{-\infty}^{\infty} du \int_{-\infty}^{\infty} I(u,v)\, W(x-u, y-v)\, dv$$

where H is the observed profile, I is the true profile, and W is the instrumental function. A formal solution of the problem may be obtained, but it is of no use in application. But we can obtain some interesting and useful results by proceeding as follows:

As a first approximation, let us assume that the instrumental function is unity in a rectangle of width 2δ and length 2ϵ and is zero outside. We may then write the above equation in the form:

$$H(r) = \frac{1}{2\epsilon\delta} \int_{0}^{\epsilon} dy \int_{r-\delta}^{r+\delta} I(\sqrt{x^2+y^2})\, dx$$

Now our profiles are all even function, i. e.

$$I(r) = I(-r)$$

We may, therefore, expand I in a series of the form

$$I = \sum_{n=0}^{\infty} c_n (x^2+y^2)^n$$

where $x^2 + y^2 = r^2$. It can then be shown that

$$H(r) = I(r) + \frac{\epsilon^2}{6}\frac{I'(r)}{r} + \frac{\delta^2}{6} I''(r) + \frac{\delta^4}{5!} I^{iv}(r) + \frac{3\epsilon^4}{5!}\left(\frac{I''(r)}{r^2} - \frac{I'(r)}{r^3}\right) + \frac{\epsilon^2\delta^2}{(3!)^2}\left(2\frac{I'(r)}{r^3} - 2\frac{I''(r)}{r^2} + \frac{I'''(r)}{r}\right) + \frac{\delta^6}{7!} I^{vi}(r) + \cdots$$

To normalize our representations, we can take the distance out to the point of inflection (or mean deviation), as the unit. We shall designate this by σ. (In all cases we have $\frac{\delta}{\sigma} < 1$.) Then if R is in σ units and $\epsilon = \delta$, we have

$$H(R) = I(R) + \frac{1}{6}\frac{\delta^2}{\sigma^2}\left[\frac{I'(R)}{R} + I''(R)\right] + \cdots$$

(Note that the first two terms of the right member constitute Bessel's equation of zero order.)

Neglecting higher order terms, we have

$$I(R) = H(R) - \frac{1}{6}\frac{\epsilon^2}{\sigma^2}\frac{H'(R)}{R} - \frac{1}{6}\frac{\delta^2}{\sigma^2}H''(R) + O\left(\frac{1}{20}\left(\frac{\epsilon}{\sigma}\right)^4 \text{ or } \left(\frac{\delta}{\sigma}\right)^4\right) I^{iv}(R)$$

$R^4 e^{-\frac{R^2}{2}}$ max valu $\frac{1}{4} H(0)$ $< \frac{1}{80}\left(\frac{\epsilon}{\sigma}\right)^4 H($

To keep within our measuring accuracies $\epsilon = \delta < 0.7\,\sigma$ (1 part in 150). In the case of use of the central intensity,

$$I(0) = H(0) - \frac{H''(0)}{6\sigma^2}(\epsilon^2 + \delta^2)$$

If we are using τ diameters, a trial and error procedure must be used. If we take some value $\bar{\tau}$, such that H (R) = $\bar{\tau}_1$H (0), we can find R, H' (R), H" (R). From these values we may derive τ

where I (R) = τI (0)

by the formula

$$\tau = \frac{\bar{\tau}_1\left[1 - \frac{1}{6}\left(\frac{\epsilon^2}{\sigma^2}\frac{H'(R)}{R\,H(R)} + \frac{\delta^2}{\sigma^2}\frac{H''(R)}{H(R)}\right)\right]}{\left[1 - \frac{\epsilon^2+\delta^2}{6\sigma^2}\frac{H''(0)}{H(0)}\right]} \qquad \text{A)}$$

Going then to our transformed tracing, we must find a value $\bar{\tau}_2$ which by the formula

$$\tau = \frac{\bar{\tau}_2\left[1 - \frac{1}{6}\left(\frac{\epsilon^2}{\sigma^2}\frac{h'(r)}{r\,h(r)} + \frac{\delta^2}{\sigma^2}\frac{h''(r)}{h(r)}\right)\right]}{\left[1 - \frac{\epsilon^2+\delta^2}{6\sigma^2}\frac{h''(0)}{h(0)}\right]} \qquad \text{B)}$$

gives τ such that $i(r) = \tau\, i(0)$. We must by trial and error in B) find the r and $\bar{\tau}_2$ which gives the τ determined by A).

If we take $H(x) = a_0 + a_1 x^2 + a_2 x^4 + \cdots$

using as many points as convenient,

$$H''(0) = 2\,a_1$$

This is a ready method for evaluating the quantities used in the correction equation.

An alternative method is to employ normal error functions to approximate the profile. The integral equation is easily soluble in this case, and the corrections can be read from error tables.

An evaluation of the foregoing technical difficulties allows us to list the following procedures as advisable ones:

1) The diameter problem is subject to all the pitfalls of photographic photometry and all the usual precautions must be taken. Sky meter readings would be a useful adjunct.

2) Only two clusters at a time should be compared, these by means of half-half plates. The nebulae in the two clusters being photographed should be of comparable brightness in order that turbidity effects may be avoided by staying below saturation. This necessitates that the work be done in a series of steps, cluster A to B, cluster B to C, et cetera.

3) Wide-field plates should be used for local background corrections.

4) For minimizing both resolution and turbidity troubles, a fine-grain emulsion should be used.

5) The largest scale feasible should be employed, the Cassegrain foci if possible.

3. The Identification and Sampling Problems:

We must now examine to what extent the foregoing theory based on a hypothetical standard nebula can be applied in practice. It is clear that what was said about globular nebulae is immediately applicable to other elliptical nebulae provided that 1) the two ellipticals are identical, 2) their axes are parallel, and 3) we compare the same profiles (e.g. major or minor axes). In general it will not be possible to know when requirements 1) and 2) are satisfied, just as in the case of globulars it is not possible to know whether a given nebula is truly globular or is an elliptical with its axis parallel to the line of sight or even a late nebula of which we see only the nucleus.

We may approach the problem in one of two ways: 1) We may seek more refined criteria whereby we may positively identify two or more nebulae as being the same, or 2) we may establish certain statistical criteria which will allow us to work with samples.

Great progress was made in stellar astronomy because of the evidence of observables which served as labels that a certain star belonged to such and such a type. These observables were notably spectral characteristics and singular variations of luminosity. Though this type of label is not available in extra-galactic studies, by studying fine structure of nebulae presenting the same projected form, we may be able to arrive at suitable identification procedures, and narrow down the comparability uncertainties.

Let us consider the globular nebulae. Whereas, as mentioned above, single plate identification is subject to many uncertainties, it is surprising how much information may be gleaned by studying images in two colors. The outer portions of SO and late nebulae are bluer than the nuclei. So SO's will tend to change size from red to blue plates: Ellipticals will not. In the case of remote clusters, experience shows that the nucleus will disappear from an SO on a blue plate. These and other changes allow a globular to be identified without too much uncertainty. But we are still left with the problem of sorting the globulars. In a study of orientation of nebulae, Hubble showed that, assuming random orientation and equal real distribution, only about 54 per cent of apparent EO's are really EO's.

A preliminary study has been made of the sizes of globular nebulae in the Coma, Ursa Major I, Corona, and Ursa Major II clusters.

The sizes were taken both from tracings and from micrometric measurements. The results showed that several globulars in each cluster were identical in size. It was possible to get complete superposition of their tracings. This fact suggested that globular nebulae may occur only in discrete sizes. This supposition is further born out by the accompanying graph on which is plotted

$$\log \theta = \log \left(\frac{1 + z}{\Delta}\right) + \log(\text{constant})$$

for the globular nebulae in the four clusters. It is possible to pass parallel lines with the required theoretical slope through groups of globulars which must possess the same diameters. This surprising result, if substantiated by more extended investigations, readily suggests a way of accurately determining the distance to a cluster: The diameters of the largest globulars are plotted on a vertical logarithmic scale, and the scale is shifted horizontally until the points lie on the theoretical parallel family which passes through the globular distribution in clusters of known distance. The abscissa will then give the value of $(1 + z)/$ Distance. Whether this "quantization" of nebulae is real, due to some selectivity effect, or is an accidental result in the sample so far studied must await studies in more clusters. Whether the supposed quantization exists also in magnitudes must ~~can readily~~ be verified.

In the absence of directly comparable objects, Hubble established certain statistical procedures. He assumed that the fifth brightest nebula in each cluster was comparable. If the luminosity functions of the clusters are all the same, then this is a safe criterion.

At the present, little is known about the luminosity functions, but the above diameter studies provide sufficient material for studying the large ends of the diameter frequency functions of the above named clusters.

If allowance is made for the decrease of both mean value and dispersion with distance, the Coma, and two Ursa Major clusters are not too dissimilar. The Corona Cluster, however, has an excess of large nebulae.

The second graph shows a comparison of the different methods of sampling. Neglecting the Corona Cluster, we see that the results using the mean of the five largest, the mean of the 10 largest and the mean of the 20 largest are in excellent agreement, as shown by the nearly equal slopes. Use of the mean of the EO's ~~in~~ among the largest 20 nebulae gives results which show more homogeneity as evidenced by the constant slope. Further the general sample and the EO sample are in good agreement. It follows that the sample we use is of minor import provided that the distributions are comparable. The large disparity of the Corona Cluster is due to the excess of large nebulae. Consequently, if distances were naively computed from the diameters, without knowledge of the diameter frequency function, we could expect sizeable errors. That the same might be said about luminosities is problbly true.

From the preliminary study outlined in this paper, it is seen that the possibility of using diameters in the cosmological problem is not to be excluded. It is true that diameters will not provide something for nothing. They point the way around the difficulties of envelope shift and evolution, but the photometric problems require the highest order of care. It must be remembered that the attack on the cosmological problem is not a restricted one, but that diameters will be used in conjunction with magnitudes and counts. Thus, with each method serving as a check on the others, we may say that the information which diameters will yield will certainly be worth the effort.

de Vaucouleur's List of Observational Errors:

1) Atmosphere
→ A. absorption — for study of 1 nebula - negligible
B. diffusion — (scattered light)
C. Seeing
→ D. Sky.

2). Instrument:
A. Diffraction — neglect:
B. Aberrations, figure -
C. Focus —
D. Guiding
E. Scattering & reflections - ghosts
→ F. Selectivity
G. Vignetting.

3). Photographic Plate
→ A. Grain. — decreases with density
→ B. Irradiation — negligible.
C. Halation

4) Photographic Processes
A. Development: Contrast
B. Eberhard Effect
C. Local Errors

5) Photometric Calibration
A. Reciprocity Law
B. Spectra of Source
C. Ambiant Conditions

6.) Microphotometer Registry
A. Resolution
B. Scattered light
C. Response function

7) Measurement of Tracings
A. Zero line establishment.
→ B. Sky Background under 3%

8) Reduction of Measures.
A. deflection vs. magnitude

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Mailed
June 6, 1963

TENTATIVE OBSERVATIONAL CONFIRMATION OF DISCRETIZATION IN GALAXIES
(Title of Paper)

A. G. Wilson
(Author)

The RAND Corporation
(Institution)
1700 Main Street, Santa Monica, California
(Address)

(Address to which publication charge bill should be sent, if different from above)

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A test of the Edelen hypothesis of elliptical galaxy discretization was made with samples of early ellipticals in clusters. It is assumed 1) that the galactic diameters defined operationally by isophotic levels are proportional to the theoretical diameters and 2) that members of the cluster are all at the same distance.

Under these assumptions, the distribution function of diameters of early ellipticals in a cluster should consist of discrete groupings, whose lower bounds are proportional to numbers of the eigen sequence, $\sqrt{n(n+1)}$.

Diameters of elliptical galaxies of small apparent eccentricity were measured on 200 inch plates of the Coma, and Corona Borealis clusters and 48 inch Schmidt plates of the Virgo cluster, using luminosity curves and/or isophotes.

Preliminary results based on samples of galaxies with apparent ellipticities of 2 or less in these clusters showed diameter groupings falling in the positions predicted by the eigen sequence within the observational uncertainties. This conformity with the predicted theoretical distribution implies that the parameter ξ may be assumed to be constant within certain clusters. Although the distributions were consistent with the theoretical values, the confirmation is held to be tentative pending increases in the number of galaxies in the samples and in the number of clusters observed.

Revised Abstract

For A.J.

Appears: A.J. vol 68 no 8 Oct. 1963 p. 547

TENTATIVE OBSERVATIONAL CONFIRMATION OF DISCRETIZATION IN GALAXIES

A. G. Wilson
The RAND Corporation
Santa Monica, California

Images of early elliptical galaxies on 200 inch plates of the Coma, Ursa Major I, Corona Borealis, Bootes, Ursa Major II, and Coma B clusters were measured and reduced to quantities qualifying as proper diameters in the sense that, considered as measures of angular diameters, they exhibit inverse proportionality to distance. The reduction of the measurements to proper diameters was made by calibration against cluster red shifts.

The theoretical distribution function of diameters of true EO galaxies, according to the Edelen Theory, will be such that they occur at discrete intervals proportional to the sequence $[n(n+1)]^{1/2}$ where n is a postive integer. The observed proper diameters, normalized to uniform distance by relative red shifts, corresponding to apparent EO's were found to exhibit a distribution structure consistent with the predicted structure to within the observational errors.

The initial operational definitions of diameter used were micrometric, based on direct measurements of the photographic images of the galaxies, and microphotometric based on luminosity profiles.

On the basis both of the importance of the implications of the Edelen prediction and of the tentative nature of this initial observational demonstration, it is felt that ~~considerable~~ adequate justification now exists for the design and implementation of further observational tests for discretization phenomena.

down to about 18th magnitude on an additional 608 stars; however, the photoelectric limit is $V=17.0$. Fine structure is exhibited by the stars above the vertical main sequence and a slightly fainter red offshoot is found. A sequence of stars is found in *apparent* gravitational contraction but with main-sequence stars extending, nevertheless, to the observational limit. At present the data are best represented by theories of gravitational contraction, hydrogen-burning, and helium-burning, together with a non-unique epoch of stellar condensation. Contraction and nuclear time scales for the different features are compatible. The differences in apparent evolutionary tracks between the Galaxy, LMC, and SMC are reconfirmed. The two-color diagram of the bluest stars is of a blackbody character. Sp vs $(B-V)_0$ is monotonic for all stars. The M supergiants are shifted redward over previous studies and form a steep two-color relation.

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The complete distribution, throughout a gray atmosphere, of the spectrum of the source function accrues from a tentative extremum principle replacing arguments from thermostatics. Minimization of the global entropy deficit of the source function (a quantity identically nought in LTE), under the constraints that strict radiative equilibrium is preserved at all levels and that the photon flux escaping at the surface remains invariant, yields the Kothari–Singh function. It differs from the Planck function by an additive parameter related to the nonvanishing free enthalpy of nonequilibrium radiation. Since the new parameter varies proportional to the second exponential integral of the optical depth, LTE is reached asymptotically. This event disposes of a conjecture (Wildt, R., *Astrophys. J.* **123**, 115, 1956) which had left the march of the parameter undetermined.

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The theoretical distribution function of diameters of true E0 galaxies, according to the Edelen theory, will be such that they occur at discrete intervals proportional to the sequence $[n(n+1)]^{\frac{1}{2}}$, where n is a positive integer. The observed proper diameters, normalized to uniform distance by relative redshifts, corresponding to apparent E0's were found to exhibit a distribution structure consistent with the predicted structure to within the observational errors.

The initial operational definitions of diameter used were micrometric, based on direct measurements of the photographic images of the galaxies, and microphotometric based on luminosity profiles.

On the basis both of the importance of the implications of the Edelen prediction and of the tentative nature of this initial observational demonstration, it is felt that adequate justification now exists for the design and implementation of further observational tests for discretization phenomena.

Astronomical Journal, vol. 68, No. 8 - Oct. 1963 p 547

From the ASTRONOMICAL JOURNAL
68, No. 8, 1963, October—No. 1313
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p 535

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Report to Director Mt. Wilson Observatorie
July 1, 1963

Studies of Galactic Diameters

D. G. Edelen of the RAND Corporation has applied the Einstein Field equations to galaxies considered as granulated aggregates of matter and found that under certain very general conditions, their ellipticities must be functions of the semi-major axes. The functional relationships are multi-branched, predicting that the distribution functions of the major axes of galaxies should possess discrete peaks distributed proportionally to the eigen sequence, $[n(n + 1)]^{1/2}$, where n is a positive integer. In order to test this prediction, A. G. Wilson of the RAND Corporation has resumed his earlier studies of galactic diameters (Carnegie Year Books No. 49 and 50) re-examining previous observational suggestions of discretization in the diameters of cluster galaxies. Using plates of clusters taken by Baade, Humason, and Sandage with the 200 inch, Wilson has constructed distribution functions of relative diameters of early ellipticals in the Coma and Corona Borealis clusters. The diameter distributions show assymetric density maxima occuring at values whose ratios are consistent with the theoretical prediction to within the observational errors. The preliminary results indicate that larger samples must be studied before there can be a definitive confirmation of the Edelen Hypothesis.

found in visible and ultraviolet spectra.

Dr. George Wallerstein of the University of California has used both the 100- and 60-inch telescopes in a search for the presence of the lithium doublet at λ6708 in F stars. The line was not present in 5 stars taken at 6 A/mm or in 11 stars observed at 30 A/mm. Three plates at 6 A/mm were taken for the measurement of the ratio of Li^6/Li^7 in 111 Tauri, a star in whose spectrum Wallerstein had found lithium at the Lick Observatory.

In 1956 Wilson and M. K. Aly reported that λ Andromedae, a K0 IV star with exceptionally strong Ca II emission, probably showed the helium line at λ5876 in absorption. This has been confirmed by Wallerstein on spectrograms of higher dispersion (6 A/mm). Wilson and Wallerstein have examined 6 A/mm spectrograms of 48 stars of types G8–K2 for evidence of λ5876 in absorption. It is possibly present in 3 stars: β Ceti, π Cephei, and HR 6791. Spectra of the first two of these in the violet show that the Ca II emission is not strong.

Recently the investigations of Wilson have disclosed some extremely interesting correlations involving the H and K Ca II emission in late-type stars. So far, however, a quantitative interpretation of both the width and intensity of the emission in terms of a series of model stellar chromospheres has not been made. To do this, reliable profiles of the emission peaks must be obtained.

Dr. Ray Weymann of the University of California at Los Angeles has obtained spectrograms of suitable quality for reliable microphotometry at 4.5 A/mm of a number of stars, mostly luminosity class III giants in the K0–K3 range. Striking differences in the intensity and character of the central self-reversal are apparent among stars of nearly identical spectral class.

These spectrograms have been supplemented by some at 8 A/mm covering Hα and the Ca II infrared triplet. It is hoped that additional plates of stars showing much more intense emission can be obtained; in particular, manifestations of chromospheric activity in the Ca II infrared triplet are being sought.

Dr. Albert G. Wilson and Dr. George Abell, cooperating with the Jet Propulsion Laboratory and the Air Force, undertook to test the feasibility of "deep space" tracking by optical methods. The Palomar 48-inch schmidt was successfully used on August 28 and 29, 1962, to photograph at distances out to 600,000 km the Agena carrier rocket which injected the Mariner II Venus probe. (These photographs were made five weeks before the Soviet announcement of their "first" photograph of a space craft on an interplanetary mission.)

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CARNEGIE INSTITUTION OF WASHINGTON
YEAR BOOK 62 1962-1963 p. 44

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~~P-2759~~ - Abstract

Images of early elliptical galaxies on 200-inch plates of six distant clusters were measured and reduced to quantities which may be considered as proper galaxy diameters in the sense that they exhibit inverse angular size with distance when calibrated against cluster red shifts. The structures in the observed distributions of the proper diameters so obtained are seen to be consistent with the structure in the distribution that is predicted by the Edelen theory.

Appears: A.J. vol. 68 No 8. Oct. 1963 p. 535

GALACTIC SCALE DISCRETIZATION

Dominic G. B. Edelen
The RAND Corporation
1700 Main Street; Santa Monica, Calif.

If it is assumed that (1) a galaxy has a definite physical boundary, (2) this boundary is stable with respect to the Einstein gravitational field, (3) as a consequence of an invariant averaging process the macroscopic Einstein field equations contain terms exhibiting small deviations from oblate spheroidal symmetry, (4) the deviations are time-independent, and (5) the physical parameter denoted by ζ is independent of the particular galaxy under consideration, then the length of the semi-major axes of E-series galaxies can assume only discrete values that are proportional to uniquely determined functions of the eccentricity. For small values of the eccentricity, proportionate lengths of the semi-major axis can be approximated by the root of $n(n+1)$, for integer n. Using n as a parameter to distinguish the various branches of the discretization curves, it is found that there is a forbidden region between the branches $n = 1$ and $n = 2$. For the branches with $n > 1$, the allowable lengths of the semi-major axis become multiple valued for values of the eccentricity corresponding approximately to the well known limit of 7 for the ellipticity. It is found that there are $n + 1$ curves in each branch, referred to as twigs, and that each twig is associated with a circulation on the bounding surface together with a polar flow. The twigs corresponding to the ellipticals are those of vanishing polar flow but nonvanishing circulation; the twigs with nonvanishing polar flows are tentatively identified with the non-ellipticals. As an immediate consequence of these results, one can obtain the true eccentricity and inclination angle of ellipticals as well as a theoretical basis for their geometric morphology.

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Astronomical Journal vol 68, No. 8
October 1963

Stan Greenfield and Ted Harris 7/10/63

Dom Edelen and Al Wilson M-4967

EXTENSION OF RSR-7086, "GALACTIC SCALE DISCRETIZATION"

W. Kellogg/M. Klappert

Memos M-9206 of 11/19/62 and M-2007 of 3/13/63 outline the initial purposes of RSR-7086. The present memo is a request for an extension to the project beyond January 1, 1964, in order to extend and refine the theoretical, observational, and epistemological results to date.

The principal results to date are described in RM-3628-RC and RM-3694-RC by D. G. Edelen and RM-3771-RC and RM-3772-RC by A. G. Wilson. Some of these results are to be reported before the American Astronomical Society, July 24 at the Fairbanks meeting.

Summarizing to date:

1. A new prediction based on the Einstein equations of general relativity has been derived by Edelen and confirmed observationally by Wilson providing a fourth observational test of the equations of the general theory.

2. A method for determining the true ellipticities and angles of inclination of elliptical galaxies from the observed ellipticities and diameters has been derived.

3. Three important new tools for observational cosmology have been discovered.

    a. A method which allows the use of diameters of galaxies in cosmology.
    b. A solution to the Hubble sampling problem through the use of a "paradigmatic set" of elliptical galaxies.
    c. Coalescence tests for world models.

In prospect or currently being studied:

1. The application of the Edelen equations to the problem of the morphology of galaxies of further types: S0's, spirals, and barred spirals.

2. Observational patterns in galactic evolution based on the r-s diagrams.

3. Identification and observational calibrations of nearby paradigmatic galaxies for analysis according to the new theoretical morphological paramaterization.

4. Examination of stellar-planetary systems by use of the relativistic eigenfunctions: Bode's Law, planetary mass distributions, planetary angular momentum, distribution of planetary systems for main sequence stars.

5. Finite difference techniques based on eigenvalue reppesentation of momentum-energy discontinuities to yield a general description of the momentum-energy tensor of a galaxy which is consistent with any pre-assigned cosmological model. This in turn to imply the appropriate dynamical processes determining galactic structure consistent with the cosmological model.

6. A possible marriage of cosmological models and galactic models such that the observables of the combined systems yield implications as to the unobservables of each constituent. In particular, implications as to the dynamical structure of galactic nuclei.

7. An examination of the evolution question by means of conformal transformations - the same idea as used by Weyl in setting up his cosmological principle - offers the possibility of describing the motion of galaxies on the r-e diagrams. This would give a rational basis for an analog of the usual quantum mechanical transition probabilities, but without the undesired indetermanism - at least as to the geometry.

8. Further development of the epistemological method of sequential and cojoint equivalence classes.

Dom Edelen Al Wilson

DE:AW/cs

Paper Presented AAS
July 24, 1963
Fairbanks

TENTATIVE OBSERVATIONAL CONFIRMATION OF DISCRETIZATION IN GALAXIES

A. G. Wilson

The RAND Corporation, Santa Monica, California

Abstract

The design of an observational test to corroborate the theoretical prediction that the diameters, r, of galaxies of low eccentricity* are proportional to an eigen-sequence:

$$[n(n+1)]^{\frac{1}{2}} \quad n = 1,2, \ldots\ldots$$

must be approached initially in its epistemological aspects. The r specified by the theory as the diameter or semi-major axis of an ellipsoid determined by a surface upon which there exists a discontinuity in some component of the energy-momentum tensor, while well defined for theoretical purposes, is not in general identifiable with any observable feature of elliptical galaxies. In fact, there is question that such a discontinuity may even exist in any observable sense. Compounding the difficulty of correctly relating such a theoretical diameter r to a diameter defined on observables is the fact that the many operational definitions of diameter, whatever the detailed method of measuring, usually involve systematic errors not easily isolated or evaluated. Hence it is extremely difficult to assign meaningful numbers to the ratios of the diameters of elliptical galaxies of different sizes and different distances.

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*In accord with present ideas concerning the morphology of galaxies, the only class of galaxies which have members with low eccentricities is the ellipticals. In this paper, therefore, galaxies will mean elliptical galaxies.

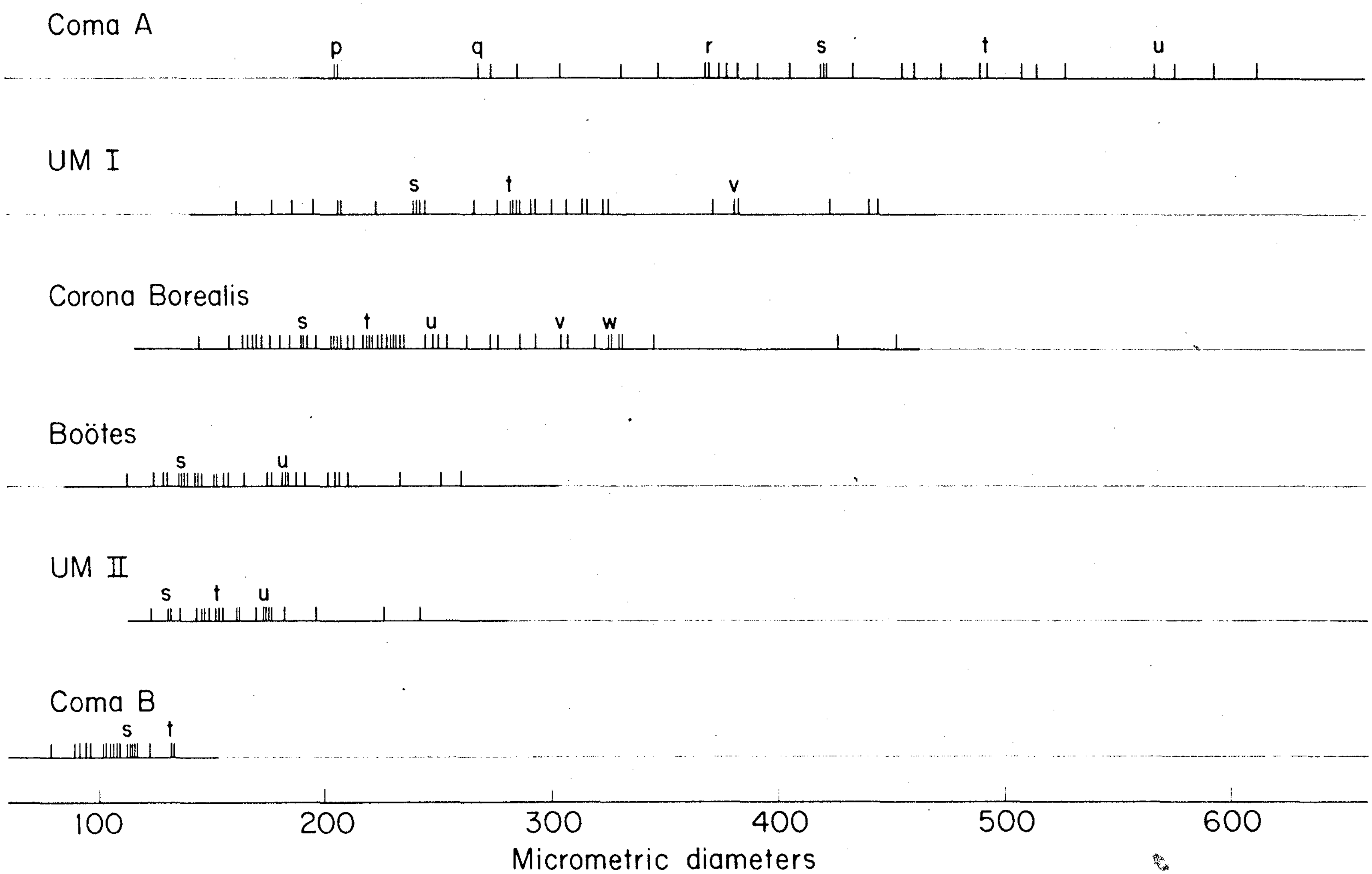
The procedure adopted in constructing the present test of the Edelen discretization hypothesis was to take several specified operational definitions of diameter all of which result in a set of measurements of linear dimensionality and convert these measurements into quantities which may be called proper diameters in the sense they possess the property, when angularly interpreted, of varying inversely with distance. This was accomplished by calibrating the measurements with red shifts, on the assumption of constant proportionality of red shift with distance.

The operational procedure selected for reporting here is that of micrometric measurements. In Fig. 1 the measurements (s) made of images of galaxies on a homogeneous set of photographs of six clusters are shown. The plates were 30-minute exposures taken by Humason with the 200-inch telescope on 103a-0 emulsions. The sample of galaxies measured was selected from probable cluster members having apparent ellipticities of three or less.

Patterns were prescribed for discernment of "signals" which might correspond to the distributions expected from the discretization eigen functions. These patterns were defined by one or more criteria based on (1) condensations at roughly periodic intervals, (2) abrupt gaps on the lower edges of the condensations, and (3) only E0 galaxies at the "band heads" (so called for obvious reasons). The more sophisticated the pattern the rarer the event and the higher the level of noise through which discernment is possible. On the bases of identification of patterns according to these specifications, values of (s) were determined which were thought likely to correspond to a band head signal emerging from the general noise of the data.

The (s) values of these signals are shown plotted against the red shifts of the clusters in Fig. 2. The fact that a set of very closely parallel lines can be unequivocally constructed passing through the (s) values of the signals allows feedback to establish further confidence in the pattern identifications. The signals from each cluster lying on the same line are identified with a corresponding letter designation such as t, u, v, etc.

If the (s) measures are proper diameters, then they must possess the property of linear variation with distance (or some suitable



Coma A
p
q
r
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UM I
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t
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Corona Borealis
s
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Boötes
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UM II
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Coma B
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100
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300
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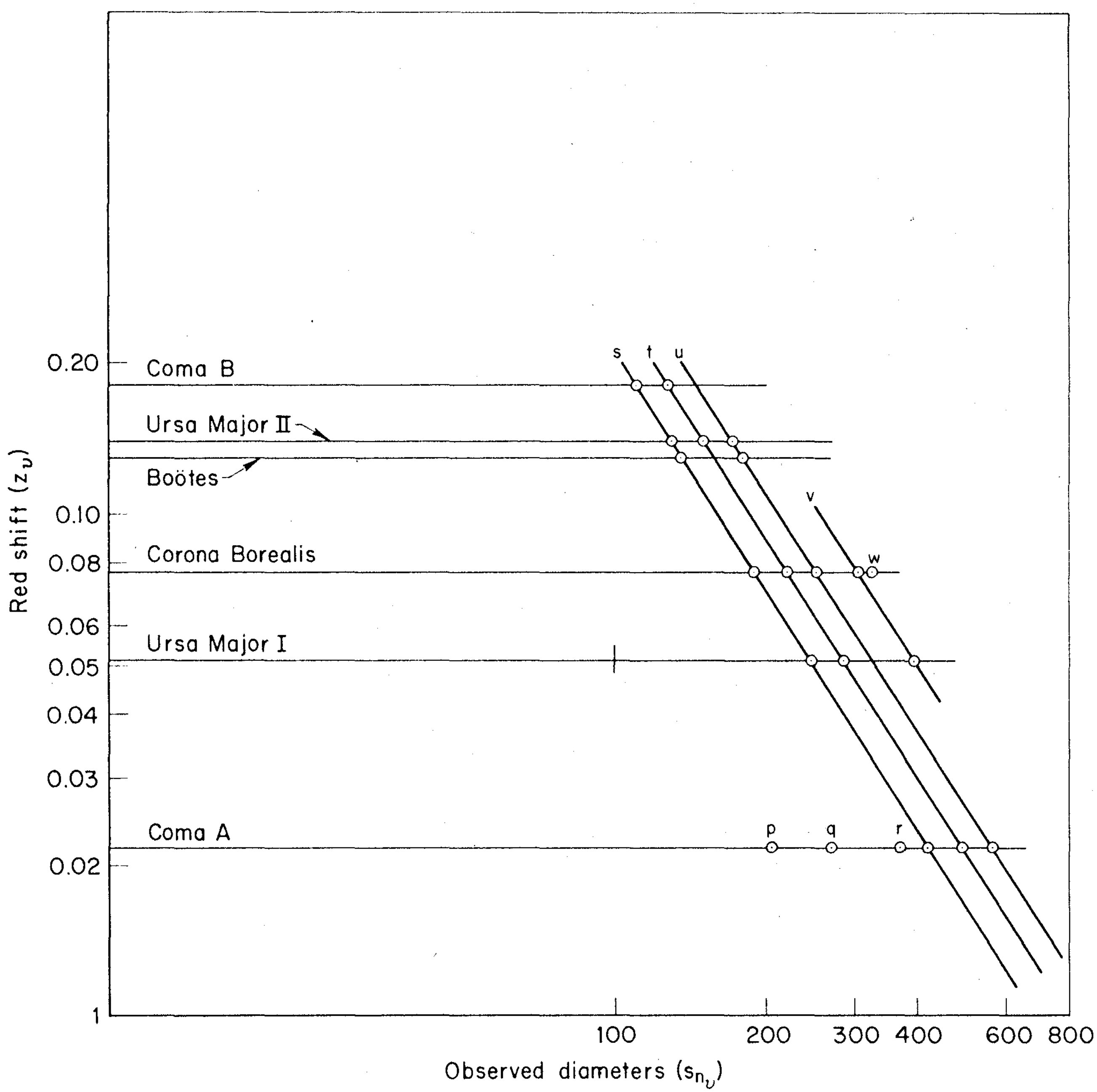
500
600
Micrometric diameters


Coma B
Ursa Major II
Boötes
Corona Borealis
Ursa Major I
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s
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0.10
0.08
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0.03
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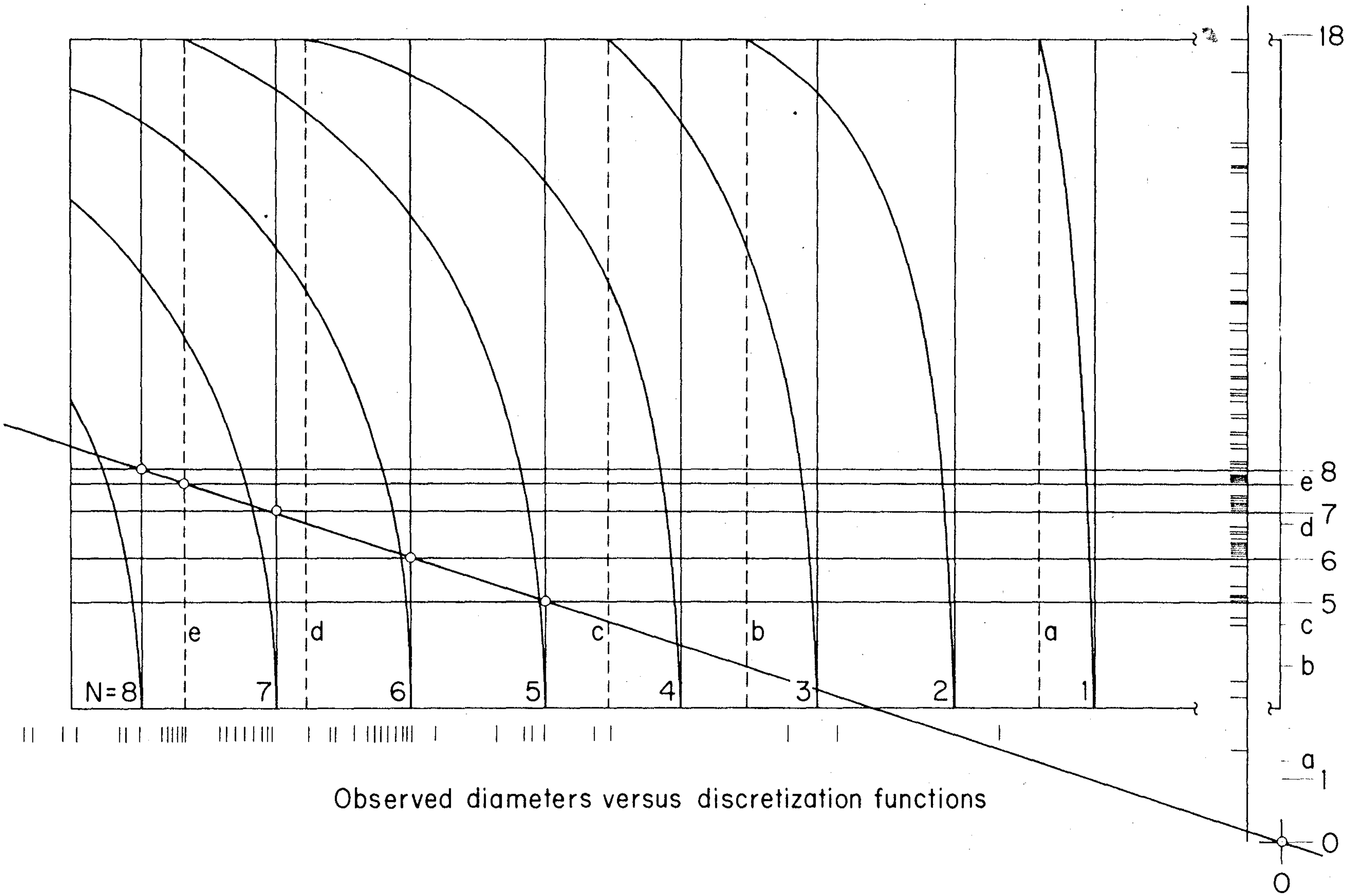
1
100
200
300
400
600
800
Red shift (z_υ)
Observed diameters (s_{n_υ})

modification of this law) demanded by our restriction on all definitions of diameter. For proper diameters the slopes of the (s) versus red-shift lines should be equal to -1. The observed slopes of these lines are all approximately equal to -1.63. This may be interpreted to mean that all of the (s) measurements are subject to systematic errors which cause small galaxies to be measured as too large and large galaxies to be measured as too small. The observed slope of -1.63 can be used to correct for this systematic error and for any possible physical variations which are proportional to distance, such as possible factors of $(1+z)$ or variations in the parameter ε, simply by raising each measurement to the 1.63 power. In other words if $\theta = s^{1.63}$, then the quantity θ may be considered to be a proper diameter.

The subset of all galaxies with observed ellipticities of one or less were then normalized to the distance of the Coma cluster by means of the red shift ratios, and the resulting distribution of the θ's are plotted in Fig. 3.

The distribution of observed diameters plotted on the right of Fig. 3 is seen to have a correspondence to an expected distribution of diameter sizes based on the eigen functions of the Edelen theory. A scale factor is assumed in Fig. 3 and a line drawn through the origin to demonstrate a possible correspondence of band heads, gaps, etc. The particular correspondence shown is <u>not</u> to be taken as the correct correspondence. It is for illustrative purposes only. Many alternate identifications are possible, and with the probable errors in the observed band heads being greater than the differences in the successive ratios of eigen numbers, no meaningful identification can be made on the basis of this data. All that is to be remarked is that the observations seem overall to be consistent with first-order effects predicted by the theory.



Observed diameters versus discretization functions

In conclusion, on the basis of this data, there appears to be no serious contradiction to the discretization prediction. The consistency of the observed results with regard to the distributions may be accepted as ample justification for initiating programs to investigate discretization phenomena, but should in nowise be considered a proof of the Edelen hypothesis. Hopefully, the present demonstration may prove to be the first of a series of demonstrations which can be used inductively to establish validity. A higher level of confidence can come both from further direct tests along the lines of the present one and through applications of other consequences of the theory. There is no question that if this theory proves true, as is presently indicated, it constitutes a major breakthrough in extra-galactic astronomy and cosmology.

Paper Presented
July 24, 1963

Abstract

GALACTIC SCALE DISCRETIZATION

If it is assumed that (1) a galaxy has a definite physical boundary in the sense that some physical quantity has a jump across this boundary, (2) the boundary is stable with respect to the Einstein gravitational field, (3) as a consequence of an invariant averaging process the macroscopic Einstein field equations contain terms exhibiting small deviations from oblate spheroidal symmetry, (4) the deviations are time-independent, and (5) the physical parameter denoted by ξ is independent of the particular galaxy under consideration, then the length of the semi-major axes can assume only discrete values that are proportional to uniquely determined functions of the eccentricity. For small values of the eccentricity, proportionate lengths of the semi-major axis can be approximated by the root of $n(n + 1)$, for integer n. Using n as a parameter to distinguish the various branches of the discretization curves, it is found that there is a forbidden region between the branches $n = 1$ and $n = 2$. For the branches with $n > 1$, the allowable lengths of the semi-major axis becomes multiple valued for values of the eccentricity corresponding approximately to the well known limit of 7 for the ellipticity. It is found that there are $n + 1$ curves in each branch, referred to as twigs, and that each twig is associated with a circulation on the bounding surface together with a polar flow. The twigs corresponding to the ellipticals are those of vanishing polar flow but nonvanishing circulation; the twigs with nonvanishing polar flows are tentatively identified with the non-ellipticals. As an immediate consequence of these results, one can obtain the true eccentricity and the inclination angle of ellipticals, as well as a theoretical basis for their geometry morphology.

Let S denote the surface of a world tube in an Einstein-Riemann space (i. e., a time-like hypersurface whose space sections are closed). The physical picture we have in mind is that the region interior to S is occupied by the world lines of a galactic structure, and that exterior to S is the background field of "free space." A complete description of this situation would be achieved by specifying the momentum-energy tensor appropriate to the dynamical processes interior and exterior to such galactic structures. In view of the fact that at present we do not have adequate detailed information concerning such dynamical processes, we cannot specify the momentum-energy tensor with any acceptable degree of certainty. We therefore adopt the viewpoint that although appropriate functions T_{AB} exist for any particular galaxy, we know them only to the extent that $T^{B}_{A;B} = 0$, as required by the Einstein theory. Since S is to represent the effective boundary of a galactic structure, we consider those models for which at least one component of T_{AB} has a jump across S. This requirement simply states that there is some physical quantity (or its derivatives) which has a jump across S, and hence it is physically as well as mathematically meaningful to speak of an interior and an exterior of a galaxy.

Of necessity, we entertain those models which represent macroscopic behavior, whereas the Einstein field equations relate local differential geometric quantities to local momentum-energy distributions. If we denote the local momentum-energy tensor by t_{AB} and the corresponding metric tensor by g_{AB}, and use an invariant averaging process (over a suitably chosen 4-dimensional volume), the Einstein equations formed on the averaged metric tensor h_{AB} have as momentum energy tensor T_{AB} the sum of the averaged t_{AB} and a quantity which may be interpreted as the momentum-energy required to equilibrate the local fluctuations. Hence, if the averaged t's exhibit oblate spheroidal symmetry, the T's will, in general, exhibit deviations from oblate spheroidal symmetry. These deviations from oblate spheroidal symmetry are assumed to be small.

Observationally, most galactic structures appear to have locally stable iso~~topes~~photes, and hence we assume that the boundary of a galaxy is geometrically stable. Mathematically, this means that there is an irrotational isometry in the bounding surface S which takes the boundary seen in three-space at one time into an identical metric replica of itself at a later time. It can then be shown that the magnitude, ϕ, of the vector field generating the irrotational isometry gives a measure of the local fluctuations of the jumps in the momentum-energy tensor across S, and that ϕ must be a bounded single valued solution of the Klein-Gordon equation on S. It must be clearly noted that the function ϕ is defined on a time-like surface in the present analysis; in contradistinction to the space-like surfaces encountered in the Cauchy problem. (This intrinsic difference is reflected in the occurrence of a discrete rather than a continuous spectrum.)

If it is assumed that ϕ is "time independent," so that the local deviations from oblate spheroidal symmetry are time independent and the boundary of the galaxy is stationary in space, the semi-major axis of the galaxy must be proportional to a uniquely determined function $\rho(n, m, \epsilon)$, where n and m are integers and ϵ is the eccentricity. The factor of proportionality is a constant times the physical quantity ξ, where ξ can, in general, be thought of as the jump in the total energy seen by an observer moving with the vector field that generates the isometry on S. A plot of $\rho(n, m, \epsilon)$ is given in Fig. 1. For obvious reasons we refer to n as the branch number and m as the twig number. The twigs corresponding to equatorial circulation without polar flow are $m = 0$.

The discretization function $\rho(n, 0, \epsilon)$ is thus appropriate to the description of ellipticals, and is given in Fig. 2. This figure is the basis from which the remaining conclusions of the first paragraph are drawn.

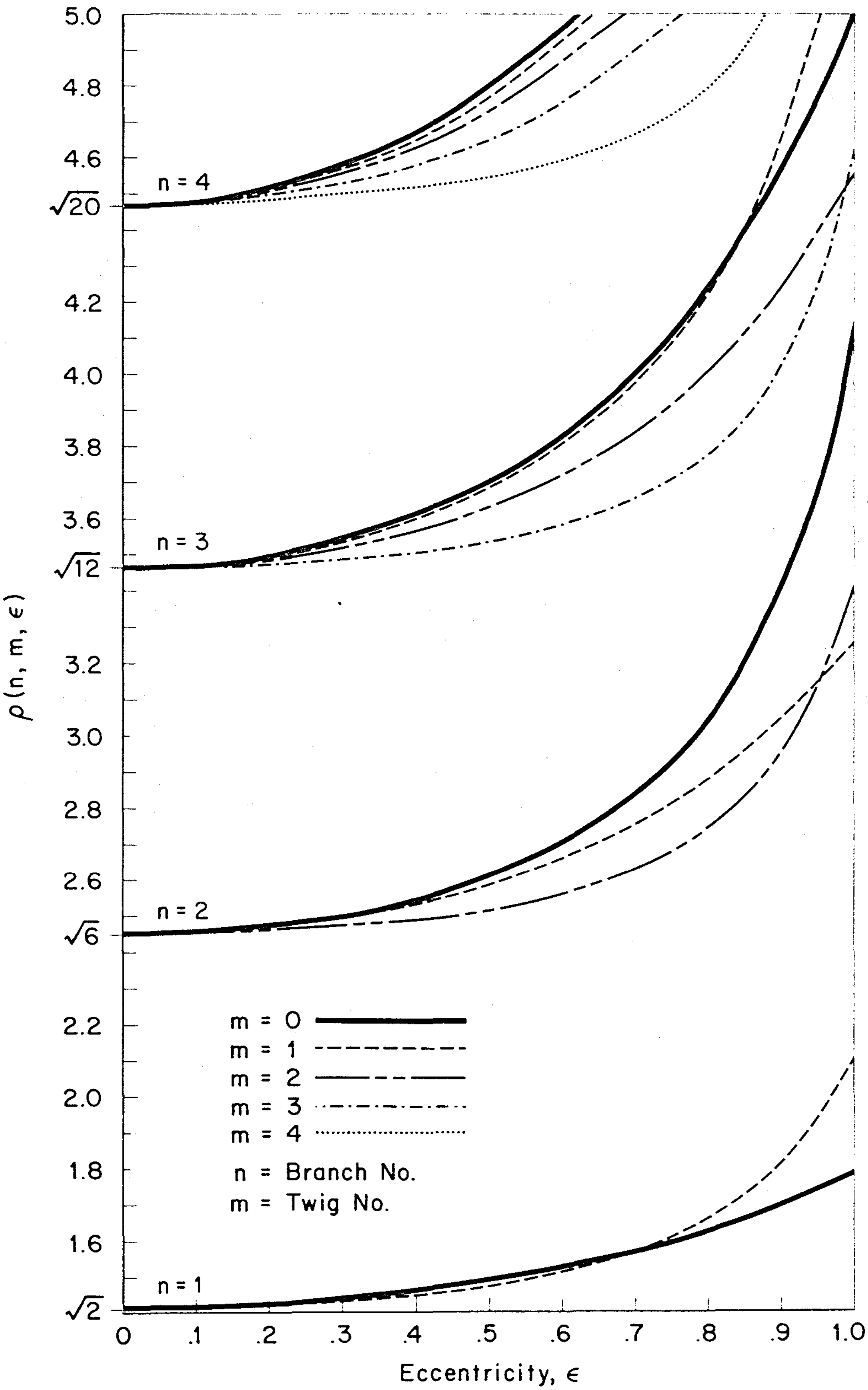


Fig. 1—The discretization function, ρ

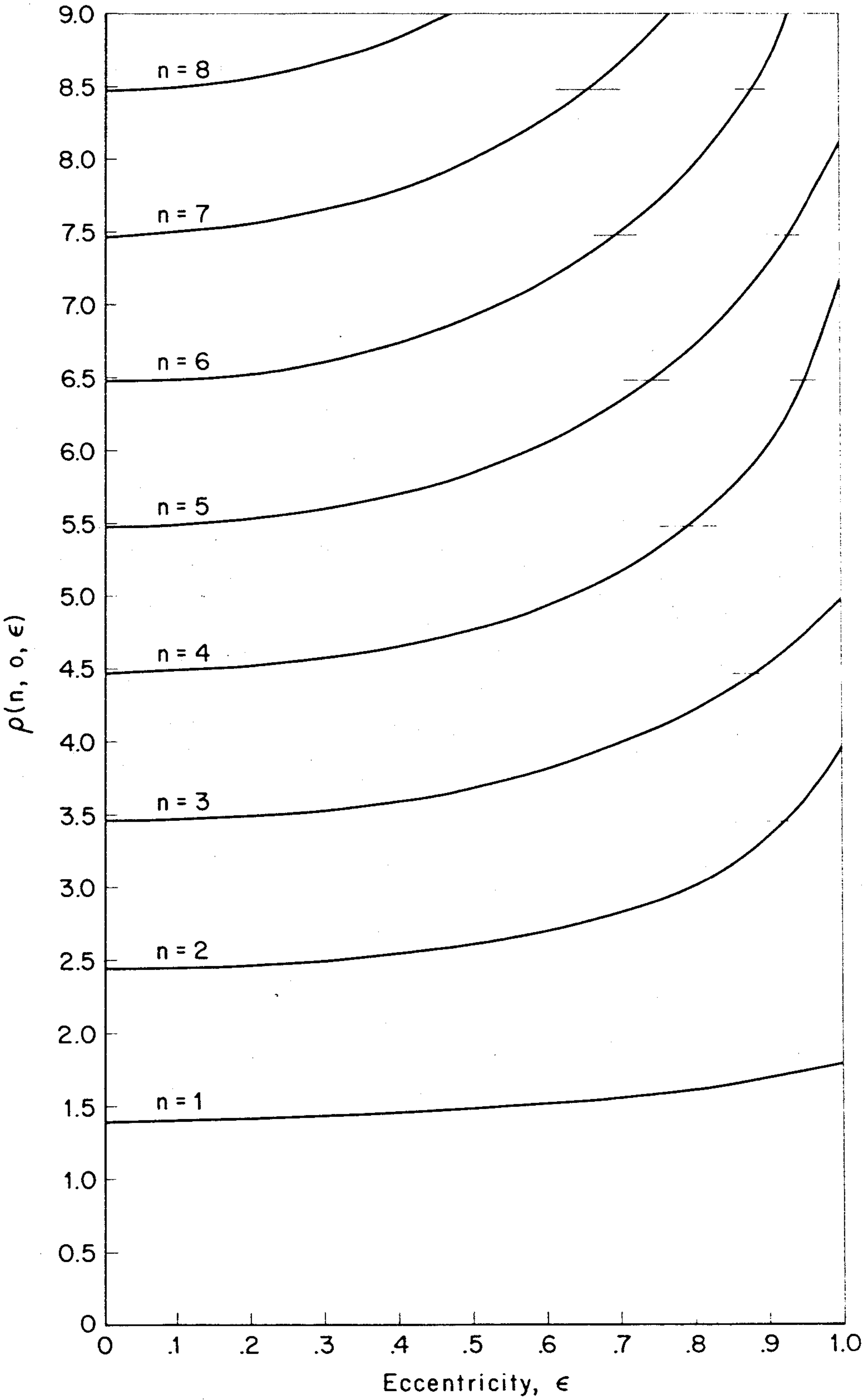


Fig. 2—The discretization function, $\rho(n, o, \epsilon)$ for E-series galaxies

July 11, 1963

T. Harris

TO: S. M. Greenfield

D. G. B. Edelen

FROM: A. G. Wilson

SUBJECT: EXTENSION OF RSP-7086, "GALACTIC SCALE DISCRETIZATION"

COPIES TO: D. Edelen, T. Harris, W. Kellogg, M. Klappert

Memos M-9206 of 11/19/62 and M-2007 of 3/13/63 outline the initial purposes of RSP-7086. The present memo is a request for an extension to the project beyond January 1, 1964 in order to extend and refine the theoretical, observational, and epistemological results to date.

The principal results to date are described in RM-3628-RC and RM-3694-RC by D. G. Edelen and P-2759 by A. G. Wilson, to be reported before the American Astronomical Society, July 24 at the ~~Fixx~~ Fairbanks meeting. Summarizing to date:

1. A new prediction based on the Einstein equations of general relativity has been derived by Edelen and confirmed observationally by Wilson providing a fourth observational test of the equations of the general theory.

2. A method for determining the true ellipticities and cycles of inclination of elliptical galaxies from the observed ~~ellipticities~~ ellipticities and diameters has been derived.

3. Three important new tools for observational cosmology have been discovered.

   a. A method which allows the use of diameters of galaxies in cosmology.

b. A solution to the Hubble sampling problem through the use of a "paradigmatic set" of elliptical galaxies.

c. Coalescence tests for world models. – means structure into cosmology
Nov 5

In prospect or currently being studies:

1. The application of the Edelen equations to the problem of the morphology of galaxies of further types, SO's, spirals, barred spirals.

2. Patterns in galactic evolution based on the r-E diagrams.

3. Identification and calibration of nearby paradigmatic galaxies for observational analysis according to the new morphological paramaterization.

4. Examination of stellar-planetary systems by use of the relativistic eigenfunctions: Bode's Law, planetary mass distributions, planetary angular momentum, distribution of planetary systems for main sequence stars.

5. Finite difference techniques based on eigenvalue representation of momentum-energy discontinuities will yield a general description of the momentum-energy tensor of a galaxy which is consistent with any pre-assigned cosmological model. This in turn will imply the appropriate dynamical processes determining galactic structure consistent with a cosmological model.

6. A possible marriage of cosmological models and galactic models such that the observables of the combined systems yield implications as to the unobservables of each constituent. In particular, implications as to the dynamical structure of galactic nuclei.

7. An examination of the evolution question by means of conformal transformations - the same idea as used by Weyl in setting up his cosmological principle - offers the possibility of describing the motion of galaxies along the discretization curves (and possibly from one to the other). This would then give a rational basis for the ~~xx~~ usual quantum mechanical transition probabilities, but without the undesired indetermanism - at least as to the geometry.

8. Further Development of the epistemological method of sequential and conjoint equivalence classes. i.e. Paradigmatic Inference
Nov 5

30 September 1963 L-19879

Professor B. Lindblad
President, Commission 28, I. A. U.
Stockholm Observatory
Saltsjobaden, Sweden

Dear Professor Lindblad:

Enclosed for inclusion in the draft report for the XIIth General Assembly is a brief summary of current galaxian research being conducted at The RAND Corporation by D. Edelen and myself. The preliminary results were reported at the meeting of the American Astronomical Society in Fairbanks, Alaska, 24 July, with abstracts to appear in the Astronomical Journal. However, some of the results reported here are as yet unpublished elsewhere and depend on this communication for establishment of priority.

With kindest personal regards.

Sincerely yours,

Albert G. Wilson
Planetary Sciences Department

AGW:cn

Edelen has shown that the Einstein theory of general relativity, when used in conjunction with the epistomological equivalents of certain well known properties of galaxies, predicts a relation between the galaxian semi-major axis r and the eccentricity ϵ (or ellipticity) of the form $r(n, m, \epsilon)$ = constant where n is a positive integer and $0 \leq m < n$. The most important of these epistomological equivalents are that one is theoretically concerned with aggregates instead of individual stellar images and that the geometrical outline of galaxies presist in time. The first of these requires the use of an invariant averaging process, in some respects similar to that introduced by Shirokov and Fisher (Shirokov, M. F. and I. Z. Fisher, 1962, Astronomicheskii Zhurnal, 39, 899 -- Soviet Astronomy, AJ, 1963, 6, 699); the second gives rise to a kind of intrinsic geometric stability. In the particular case $\epsilon = 0$, the relation between r and n takes the form

$$r^2 \xi = n(n + 1),$$

independent of m, where ξ is a physical parameter corresponding to the jump in the total energy density across the surface of the world tube representing the galaxy, as seen by an observer moving along an intrinsic time line of the surface.

If ξ is constant, or of limited variation, or constrained to a small number of possible values, it follows that the diameters of E0 galaxies should exhibit discretization in size. In particular, if ξ is a constant, the diameters of E) galaxies should be proportional to the discrete values of the eigen sequence $(n(n + 1))^{\frac{1}{2}}$. In addition, the Edelen theory provides an immediate method for determining the true eccentricity of elliptical galaxies; and the inclination of their axes from observations of their apparent eccentricity and size. When multiple values of n are theoretically available, one obtains the prediction of the observed limit of ellipticity 7, if a galaxy always selects the minimum energy state. It can also be shown that the ellipticity limit obtained in this way decreases with size (i.e., with increasing values of n) so that one would expect to find the larger ellipticals more spherical than the smaller ones. In addition, the eigen functions associated with the eigen relations between n, m, ϵ, and r, when combined with the theoretical geometry indicate a possible basis for a rigorous and self-consistent explanation of several features of observed galaxtic morphology.

The earlier data of Wilson (Directorâs Report, Mt. Wilson and Palomar Observatories, Carnegie Year Books 49 and 50) which first suggested discretization among globular galaxies has been re-assessed and combined with new measures to study the independent Edelen discretization prediction.

The present observational confirmation of the galaxian diameter eigen sequence rests on three independent sets of data. 1) Wilson's angular diameters of E0 galaxies in six clusters re-measured on a homogeneous set of 200 inch plates. 2) The log D's of all E0 galaxies in the new de Vaucouleurs catalogue of galaixes (Reference Catalogue of Bright Galaxies, University of Texas) and 3) the fine structure in Abell's luminosity function of the Coma cluster (G. O. Abell, Membership of Clusters of Galaxies, p. 233, McVittie: Problems of Extragalactic Research). The cluster angular diameters and the field (from de Vaucouleurs' catalogue) angular diameters are reduced to linear diameters by assuming distance is proportional to $z/(1 + z)$ where z is the redshift, $\delta\lambda/\lambda$. Thirty-six out of the sample of 40 cluster E0's fit the predicted eigen-sequences to within the internal error of measurement. Twenty-nine out of 32 field galaxies fit one of four eigen-sequences, with a precision that leads to the conclusionsthat the peculiar motion component of all the individual redshifts are very small. The diameter redshift relation for cluster galaxies confirms the Hubble law and reveals the hitherto unsuspected relation that the redshifts of all clusters so far published (Humason, Mayall, Sandage, Redshifts and Magnitudes of Extragalactic Nebulae, A.J., p. 97, 1956) obey the empirical relationship $\mu n(n + 1) = K_\sigma$, where $\mu = \frac{1 + z}{z}$, ni is a positive integer, and K_σ is a limited set of discrete constants related to the parameter ε of the Edelen discretization function. Using Hubble's relationship, log (diameter) = -0.2 m = constant, the fine structure of Abell's Coma cluster luminosity function is explained quantitatively by the discretization eigen sequence, providing a third observational confirmation.

If confirmed, for larger samples, the observed "bi-discretization" of galaxy size and redshifts for E0's in clusters raises some entirely new cosmological problems.

A. G. Wilson
September 30, 1963

7 October 1963 L-20237

Dr. I. S. Bowen, Director
Mt. Wilson and Palomar Observatories
813 Santa Barbara Street
Pasadena, California

Dear Dr. Bowen:

Recent work on the diameters of elliptical galaxies gives a preliminary indication of the validity of the theoretical prediction of Edelen that the sizes of the major axes of E0's are discretized in proportion to an eigen sequence of the form $[n(n+1)]^{1/2}$, where n is a positive integer.

Since the preliminary indication is based on a rather small sample, consisting for the most part of measurements of diameters of elliptical galaxies in six clusters on a set of homogeneous 200 inch plates kindly provided by Sandage and Humason, I feel it necessary to extend the sample before a definitive conclusion concerning discretization can be drawn. For this purpose I would like to propose the following guest investigator program for the observatory council's and your consideration:

1) Measurements of diameters of globular and near globular galaxies on tracings made from original sky survey plates. This would involve some 30 galaxies, many in the Virgo Cluster.

2) Investigation of a new technique for determining diameters of elliptical galaxies which consists of "sharpening" their profiles by making positives of several existing 100 inch and 200 inch cluster plates on plates coated with Eastman 649 emulsion. This emulsion has an almost "step function" H and D curve and would effectively provide images of cluster galaxies reduced to a single common isophote. Micrometric diameters of such images should have greatly reduced internal errors.

3) Transfer plates of several field globular galaxies and near by clusters made with the 48 inch Schmidt for homogeneity checks and calibration. About two clear nights per month in the winter and spring seasons would be required covering the 10 hour to 16 hour regions.

4) A two hundred inch transfer plate between the centers of the Coma and the Corona Borealis Clusters for calibration and homogeneity check

Plate requirements would consist of about three dozen 10x10, 103a-O, 48 inch Schmidt plates and about three dozen 649 emulsion 5x7 plates for cluster and transfer positives.

Sincerely yours,

A. G. Wilson
Planetary Sciences Department

AGW:cn

cc: Dr. Allan R. Sandage

MOUNT WILSON AND PALOMAR OBSERVATORIES
CARNEGIE INSTITUTION OF WASHINGTON
CALIFORNIA INSTITUTE OF TECHNOLOGY

813 SANTA BARBARA STREET
PASADENA, CALIFORNIA

December 19, 1963

Dr. A. G. Wilson
Planetary Sciences Department
The Rand Corporation
1700 Main Street
Santa Monica, Calif.

Dear Al:

At its meeting yesterday the Observatory Committee voted to invite you to carry ou the program on the 48-inch as outlined in your letter of October 7, 1963.

Has there been any further discussion of the continuation of the Air Force program that you and George Abell were carrying out on the 48-inch?

With best regards,

Sincerely yours,

I S Bowen

I. S. Bowen

IB:WB

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LETTER No. ....
DATE ....

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GREENFIELD		cy
HARRIS		cy
HAYDON		
JEFFRIES		
KLEIN		
KING		
A. LATTER		
R. LATTER		
LEDERER		
MATTOX		
MENGEL		cy
NOVICK		
OYSTER		
RESCO		
RUMPH		
SCHAMBERG		
SPEIER		
VOGEL		
WALDRON		
WARE		
WHITE		
WILLIAMS		
WEIHE		
ZWICK		
Wilson	✓	
FILE NO.	10854	

AG

EVIDENCE FOR DISCRETIZATION DERIVED FROM THE LUMINOSITY FUNCTIONS OF THE COMA CLUSTERS

Oct. 23, 1963

A. WILSON

Abell (Membership of Clusters of Galaxies, p. 233, Problems of Extragalactic Research - McVittie, editor) has published a luminosity function of the Coma Cluster which exhibits several features of fine structure, most prominent of which is a "break" at magnitude 14.7. Similar breaks are observed in the luminosity functions of other clusters which, if assumed to occur at the same absolute magnitude in each cluster, may be used as a distance indicator of the cluster. When considered in ~~the~~ this way the breaks for five clusters are in good agreement with Hubble's law of red shifts. For this reason Abell feels that the fine structure exhibited by his luminosity functions is real and not due to some systematic effect in the method of reducing the observations.

A more detailed analysis of the published luminosity function of the Coma Cluster shows that the fine structure features seem to be quantitatively consistent with the theoretical eigen sequence for discretization in <u>sizes</u> of elliptical galaxies derived by Edelen (Discrete Galactic Structure --- RAND RM).

The discretization eigen functions for elliptical galaxies according to the Edelen theory, are nearly constant with eccentricity (or ellipticity) for low values. If a general sample of cluster galaxies happens to consist for the most part of early ellipticals it is to be expected that the distribution function of <u>size</u> in the sample should exhibit the discretized branches of the eigen functions, although blurred by the presence of galaxies of larger ellipticity. The loss of detectability of the eigen branches will depend on the distribution function of the galaxies with respect to their eccentricity. But nothing can be said about this a priori.

Although there is chance of misidentification of morphological type, especially between certain ellipticals and SO's, and correct identification becomes increasingly difficult with increasing cluster distance, there is good evidence that the membership of clusters of galaxies consists predominately of ellipticals.

Accordingly, if Hubble's relation between magnitudes and sizes of ellipticals is generally applicable (Hubble - Realm of the Nebulae), as has been recently confirmed by Neyman and Scott (Problem of Selection Bias in the Structure of Galaxies), and if the distribution function of the cluster galaxies in eccentricity is such that for the most part they are E_2 or less, the Edelen theory would predict regularized fine structure in the luminosity function of clusters.

Assuming the Hubble relation

$$\log D = - 0.2m + \text{const.} \qquad (1)$$

where D is the diameter and m the magnitude, the Edelen discretization sequence for diameters,

$$\log D = \log J + \frac{1}{2} \log [n(n+1)] \qquad (2)$$

where J is a parameter, which may assume a limited number of values and n is a positive integer, may be transformed into a discretization relation for magnitudes

$$\log [n(n+1)] = - 0.4m + \text{const.} \qquad (3)$$

Under the preceding assumptions, the fine structure "bumps" in the luminosity function should be in agreement with equation (3). Table I is a difference table based on equation (3) which gives the theoretical differences in magnitudes, Δm corresponding to differentvalues of n.

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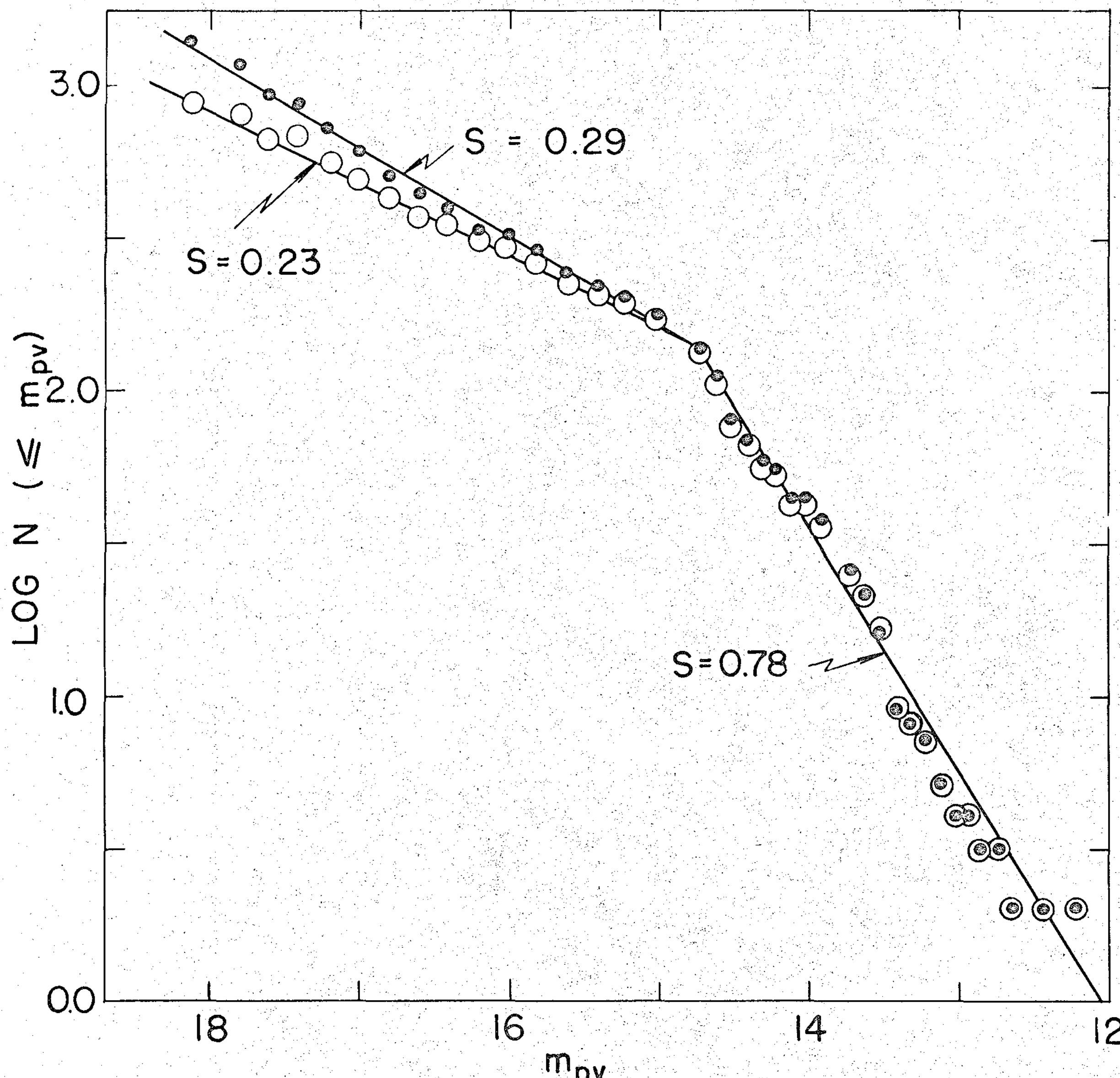
FORM 18-2 (10-48) AP

TABLE ~~II~~ I

$$\Delta m = \frac{5}{2}\left(\log[n_1(n_1+1)] - \log[n_2(n_2+1)]\right)$$

$n =$	1	2	3	4	5	6	7	8	9	10
1	0									
2	1.193	0								
3	1.945	0.752	0							
4	2.500	1.307	0.555	0						
5	2.940	1.747	0.995	0.440	0					
6	3.306	2.113	1.360	0.806	0.365	0				
7	3.618	2.425	1.473	1.118	0.678	0.313	0			
8	3.891	2.698	1.946	1.391	0.951	0.585	0.273	0		
9	4.133	2.940	2.188	1.633	1.193	0.828	0.515	0.242	0	
10	4.351	3.158	2.430	1.851	1.411	1.045	0.733	0.460	0.218	0


S = 0.29
S = 0.23
S = 0.78
3.0
2.0
1.0
0.0
LOG N (≤ m_{pv})
18
16
14
12
m_{pv}

Magnitudes at which fine structure bumps occur in the luminosity function of the Coma Cluster are given in Table II. (Abell, loc. cit.) These values were measured at the faint end of each group See Fig I This would correspond to the sharp cut-off and gap on the small side of each eigen branch.

TABLE II

Observed Magnitudes
(17.6) or 17.55
16.8
16.2 (or 16.17)
15.6
15.0
14.7
14.5 (or 14.55)
(14.1) or 14.15
13.7
(13.4) or 13.33
12.6 (or 12.65)

A Δm difference table based on the observed values of Table II is given in Table III.

Comparisons of Tables I and III lead to different sets of sequences (i.e., different values of J). One such set, A is given in Tables IV-A. The observed differences are in black, the theoretical differences in red, the residuals in parenthesis. If the mean of the absolute value of the residuals is constrained to less than 0.1 magnitude only five sets C, E, G, and I' (with A a borderline case) may be considered as fits to the theoretical values. The branch numbers corresponding to the magnitudes are summarized in Table V, together with the corresponding means of the absolute values of the residuals (ϕ called the "index").

Subtracting magnitudes corresponding to the same branch number, as given in Table V and re-converting Δm to Δ log D by equation (3), we obtain:

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TABLE III

Δm's From TABLE II

m_{11} \ $m=$	17.55	16.8	16.2	15.6	15.0	14.7	14.5	14.15	13.7	13.33	12.6
17.55	0										
16.8	0.75	0									
16.2	1.35	0.6	0								
15.6	1.95	1.2	0.6	0							
15.0	2.55	1.8	1.2	0.6	0						
14.7	2.85	2.1	1.5	0.9	0.3	0					
14.5	3.05	2.3	1.7	1.1	0.5	0.2	0				
14.15	3.40	2.65	2.05	1.45	0.85	0.55	0.35	0			
13.7	3.85	3.1	2.5	1.9	1.3	1.00	0.8	0.45	0		
13.33	4.22	3.47	2.87	2.27	1.67	1.37	1.17	0.78	0.37	0	
12.6	5.05	4.3	3.6	3.0	2.4	2.1	1.9	1.55	1.1	0.73	0

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FORM 18-2 (10-48) AP

TABLE IV - A

Residual = Theoretical - Observed

	m =	17.55		16.2		15.6		15.0		14.5		14.15		13.7	13.33
	n =	1		2		3		4		5		6		8	9
1	17.55	0													
		0													
2	16.2	1.35	-0.157	0											
		1.193		0											
3	15.6	1.95	-0.005	0.6	0.152	0									
		1.945		0.752		0									
4	15.0	2.55	-0.050	1.2	0.107	0.6	-0.045	0							
		2.500		1.307		0.555		0							
5	14.5	3.05	-0.110	1.7	0.047	1.1	-0.105	0.5	-0.060	0					
		2.940		1.747		0.995		0.440		0					
6	14.15	3.40	-0.094	2.05	0.063	1.45	-0.090	0.85	-0.044	0.35	+0.015	0			
		3.306		2.113		1.360		0.806		0.365		0			
8	13.7	3.85	+0.041	2.5	0.198	1.9	+0.046	1.3	+0.091	0.8	+0.151	0.45	+0.135	0	
		3.891		2.698		1.946		1.391		0.951		0.585		0	
9	13.33	4.22	-0.087	2.87	0.070	2.27	-0.152	1.67	-0.037	1.17	+0.023	0.82	+0.008	0.37	-0.128 0
		4.133		2.940		2.118		1.633		1.193		0.828		0.242	0

index = 0.083 ÷ 8 = .0104

net = -0.017

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FORM 18-2 (10-48) AP

TABLE V

Set		17.55	16.8	16.2	15.6	15.0	14.7	14.5	14.15	13.7	13.33	12.6		index
														0.0104
A		1		2	3	4		5	6	8	9			
C		3		6	8									0.003
E			2				6	8		10				0.009
G				1		2				4	5	7		0.007
I'							3		4	5	6	~~8~~		0.002

TABLE VI

	A	C	E	I'	G
A	0				
C	0.820	0			
E	0.880	0.60	0		
I'	1.428	1.144	0.548	0	
G	1.592	1.308	0.720	0.164	0

The group generated by the 3-4-6 ratios (see Virgo Cluster Data) has the two basic elements

$$S = .111$$
$$T = .272$$

In terms of these bases,

$$\{A\} - \{C\} = 0.820 = 3T + .004$$
$$\{C\} - \{E\} = 0.600 = T + 3S - .005 = 7S - T$$
$$\{E\} - \{I'\} = 0.548 = 2T + .004$$
$$\{I'\} - \{G\} = 0.164 = T - S + .003$$

The intervals between sets (the ΔJ's) in excellent agreement with those in the Virgo Cluster.

Sets A, E, and G may span the break at m = 14.7. Sets C and I' appear to be divided by the break. The relative populations of each set determine the slope on both sides of the break.

M E M O

TO:

M. H. Davis
D. Deirmendjian
D. Edelen
S. M. Genensky
S. M. Greenfield
O. Gross
T. E. Harris
W. W. Kellogg
R. Kirkwood
G. E. Kocher
C. Lindholm
A. Mandansky
R. R. Rapp
G. F. Schilling
D. S. Scott
R. Specht
E. H. Vestine
J. D. Williams

G. Abell, UCLA
H. Bramsom, Hughes

* * * * * * * * * * *

LARGE SCALE DISCRETIZATION PHENOMENA

Al Wilson

Wednesday, 6 November 1963, 10:30 A.M.

Planetary Sciences Conference Room

For further information or questions, please call Monta Klappert, X 7246.

Original Abstract

Sent Nov. 6, 1963
Paper for AAS

DISCRETIZATION IN EO FIELD GALAXIES

Albert G. Wilson
The RAND Corporation
1700 Main Street, Santa Monica, California 90406

Diameters of all galaxies identified as EO's in de Vaucouleurs' *Reference Catalogue of Bright Galaxies* fall into four sets, each of which exhibit the Edelen eigen-sequence, $[n(n+1)]^{1/2}$, where n is a positive integer. The synoptic red shifts, $u = \frac{c+v}{v}$, are related to the diameters (D) by the relation,

$$[\log D - \frac{1}{2}\log(n(n+1))] = C_i + \alpha \log u$$

where $\alpha = 0.44$ and the four values of C_i satisfy the equation

$$C_i + \frac{1}{3}\log[i(i+1)] = \text{constant, for } i = 6,7,8, \text{ and } 9$$

The sharp linear relations between ~~log D~~ $[\log D - \frac{1}{2}\log n(n+1)]$ and log u imply that any peculiar velocities which might be associated with the red shifts are very small.

6a. Discretization in EO Field Galaxies

Albert G. Wilson
The Rand Corporation, Santa Monica

Diameters of all galaxies identified as EO's in de Vaucouleurs' Reference Catalogue of Bright Galaxies fall into four sets, each of which exhibits the Edelen eigen-sequence, $[n(n+1)]^{\frac{1}{2}}$, where n is a positive integer. The synoptic red shifts, $u = \frac{ctv}{v}$, are related to the diameters (D) by the relation,

$$[\log D - \tfrac{1}{2} \log(n(n+1))] \doteq C_i + \alpha \log u$$

where $\alpha = 0.44$ and the four values of C_i satisfy the equation

$$C_i + \frac{1}{3} \log [i(i+1)] = \text{constant, for } i = 6,7,8, \text{ and } 9$$

The sharp linear relations between $[\log D - \tfrac{1}{2}\log n(n+1)]$ and $\log u$ imply that any peculiar velocities which might be associated with the red shifts are very small.

7a. Further Measurements of [illegible] Sources at 8000 Mc/s

F. T. Haddock and R. W. [illegible]
Radio Astronomy Observatory

Plane-polarized emission of [illegible] at 8000 Mc/s, using a 1000-Mc/s [illegible] and an 85-ft telescope. On clear [illegible] is typically 0.01 °K r.m.s. with [illegible] IBM 7090 processed data.

The flux density and posit[illegible] ponent of the following sources ha[illegible] 3C111, 3C123, 3C144, 3C196, 3C218, [illegible] 3C295, 3C345, 3C348, 3C353, 3C380, [illegible]

All 16 radio galaxies and [illegible] definite polarization.

A G Wilson

90406

31 December 1963 L-25828

Editor
Astronomical Journal
Box 2023 Yale Station
New Haven, Connecticut

Dear Sir:

Enclosed are two copies of A. G. Wilson's "Discretization in EO Field Galaxies." This Abstract should replace the pre-print version. The full paper was presented at the American Astronomical Society meeting in Washington, D. C., 27 December 1963.

Sincerely,

D. H. Scott, Assistant Editor
Planetary Sciences Department

DHS:be

Enclosure
A-745

bcc: Reports

November 13, 1964

Abstract No. 848

ON SUPER-ORGANIZATIONS AMONG CLUSTERS OF GALAXIES

A. G. Wilson

The mean redshifts of clusters of galaxies do not appear to be distributed randomly, but rather show a tendency to be distributed in accordance with functionally related discrete values (Wilson, A. G., Proc. Nat. Acad. Sci. Vol. 52, 847, 1964). This may be interpreted as implying that clusters are located on a set of shells which possess a definite relation between successive radii. The cosmological principle requires that all equivalent observers (observers located at clusters) should view the clusters as similarly distributed. (For present purposes we may be associated with the Virgo Cluster.) Structured radial distribution of clusters observed by equivalent observers requires structured angular distribution of clusters observed by the equivalent observers. Hence if the regularity in radial distribution of clusters is real, angular structure in the distribution of clusters should also be in evidence.

The large numbers of clusters observed in all unobscured directions in the sky renders statistically meaningless any patterns selected ab initio on the basis of angular distribution criteria. This difficulty may be avoided by invoking an independent selectivity factor. A study was made of the clusters in Abell's catalogue (Abell, G. O., Ap. J. Suppl. 3, No. 31, 1958) selected on the basis of membership in the richest classes (4 and 5). Though widely separated, these clusters have angular positions consistent with structured rather than random distribution (the details to be published elsewhere). In addition, the same distribution properties observed for the richest clusters obtain in the subset of the rich nearby clusters. These non-random angular distribution patterns lend confirmation to the hypothesis of the existence of some sort of super-organization of which the clusters of galaxies are members.

In view of the same difficulties which arise in explaining super or second order clusters as dynamic systems (Zwicky, F. Pub. Ast. Soc. Pac. 69, 518, 1957), it is completely unsupportable to postulate the existence of a dynamic system with a diameter of the order of 10^9 parsecs, the value consistent with the distances and angular separation of the clusters involved. Consequently, if the apparent super-organizations to which these clusters belong are real, they must originate through physical communication processes other than those presently recognized.

AG Wilson

DISCRETIZATION IN EO FIELD GALAXIES

A. G. Wilson

December 1963

DISCRETIZATION IN EO FIELD GALAXIES*

A. G. Wilson

The RAND Corporation, Santa Monica, California

The Edelen discretization prediction (D. G. B. Edelen, A. J., 68, 535, 1963) states that the linear diameters S_n of EO galaxies are given by a relation of the form

$$S_n = S_o \xi^{-1/2} [n(n + 1)]^{+1/2} ,$$

where S_o is a constant and ξ is an energy parameter with n = 1, 2, 3.... This prediction is tested against the sample of all EO galaxies for which diameters and redshifts are given in G. and A. de Vaucouleurs' Reference Catalogue of Bright Galaxies (University of Texas Press, in press). The sample, consisting of 31 galaxies divides itself into five different "ξ classes." Within each class (ξ being constant) a relationship of the type

$$(1) \qquad 1 + \log D_n - \frac{4}{9} \log u - \frac{1}{2} \log n\,(n + 1) = C_\nu$$

is observed. In Eq. (1), D_n is the catalogue angular diameter, $u = \frac{1 + Z}{Z}$ is the synoptic redshift of the galaxy whose diameter is D_n, and C_ν is a constant for all members of the class. The two largest classes have 10 and 11 members with the mean spread (difference between the minimum and maximum values of C_ν) in each class being

*This research was sponsored by The RAND Corporation.

0.02. Attempts to generate similar fits based on random numbers to within a spread of 0.02 failed in 100 tests on the RAND 7090 computer.

The close fit to the Edelen eigensequence suggests that Eq. (1) determines the principal component of the redshifts. Residual components representing peculiar velocities are small as deduced by G. de Vaucouleurs (A. J., 63, 253, 1958) and Neymann and Scott (A. J., 66, 148, 1961).

It is further found that the constants C_ν belonging to each class are related by an equation of the form

$$C_\nu + \frac{1}{3} \log \nu \ (\nu + 1) = \text{a constant.} \qquad (2)$$

The mean color of the galaxies of each class is correlated with the class index ν, suggesting that the parameter ν represents an evolutionary or age parameter. If this interpretation is correct, the cube-root discretization relation of Eq. (2) allows the concept of discrete ages and, hence, of discrete epochs of creation for EO galaxies. Enlarged samples should affirm or deny this interpretation's consistency and also its applicability to other morphological types.

To Stan for annual report

Galactic Scale Discretization--Summary December 6, 1963

The theoretically predicted relation between sizes of E0 galaxies was checked against samples of both cluster and field galaxies and found to be consistent with observations. The relation, then assumed to be valid, was used to calibrate scales and dimensionality of relations discovered to exist between apparent angular diameters and red shifts. The following empirical results were obtained:

1) Discovery of two additional discretization parameters, one governing red shifts and distance, the other governing time.

2) Observational evidence of a Machian relation between structural and cosmological parameters.

3) Observational evidence of the cosmological principle.

These results lead to an entirely new non_homogeneous ~~cosmology~~ universe differing from both the evolutionary (big bang) and steady state models. The new discretized ~~cosmology~~ universe approaches the steady state model in the limit.

A.G.Wilson

90406

13 December 1963 L-24997

Editor of Nature
Macmillan and Company, Ltd.
St. Martin's Street
London, W. C., 2
ENGLAND

Dear Sir:

Enclosed is a Letter to the Editor, in duplicate, entitled, "A Discretized Cosmology: Theoretical and Empirical," by D. G. B. Edelen and A. G. Wilson, which is submitted for your consideration for publication in Nature. The authors would greatly prefer that this be printed as a single, co-signed letter. If this should prove impossible, your editors can create two, individually signed letters, as follows:

1) The first letter (signed by D. G. B. Edelen) should include all material through line 3 of page 7; it should then conclude with the last paragraph on page 14, so rewritten as to begin, "It can be shown that the observational cosmology summarized in the following letter has a theoretical...."

2) The second letter (signed by A. G. Wilson) should include all the remaining material, beginning "Of primary interest..."; eliminate the last paragraph beginning on page 14, and conclude with the final paragraph of page 15.

Sincerely,

D. H. Scott, Assistant Editor
Planetary Sciences Department

DHS: be
bcc: Reports

A DISCRETIZED COSMOLOGY:

THEORETICAL AND EMPIRICAL

D. G. B. Edelen and A. G. Wilson

December 1963

Letter to Nature: Sent Dec. 13, 1963
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A DISCRETIZED COSMOLOGY: THEORETICAL AND EMPIRICAL*

D. G. B. Edelen and A. G. Wilson

The RAND Corporation,* Santa Monica, California

A theoretical prediction and its observational confirmation are reported here. The theoretical prediction derives from and provides a new and possibly fundamental means of testing the general theory of relativity. We have used the term "general theory" advisedly -- the predictions being a direct consequence of the fundamental equivalence between physics and geometry and having no Newtonian analog. Validating these predictions thus provides more than just a verification of the geodesic hypothesis, the Schwartzschield line element, or the principle of equivalence, each of which is explained by a number of theories without the explicit equivalence of geometry and physics.

*This research was sponsored by The RAND Corporation.

Let Σ denote the surface of a world tube in an Einstein-Riemann space (i.e., a time-like hypersurface whose space sections are closed). The physical models considered are those for which the region interior to Σ is occupied by the world lines of a galactic structure, and for which the region exterior to Σ is the background field of "free space." Such models would be described mathematically by a statement of the momentum-energy tensor T_{AB} appropriate to the dynamical processes interior and exterior to such galactic structures. Unfortunately, without adequate detailed information on the dynamical processes of galaxies, we cannot specify the momentum-energy tensor with any acceptable degree of certainty. We therefore adopt the viewpoint that although appropriate functions T_{AB} exist for any particular galaxy under consideration, we know only that their covariant divergence vanishes, as required by the Einstein theory. Since Σ is to represent the effective boundary of a galactic structure, we consider those models for which at least one component of the momentum-energy tensor has a jump across Σ. This requirement simply states that there is some physical quantity (such as density) that has a jump across Σ; hence, for the models we are considering here, it is physically as well as mathematically meaningful to speak of an interior and an exterior of a galaxy.

We have selected models that represent macroscopic behavior, since observations of galaxies show composite or averaged data, rather than resolved images of individual stars. On the other hand, the Einstein field equations relate local differential geometric quantities and local momentum-energy complexes. If we denote the local momentum-energy tensor by $\underline{t}$ and the corresponding metric tensor by $\underline{g}$, an invariant averaging process leads to Einstein equations formed on the averaged metric tensor $\underline{h}$ and an associated momentum-energy tensor $\underline{T}$; the latter is the sum of the averaged $\underline{t}$ and a tensor that may be interpreted as the momentum-energy required to equilibrate the local fluctuations. It also follows that if the averaged $\underline{t}$ exhibits oblate spheroidal symmetry, the resulting $\underline{T}$ will generally exhibit deviations from oblate spheroidal symmetry. These deviations are assumed to be small and spatial. In retrospect, we may view the local fluctuations as arising from the galaxies' local inhomogeneities--their granular structure.

We assume over a period of time that most galaxies will have persistent geometric shapes. Their boundaries, we therefore assume, are geometrically stable. Mathematically, this means that there is an irrotational isometry in the bounding surface Σ--namely, that the boundary seen in three-space at one time is taken into an identical metric replica of itself at a later time. This requirement, together with the conditions necessary for solving the Einstein field equations, leads to a system of linear differential and algebraic equations for determining the jump strengths of the momentum-energy tensor $\underline{T}$. These equations show that the magnitude $e^{-\varphi}$, of the vector field generating the irrotational isometry gives a measure of the local fluctuations of $\underline{T}$, and that φ must be a single-valued bounded solution of the Klein-Gordon equation on Σ.

One should clearly note that the function φ is here defined on a time-like surface, unlike the space-like surfaces encountered in the Cauchy problem. This intrinsic difference is related to the occurrence of a discrete rather than a continuous spectrum. Furthermore, since $\varpi = 0$ in Newtonian mechanics (i.e., the time orientation is absolute, the magnitude of the proper time vector being uniformly one everywhere), a nonvanishing ϖ has no direct Newtonian analog.

If we assume that the eccentricity ϵ of a galaxy is approximately constant for the time interval of interest, the above assumptions imply the existence of a class C of models of quasi-oblate spheroidal galactic structures, for which the effective lengths r of the semi-major axes are given by

$$\xi^{1/2} r = \rho(n, m, \epsilon) \quad . \tag{1}$$

The explicit circumstances for this result are, however, rather general. All that is basically required is that a galactic structure have a physically realizable and geometrically stable boundary and that there be deviations from oblate spheroidal symmetry that are both time-independent in the convected coordinate system of R(t) and small. There are, of course, other assumptions, but they are primarily mathematical, since we have disclaimed knowledge of both the momentum-energy tensor and the metric tensor.

Let us first note that, owing to the curvature of space-time, the effective length of the semi-major axis will not be equal to the corresponding metric length calculated from the metric properties of the space. The Einstein field is known to be weak, however, numerical deviations from Newtonian calculations being small. Further, we optically observe the projective geometry of a quantity, not its actual metric geometry. Thus, since the gravitational lens effect for sources on the periphery of galaxies is extremely small (if not altogether nonexistent to the observer, who cannot perceive individual stellar images in distant galaxies), and since the theoretical r is what would appear under projection, we assume that the theoretical r and an observational r are numerically equivalent. In addition, although the quantity ξ is constant on the bounding surface of any one galaxy to within the approximations considered, it could vary from galaxy to galaxy. It could, and probably does, depend on the age of the galaxy as well as on the force fields present during the formation of the galaxy. Equation (1) shows that if ξ varies in a continuous fashion from galaxy to galaxy, so does r. We now make a conjecture that can be validated only by direct observation: We can divide the class C of galactic structures into a finite number of subclasses $C(\nu)$, $\nu = 1,2,3...$, each of which has more than one element and a unique value of ξ. Combining Eq. (1) with this conjecture, we are led to the following prediction: <u>The lengths of the semi-major axis of the quasi-oblately spheroidal galaxies of class $C(\nu)$ are given by $\ell(\nu)\rho(n,m,\varepsilon)$ where $\ell(\nu)$ is a unique constant depending only on $C(\nu)$.</u>

What does the number m signify? If $m = 0$, the vector U defines a circulation on $\Sigma(t)$, whose stream lines are parallel to the equatorial plane. If $m \neq 0$, on the other hand, the vector U defines a rotation combined with a polar flow. Since the distinction between pure rotation and one with an accompanying polar flow is fundamental, and since the usual equilibrium models of elliptical galaxies assume circulation without polar flow, <u>we conjecture that the case</u> $m = 0$

corresponds to $C(\nu)$ galaxies in the E-Series, while the cases $m \neq 0$ correspond to nonelliptical $C(\nu)$ galaxies.

The first fact to be noted is that as ϵ goes to zero, we have

$$\rho(n, m, 0)^2 = n(n + 1), \quad 0 \leq m \leq n \quad , \tag{2}$$

In addition, all branches of $\rho(n, m, \epsilon)$ have horizontal tangents at $\epsilon = 0$. Thus,

$$r = \xi^{-1/2}\sqrt{n(n+1)} \quad , \tag{3}$$

for small values of ϵ_1 as well as for $\epsilon = 0$.

With observational confirmation of Eq. (3), we can determine the actual eccentricity ϵ and inclination angle, i, of the galactic axis by the well known formula

$$\epsilon_0 \cong \epsilon \sin i \quad , \tag{4}$$

since ϵ_0, the observed eccentricity, is known and since ϵ is determined by a projection of a data point of constant r onto the appropriate curve $\rho(n, 0, \epsilon)$.

Our primary concern has been with the discretization sequence for E_0 galaxies -- the eigenvalues of a certain elliptic partial differential operator. There are, however, equally interesting and significant predictions from the eigenfunctions themselves. With a consistent theoretical interpretation of the eigenfunctions as generators of a conformal coordinate transformation, we can calculate the resulting isophotal contours of ellipticals. The geometric morphology that is thereby predicted from $m = 0$ corresponds to observed morphology with surprising accuracy.[1] Although the mathematical and interpretative problems are significantly compounded in the case of a nonvanishing m, preliminary results reproduce certain of the morphological properties of spiral galaxies.

But of primary interest is the limiting form of Eq. (1) given by Eq. (3) which is readily amenable to observational tests. To observationally test the discretization prediction of Eq. (3), two samples of E0 galaxies were employed. The first sample was measured on a set of homogeneous plates taken with the 200-inch telescope of the Coma, Ursa Major I, Corona Borealis, Boötes, Ursa Major II, and Coma B Clusters. The diameters of galaxies identified as E0 on this set of plates were measured by several methods including direct micrometric measurements, isophotometric tracings, luminosity profile tracings, and calibrated adaptations of a stellar astrophotometer. A second set of diameter data was provided by the sample of "log D" entries of nearby E0 galaxies in de Vaucouleurs' Reference Catalogue of Bright Galaxies.[2]

The catalogue sample identified only 31 galaxies as being E0 and at the same time possessing observed redshifts, while 32 such galaxies were obtained in the cluster samples. Morphologically typing galaxies in the more distant clusters is uncertain.

By comparing the diameters of the galaxies in our two samples with the theoretically predicted values from Eq. (3), we found, with five exceptions, a fit to within the observational error of 3%.

Tests made on random-number samples indicate a small probability of accidental fit to the eigen-sequence.

Comparison between observed and predicted values in the clusters presents no difficulties, since cluster members may be presumed to lie at approximately the same distance and since ratios of observed angular diameters may be substituted for the ratios of the linear diameters as required in Eq. (3). The conjecture that there exists a small number of different classes $C(\nu)$ to which all E0 galaxies belong is seen to be verified in Table I, in which the ν's were determined from size residuals. The results of all the cluster data are summarized in the empirical equation:

$$(5) \qquad \log\theta(n,\nu) - \frac{2}{3}\log\bar{u} = C_o + \frac{1}{2}\log n(n+1) - \frac{1}{3}\log\nu(\nu+1) \qquad ,$$

where θ is the observed angular diameter, $\bar{\mu}$ is the mean synoptic redshift of the cluster (synoptic redshift is defined as $u = \frac{1+z}{z}$ and $\bar{u} = \frac{1+\bar{z}}{\bar{z}}$ where $z = \frac{\delta\lambda}{\lambda}$), C_o is a constant with the numerical value 1.067, n is the integral parameter of Eq. (3), and ν is an integral parameter which characterizes the different classes $C(\nu)$. Logarithms are all to base 10. Equation (5), applies not only because to every θ there corresponds a pair of small integers n and ν which satisfy Eq. (5) in each of the six clusters, but also because, for fixed ν, the numerical value of $(\log\theta(n,\nu) - \frac{1}{2}\log n(n+1) - \frac{2}{3}\log\bar{u})$ is the same from cluster to cluster.

Comparing the angular diameters of the bright galaxies given in the de Vaucouleurs Catalogue is complicated by the spread in relative distance, which disallows the substitution of angular diameters for linear diameters. An empirical relationship of discretization found to hold for the nearby galaxies is

Table I
CLUSTERS

Cluster	ν	A_o	$-\log \bar{u}$	$A_o-2/3\log\bar{u}$	$A_o-2/3\log\bar{u}+1/3\log\nu(\nu+1)$
Coma			(1.594)	(1.063)	
	3	1.768	0.174	0.705	1.065
	4	1.694	0.100	0.631	1.065
	5	1.640	0.046	0.577	1.069
U.M. I			(1.315)	(0.877)	
	5	1.454	0.139	0.577	1.069
Cor Bor			(1.172)	(0.781)	
	4	1.416	0.244	0.635	1.069
	5	1.358	0.186	0.577	1.069
	6	1.305	0.133	0.524	1.065
	7	1.267	0.095	0.486	1.069
Boötes			(0.936)	(0.624)	
	4	1.262	0.326	0.638	1.072
	9	1.040	0.104	0.416	1.067
U.M. II			(0.921)	(0.614)	
	6	1.143	0.222	0.529	1.070
	9	1.026	0.105	0.412	1.063
			0.810	0.540	
(Coma B)			(0.810)	(0.540)	
	6	1.064	0.254	0.524	1.065
	9	0.956	0.146	0.416	1.067
					mean 1.0674

$$\log \theta_n - 2/3 \log \bar{u} = C_o + 1/2 \log n(n+1) - 1/3 \log \nu(\nu+1)$$

$$A_o = \log \theta_n - 1/2 \log n(n+1)$$

$$C_o = \log(2 \cot \gamma) = \log (35/3) = 1.0669$$

(6) $$\log\theta(n,\nu) - \frac{4}{9}\log u = \text{constant} + \frac{1}{2}\log n(n+1) - \frac{1}{3}\log \nu(\nu+1)$$

The quantities of Eq. (6) are the same as those of Eq. (5), except that u in Eq. (6) is the individual redshift of the galaxy whose angular diameter is $\theta(n,\nu)$. All 31 galaxies of the sample fit the relation of Eq. (6) to within 3%. The values of ν identified in the nearby galaxies were 6, 8, and 9; in the clusters, values of ν ranged from 3 to 9. The three values of ν in the bright galaxies were found to be correlated with colors of individual galaxies, so that the set $\nu = 9$ corresponds to a mean color of 0.86, $\nu = 8$ to a mean color of 0.91, and $\nu = 6$ to a mean color of 0.76 (colors from [2]). In terms of qualitatively interpreted, evolutionary H-R star tracks, this would indicate that the ν parameter may characterize the ages of galaxies, with $\nu = 9$ being the index of a recent age or epoch of creation, and $\nu = 8$ and $\nu = 6$ the indices of progressively earlier epochs. From the evolutionary tracks of a population two sample, we qualitatively would expect first a mean reddening of the galaxy as stars move into the giant branches, and finally a drift of the mean color to the blue as the bulk of stars pass to the left of the zero-age main sequence. This pattern is manifested by the mean colors of sets 9, 8, and 6. Tentatively, then, the parameter ν is associated with age; and, since it is discretized, it may be identified with discrete epochs of creation.

Together, Hubble's law and the 2/3 relation between angular diameter and redshift imply the existence of yet another discretization parameter. Such a necessary discretization is found in the observed mean redshifts for clusters (or for groups of clusters). These redshifts may be approximated by the relation:

$$3C_o - \log N(N+1) = Q_N \quad , \tag{7}$$

where N is a positive integer. Table II gives the observed means of redshifts, $\log \bar{u}$ or $\overline{\log \bar{u}}$, the calculated quantities Q_N, and the residuals, for the clusters whose redshifts are reported in the Humason-Mayall-Sandage catalogue.[3] The constant C_o is the same as that of Eq. (5).

In the Virgo Cluster, where the residual is the smallest, the value of $\log \bar{u}$ is derived from 73 individual redshifts. The values for other clusters are presumably subject to an observational bias which results from the selection of brighter and generally closer galaxies of the cluster. An exception to this is the mean value of 50 observed redshifts in the Coma Cluster, which is poorly fitted by Eq. (7). A sample of Coma galaxies analogous to the sample of observed redshifts in the more distant clusters, which consist of the few brightest objects, is the brightest seven galaxies in Coma with observed redshifts having values >6000 km/sec (to assure they are not foreground objects). This sample fits the calculated value with the same order of discrepancy found for other, more distant clusters.

A tentative interpretation of the foregoing empirical results summarized in Tables I and II leads to the postulation of a universe that is both spatially and temporally discretized. This universe, based purely on observations, is hierarchically arranged. The clusters designated by Hubble as "great clusters" such as Virgo, Coma, and Corona Borealis seem to be located at or near "nodal shells," whose radii are discretized according to an empirical relation with the form of Eq. (7). The mean redshifts of some of the smaller clusters also seem to well approximate nodal shells in the N(N+1) sequence.

Table II

NODAL DISCRETIZATION

Nodal Velocities: $Q_N = 3C_o - \log N(N+1)$

Cluster	log $\bar{u}$	N	Q_N	Q_N-logu
Virgo (73)	2.4234	2	2.4225	-0.0009
Perseus	1.7499	5	1.7236	-0.0263
Coma (50)	1.6780	6	1.5774	-0.1006
Coma (7)	1.5942	6	1.5774	-0.0168
Hercules	1.4749	7	1.4524	-0.0225
Pegasus II	1.3874			
Cl 2322+1425	1.3757			
U.M.I	1.3149			
Haufen A	1.3012			
Mean of (4)	1.3448	8	1.3433	-0.0015
Leo	1.2147	9	1.2464	0.0317
Cl 1239+1852	1.1741			
Cor Bor	1.1719			
Gemini I	1.1360			
Mean of (3)	1.1607	10	1.1593	0.0014
Cl 1513+0433	1.06401	11	1.0801	-0.0161
Boötes	0.9356			
U.M. II	0.9213			
Mean of (2)	.9285	13	0.9406	0.0121

Cluster	log u	N	Q_N	Q_N-logu
Cl 1153+2341	0.9037			
Cl 1534+3749	0.8767			
Cl 0025+2223	0.8616			
Cl 1228+1050	0.8487			
mean of (4)	0.8727	14	0.8785	-0.0058
Cl 0138+1840	0.8312			
Cl 1309+0105	0.8280			
Coma B	0.8104			
Mean of (3)	0.8232	15	0.8205	-0.0027
Cl 0925+2044	0.7937			
Cl 1253+4422	0.7819			
Hydra	0.7730			
mean of (3)	0.7828	16	0.7661	0.0167

The clusters themselves, in turn, may be regarded as consisting of sets of sub-clusters designated by the ν index, which appears to be related to age or to a temporally discretized set of epochs of creation belonging to a $\sqrt[3]{\nu(\nu+1)}$ type sequence. If this interpretation is correct, the interval between successive "ν-Epochs" is changing as $\sqrt[3]{\frac{\nu}{\nu+2}}$, and the discretized universe would approach in the limit a universe described by a steady-state cosmological model.

The diameters of EO galaxies belonging to a sub-cluster are themselves discretized according to a $\sqrt{n(n+1)}$ relation which is <u>theoretically</u> predicted by the modified general theory.

If we consistently interpret the redshifts as being velocity shifts and angular diameter as being an indicator of distance, each system seems to be expanding according to a different law: The nodal system is expanding in accordance with the Hubble relation; the sub-clusters of the same ν-index are expanding relative to one another in accordance with a 2/3 law, and the members of a sub-cluster are expanding in accord with a $(2/3)^2$ law.

The constant C_o, appearing independently in Eqs. (5) and (7), which is derivable from diameter and redshift measurements and from redshifts alone, may be a new universal constant.

The observed part of the universe thus seems to agree less with either an evolutionary (single-explosion) or a continuous-creation model than with one that superposes several epochs of creation. Speculatively, we may surmise that matter comes into being at discrete epochs in discrete, roughly equally-spaced locations, termed nodes. The redshifts to these nodes are discretized by an N(N+1) relation, while the distances may be such as to assure conformity with the Cosmological Principle.

The parameter ξ, which appears in the original theoretical discretization relation, seems to be associated with the epochal parameter ν as follows

$$(8) \qquad -\frac{1}{2}\log\xi = \text{const} + \frac{1}{3}\log\nu(\nu+1) \quad .$$

Though other parameters may later prove to be involved, Eq. (8) suggests that the energy-jump parameter is temporal and that an epoch of creation occurs when the energy-jump condition reaches an integral value of ν satisfying Eq. (8).

That clusters are composed of sub-clusters is also indicated by the differences in colors and spreads in redshifts found within clusters. It is surmised that the spreads in cluster redshifts will prove explicable by cluster and sub-cluster expansion, rather than by dynamical models similar to those of star clusters. In fact, the very low scatter observed locally, when the expansion component has been accounted for, indicates that redshifts attributable to other dynamic causes are very small. (The exception to this is in compact groups and in double galaxies, where the differences in redshifts are probably dominated by the dynamical relationships resulting from physical proximity.)

It can be shown that the observational cosmology summarized here has a theoretical counterpart consistent with both relativity theory and observation. As is well known, both the steady-state and the evolutionary cosmologies assume that which has not been observed -- uniformly smeared distributions. The cosmology reported here, however, assumes a significant spatial inhomogeneity around nodes, and that it is from this inhomogeneity that arise the observables, i.e., galaxies and clusters of galaxies. As perturbations on the classical cosmology or as a sequence of bumps on a uniform field, then, the basic observational data can be shown at least phenomenologically to be consistent and theoretically predictable.

The importance of E0 galaxies as instruments of cosmological exploration derives from the simplicity of their discretization relation. While the observational results reported here were begun in order to test Eq. (3), the additional results discovered as by-products of the test are believed, because of interlocking paradigmatic inferences, to have a high level of confidence far exceeding that which in any way could be justified as statistical inference from a sample of 60 pairs of measurements. Accordingly, we feel that this picture of the universe should be placed alongside other current cosmologies for serious consideration and further observational testing.

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a Wilson

DISCRETIZATION PHENOMENA

IN SYNOPTIC REDSHIFTS

A. G. Wilson

December 1963

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DISCRETIZATION PHENOMENA IN SYNOPTIC REDSHIFTS*

A. G. Wilson

The RAND Corporation, Santa Monica, California

(Received 30 December 1963)

From the basic Einstein equations relating the geometric and physical tensors $\left(R_{AB} - \frac{1}{2} Rg_{AB} = \varkappa T_{AB} \right)$, Edelen (1) (2) has derived a discretization prediction concerning the geometries of galaxies, by introducing an averaging operator which leaves all tensor structure invariant. This theoretical prediction is a necessary but not sufficient condition of the general relativistic equations and some very general assumptions concerning galaxies (1) (2). For EO and near EO galaxies the prediction assumes the simple form

$$(1) \qquad S_n = S_o \, \xi^{-1/2} \, [n(n + 1)]^{1/2} \quad ,$$

where S_n is the linear diameter or semi-major axis of the elliptical galaxy, S_o is a constant, ξ is a parameter representing the jump in energy density across the surface of the galaxy, and n is a positive integer.

Under the assumption that ξ is a constant, Eq. (1) was observationally tested (3) using diameters of sample EO galaxies measured on 200-inch telescope plates of six distant clusters (marked with an

*This research was sponsored by The RAND Corporation.

asterisk in Table I). The results of the test may be considered as a tentative confirmation of Eq. (1), in that the fit was within the observational uncertainties, although the size of the sample (42 galaxies) was too small for statistical confidence. It was found, however, that instead of being a single constant, ξ took on six different values. From this, followed the discovery that in terms of $\tilde{\theta}$ (defined below) an empirical relation of the form

$$\log \tilde{\theta} - \frac{2}{3} \log \bar{u} = C_o + \frac{1}{3} \log \nu(\nu + 1) \tag{2}$$

obtained between the angular diameters and the redshifts of those galaxies in the tested sample (4). Equation (2) was derived by noting that the galaxies are distributed into sets or sub-clusters which satisfy Eq. (1) for a fixed value of ξ. That is to say, for each galaxy belonging to a set, a positive integer n corresponds to each observed angular diameter θ_n, such that a contracted diameter $\tilde{\theta}$ exists with

$$\log \tilde{\theta} = \log \theta_n - \frac{1}{2} \log n(n + 1) = \text{a constant} \quad .$$

In comparing the sets within a cluster, the logarithms of the parameters $\tilde{\theta}$ that characterize the set were found to be similarly related themselves through a new discretization sequence of the form $\frac{1}{3} \log \nu (\nu + 1)$, with ν a positive integer. Writing the $\tilde{\theta}$ with appropriate subscripts ν, one could then characterize the cluster itself by a new parameter θ^*, with

$$\log \theta^* = \log \tilde{\theta}_\nu + \frac{1}{3} \log \nu(\nu + 1) = \text{a constant} \quad .$$

Finally all the $\log \theta^*$ of the different clusters were found to be related through their mean synoptic redshifts, $\bar{u}$ to a single constant

C_o whose value seems close to (log 35 - log 3) = 1.0669, i.e.,

$$\log \theta^* - \frac{2}{3} \log \bar{u} = C_o \quad .$$

The synoptic redshift used here is defined by $u = \frac{v+c}{v}$, where $v = c\frac{\delta\lambda}{\lambda}$, c being the velocity of light and $\frac{\delta\lambda}{\lambda}$ the observed wavelength displacement ratio. The mean synoptic redshift $\bar{u}$ of a cluster equals $\frac{\bar{v}+c}{\bar{v}}$, where $\bar{v}$ is the arithmetic mean of the individual redshift of the galaxies belonging to the cluster. The operations in defining the discretization Eq. (2) may be performed in any order. That is to say, if all the log $\tilde{\theta}_\nu$ of the individual sub-clusters in each cluster are reduced by the appropriate 2/3 log $\bar{u}$ for the cluster, the values of (log $\tilde{\theta}$ - 2/3 log $\bar{u}$) for the same ν are the same from cluster to cluster.

Equation (2) asserts that the quantities (log $\tilde{\theta}$ - 2/3 log $\bar{u}$), characterizing the sub-clusters, may assume only those values determined by an integer ν; Fig. 1 graphically represents (log $\bar{u}$) versus (log $\tilde{\theta}$ - 2/3 log $\bar{u}$); the sub-clusters will occur only when their characteristic parameters lie on the discrete vertical lines corresponding to integer ν's with values given by $[C_o - 1/3 \log \nu(\nu+1)]$. But sub-clusters and clusters are subject not only to Eq. (2), but also to Hubble's Law, which in terms of contracted angular diameters and mean synoptic redshifts has the forms

$$\log \theta^* - \log \bar{u} = \{H^*\} \text{ for clusters}$$

(3) or

$$\log \tilde{\theta} - \log \bar{u} = \{\tilde{H}\} \text{ for sub-clusters,}$$

where $\{H^*\}$ and $\{\tilde{H}\}$ are constants or, rather, sets of allowable constants.

In Fig. 1, Eq. (3) defines a family of "Hubble lines" (drawn dotted) having slopes of 3: 1 and intercepts determined by the set $\{H^*\}$. The validity of Hubble's Law as established by observation requires that clusters be located along the Hubble lines. It follows that if Eqs. (2) and (3) are both true, sub-clusters and clusters must then lie at the intersections of υ-lines with Hubble lines. This can be tested if we are able to derive the allowable values for the set of constants $\{H^*\}$ or $\{\tilde{H}\}$. Since mean redshifts for clusters (but not for sub-clusters) are known, we shall here base our test on clusters; this restriction is assumed throughout the remainder of the paper.

From the simultaneous validity of Eqs. (2) and (3), we may infer the existence of yet another discretization. As an illustration, if it is known that an orchard is planted in columns (υ-lines) and observed to be lined up along diagonals (Hubble-lines), then one may infer that it is also planted in rows. The discretization implied is in <u>the redshifts themselves</u>.

An examination of the means of the mean synoptic redshifts of the clusters reported in the Humason-Mayall-Sandage catalogue (5) indeed confirms the existence of the inferred sequence. The means of the cluster log $\bar{u}$'s (written $\overline{\log \bar{u}}$) are found to be grouped in a way suggesting that the clusters may be located near the surfaces of a set of concentric spherical shells or nodes, so that

$$(4) \qquad \overline{\log \bar{u}} = 3C_o - \log N(N + 1) \quad ,$$

where C_o is the same constant that appears in Eq. (2) and N is a positive integer.

Equation (4), which describes the nodal redshifts, gives us the clue for determining the constants $\{H^*\}$ which, in turn, allow us to derive the mean redshifts of the <u>individual</u> clusters. The horizontal nodal lines defined by Eq. (4) complete the grid in the representation of Fig. 1. By noting the intersections at which clusters occur (marked with solid circles), one discovers that Hubble lines containing clusters pass through the intersections of the nodal- and υ-lines that are marked by circumscribed squares in Fig. 1.

This set of intersections or "Hubble points" seems to follow a regular pattern: starting on line $\upsilon = 1$, it includes both the intersection with the top or $3C_o$ line (equivalent to $\upsilon = 3$, $N = 2$), and intersections with $N = 1$ and $N = 2$; it then jumps to $\nu = 2$ for $N = 2, 3,$ and 4; then jumps to $\upsilon = 3$ for $N = 4, 5,$ and 6. All twenty-six clusters whose redshifts are reported in Reference (5) lie at the intersections of one of these eight Hubble lines with a υ-line. A cluster is located by the intersection of a Hubble line with a single υ-line, although the sub-clusters into which the cluster may be decomposed possess several different υ values. The relation between the various υ's of the sub-clusters and the cluster υ is not known at present.

If we designate the Hubble points by $[\nu_o, N_o]$, the corresponding numerical values being $\log \nu_o(\nu_o+1) - \log N_o(N_o+1)$, we should be able to calculate the mean synoptic redshifts for any cluster by the numerical equation:

$$(5) \qquad \log \bar{u}_c = 3C_o - \log \upsilon(\upsilon + 1) + [\upsilon_o, N_o] \quad .$$

With this equation, we have calculated mean synoptic redshifts for all Humason-Mayall-Sandage clusters. It is to be emphasized that the right member of this equation involves only the constant C_o and positive integers! Table I compares the calculated log $\bar{u}_c$ with observed catalogue value, log $\bar{u}_o$.

One must reserve physical interpretations, but one is still struck by the similarity with Ritz-Rydberg and Balmer-type results describing energy levels and wave lengths in atomic spectra in terms of quantum numbers, especially in view of the energy relationships between the parameter ξ of Eq. (1) and the integer ν.

The immediate question is whether the redshift discretization is a property of the spectra or whether it is a cosmological reality. Two factors suggest the latter: first, the functional relationships between the redshifts and the angular diameters that are observed independently of spectral measurements; and second, the residuals between observed and calculated values become very small in clusters like Virgo and Coma, where a great many redshifts of individual galaxies have been averaged, indicating statistical dependence on the individual spectra.

It is observed that most clusters of galaxies have one or two large galaxies located near their centers. This large central galaxy is the easiest, and therefore usually the first, to be observed in a cluster. The low residuals of Table I, even when only one or two galaxies have been observed, suggests that the mean redshift of the cluster is well approximated by the redshift of the large central galaxies. (The residuals become poorer when the central redshift is diluted with redshifts of a small additional sample.)

With Eq. (1) observationally confirmed, Eqs. (1) through (5) rest on observation alone. Unlike many results currently used in observational cosmology, they involve no assumptions concerning a particular cosmological model.

These summarized results are based on too small a sample of original observations to be inductively conclusive. Since, however, the elements of the structure are not only direct observations but also derived relations between the members of the galaxian sample, and since the structure agrees with observations not entering into the original analysis (Table I), we conclude that the validity of the structure surpasses what may be inferred statistically. Many details are undoubtedly in error and subject to refinement. Before comprehensive physical interpretations are advanced, the sample should be enlarged in order to detect exceptions and explore the more detailed nature of the structure.

TABLE I Comparison of observed synoptic redshifts (column 3) with synoptic redshifts calculated from discretization integers (column 6). The observed mean synoptic redshifts are derived from the formulae:

$$\bar{u} = 1 + 1/\bar{z} \quad \text{with} \quad \bar{z} = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{\delta\lambda}{\lambda}\right)_i \text{, where } \left(\frac{\delta\lambda}{\lambda}\right)_i \text{ is the redshift of an individual galaxy}$$

in the cluster. All redshifts are taken from Humason-Mayall-Sandage (5).

Cluster	Number of Red-shifts	Observed $\log u_o$	ν	$[\nu_o, N_o]$	$\log \bar{u}_c$ Calculated	O-C Residual
Virgo	73	2.4234	2	1,1	2.4225	0.0009
Perseus	5	1.7499	7	3,2	1.7534	-0.0035
Coma*	50	1.6780	4	3,4	1.6779	0.0001
Hercules	7	1.4749	10	3,2	1.4603	0.0146
Pegasus II	3	1.3874	4	2,4	1.3769	0.0105
Cl 2322 + 1425	2	1.3757	4	2,4	1.3769	-0.0012
Ursa Major I*	4	1.3149	5	3,5	1.3257	-0.0108
Haufen A	2	1.3012	12	3,2	1.3086	-0.0074
Leo	1	1.2147	5	2,4	1.2008	0.0139
Cl 1239 + 1853	2	1.1741	6	3,5	1.1795	-0.0054
Corona Borealis*	8	1.1719	6	3,5	1.1795	-0.0076
Gemeni	2	1.1360	7	2,3	1.1514	-0.0154
Cl 0348 + 0613	1	1.1038	6	1,2	1.1002	0.0036
Cl 1513 + 0433	1	1.0640	6	2,4	1.0546	0.0094
Boötes*	2	0.9356	10	3,4	0.9375	-0.0019
Ursa Major II*	2	0.0213	7	2,4	0.9296	-0.0083
Cl 1153 + 2341	2	0.9037	7	3,6	0.9083	-0.0046
Cl 1534 + 3749	3	0.8767	14	1,1	0.8785	-0.0018
Cl 0025 + 2223	2	0.8616	11	3,4	0.8583	-0.0033
Cl 1228 + 1050	2	0.8487	9	3,5	0.8485	0.0002
Cl 0138 + 1840	1	0.8312	8	2,4	0.8205	0.0107
Cl 1309 - 0105	1	0.8280	8	2,4	0.8205	0.0075
Coma B*	1	0.8104	8	2,4	0.8205	-0.0101
Cl 0925 + 2044	1	0.7937	12	3,4	0.7858	0.0079
Cl 1253 + 4422	1	0.7819	12	3,4	0.7858	-0.0039
Hydra	3	0.7730	11	2,3	0.7791	-0.0061

$$\text{Calculated } \log \bar{u}_c = 3C_o - \log \nu(\nu+1) + [\nu_o, N_o]$$

$$\text{where } [\nu_o, N_o] = \log \nu_o(\nu_o+1) - \log N_o(N_o+1)$$

$$\text{and } 3C_o = 3.2007$$

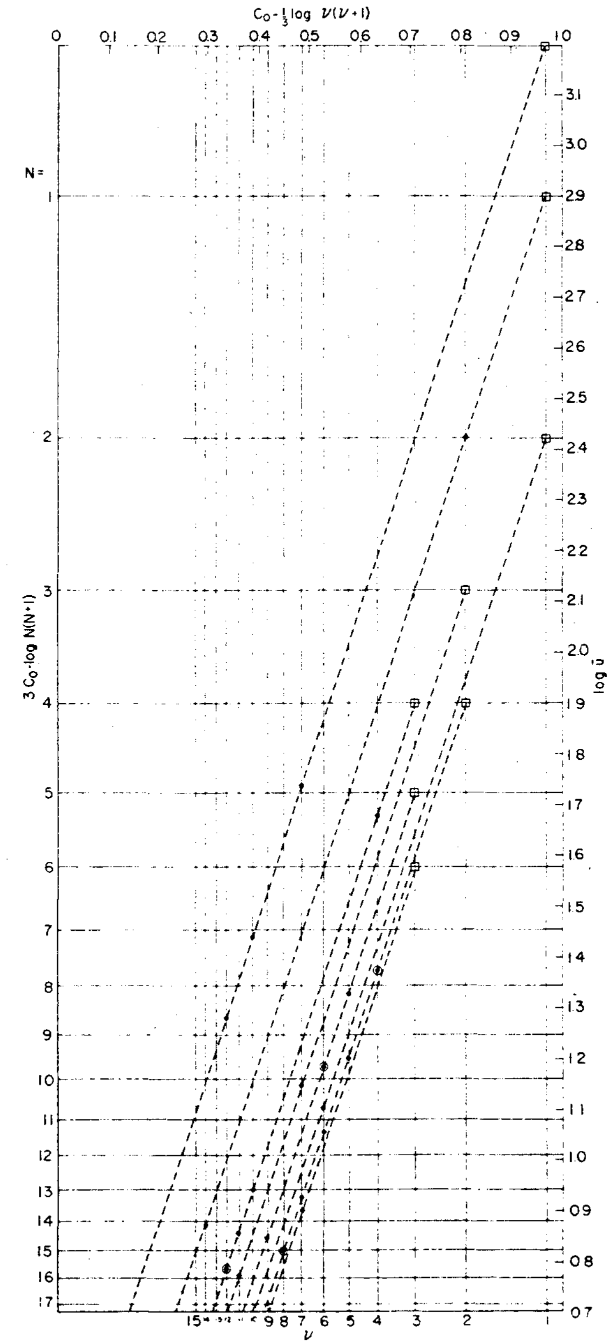

Fig. 1 Values of mean redshifts based on the ν - N discretization grid. Mean synoptic redshifts for clusters may occur at the intersection of Hubble lines (dotted) with vertical ν- lines. Observed intersections are denoted by solid circles. (A double circle indicates a multiple occupancy.) The Hubble lines required for all clusters in Reference (5) pass through eight ν - N intersections circumscribed by squares.

REFERENCES

1. D. G. B. Edelen, Astron. J., 68, 535 (1963).

2. D. G. B. Edelen, Galactic Scale Discretization--I: Theory, The RAND Corporation, RM-3941-RC (in press).

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Figure legend for "Discretization Phenomena in Synoptic Redshifts," by A. G. Wilson

Figure 1 Values of mean redshifts based on the ν - N discretization grid. Mean synoptic redshifts for clusters may occur at the intersection of Hubble lines (dotted) with vertical ν-lines. Observed intersections are denoted by solid circles. (A double circle indicates a multiple occupancy.) The Hubble lines required for all clusters in Reference (5) pass through eight ν - N intersections circumscribed by squares.

MEMORANDUM

15 Rev. (7-55)

TO: Stan Greenfield and Ted Harris

DATE: 1/7/64

MEMO NO.: 99

FROM: Dom Edelen and Al Wilson

SUBJECT: Progress Report

COPIES TO: Brownlee Haydon, Bob Specht, John Williams

RSR 7086: Galaxtic Scale Discretization

Description:

This project is a joint theoretical and observational investigation of the various discretization phenomena which have been predicted and/or discovered in the large scale aggregates of the universe: galaxies, clusters of galaxies, and the observable sample of the universe itself.

Project Personnel:

The two principal investigators are D. G. B. Edelen of the Mathematics Department (Theoretical studies) and A. G. Wilson of the Planetary Sciences Department (Observational studies). In addition, the consulting services of T. Y. Thomas, and G. de Vaucouleurs have been employed in the past year. Supporting work in RAND has been contributed by Oliver Gross.

Activity and Results: Observational

1) The initial Edelen theoretical prediction concerning discretization in galaxian diameters has been quantitatively confirmed in two samples of data: (1) EO galaxies in six clusters as measured on 200 inch plates, (2) EO bright galaxies from G and A, de Vaucouleur's new catalogue.

2) Two additional discretization parameters have been discovered, one probably related to ages of galaxies; the other, a discretization in redshifts. Patterns in discretization sequences have led to the formulation of several new empirical relations between cosmological observables which unlike many current cosmological results are purely empirical, independent of all theoretical models.

3) A new absolute constant has emerged from the diameter and redshift data which is numerically close in value to the fine structure constant of quantum physics. The existence of the fine structure constant in cosmology has been predicted by Eddington and Jordan. Its observational discovery, if definitively identified with the new empirical constant discovered in the current work, would constitute a proof of Mach's Principle and bridge some of the gaps between relativistic and quantum physics.

4) Some curious properties of the discretization sequences lead to the possibility of an entirely new construct for describing certain classes of physical phenomena. Already some of these properties have successfully predicted the numerical values of the mean cluster redshifts of all published cluster data. The possibility exists that certain phenomena currently stochastically described may be accounted for equally well by discretization "packets."

B. Theoretical

1) After the prediction of discretization in the diameters of EO galaxies, the analysis was extended to the entire E-series and the number of necessary assumptions was reduced. Additional predictions were made as to the inclination angles of the axes of ellipticals and the distribution of the semi-major axes as a function of the true eccentricity.

2) The general theory was extended by considering what are termed conformally related metric spaces. This allows us to predict the actual shapes (morphology) of ellipticals. The results of these predictions will be put to the test by de Vaucouleurs.

3) Preliminary investigations have been completed on what is termed conformally homogeneous cosmological models. The results show that we can predict the nodal properties of clusters and the dispersion velocity relations. A general paper on this subject is under preparation.

4) Preliminary investigations on the properties of invariant averaging operators have given very promising results. Since such operators are at the heart land of epitactic considerations, the results will be extended and written for external publication.

5) T. Y. Thomas has derived the epocal discretization as well as the radial discretization from considerations in which the galaxies are imbedded in an overall cosmology in which the mean mass density and radius of the universe appear as a ratio which only assumes discrete values.

Future Work:

The direction which this work will take should not be predicted. The accompanying sheet of proposed papers describes research anticipated at this writing. The unfinished portions of observational and theoretical results to date will be given the first priority.

In view of the emergence of a new epistemological construct, which seems consistent with both the observational findings and theoretical advances, it is proposed that the name of the project be changed to EPITACTIC COSMOGRAPHY. This term is felt to be one underwhich all anticipated studies might justifiably be grouped.

Personnel:

For the coming year the following level of effort is anticipated:

Edelen	1/2
Kocher	1/2
Wilson	2/3

In addition, Thomas, de Vaucouleurs, and Page will be brought in as consultants.

Dom Edelen

Dom Edelen

Al Wilson

Al Wilson

AW:cn

PROPOSED PAPERS FOR THE RAND SERIES IN EPITACTIC COSMOGRAPHY

1.	Galactic Scale Discretization - I: Theory RM-3941-RC, Nov. 1963	Edelen
2.	Galactic Scale Discretization - II: Observations RM-3771-RC	Wilson
3.	Diameters of Elliptical Galaxies RM-3772-RC	Wilson
4.	Morphology of Elliptical Galaxies - Theory	Edelen
5.	Fine Structures in Galaxian Morphology	de Vaucouleurs
6.	Galaxian Masses, Densities and Luminosities	
7.	The Hubble Effect and Discretization Algebras	Wilson
8.	A Discretized Cosmology - Observations	Wilson
9.	A Discretized Cosmology - Theory	Edelen
10.	Local and Global Discretization	Thomas
11.	Cluster Structure I - The Coma Cluster	Kocher and Wilson
12.	Clus ter Structure II - The Virgo Cluster	Kocher and Wilson
13.	Statistical Problems of Discrete Null Functions	
14.	Epitemological Aspects of Discretization	
15.	Invariant Averaging Operators	Edelen

D No: D-11954-PR

D(L) No:

Assigned to: Engr.
A. G. Wilson

Project No: 1000

Contract No: AF 49(638)-700

Task Order No:

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REPORTS OF OPERATIONS IN THE DEPARTMENT OF PLANETARY SCIENCES

FOR THE YEAR 1963

STAFF, DEPARTMENT OF PLANETARY SCIENCES

January 13, 1964

RSR 7086: GALACTIC SCALE DISCRETIZATION

DESCRIPTION:

This project is a joint theoretical and observational investigation of the various discretization phenomena which have been predicted and/or discovered in the large-scale aggregates of the universe: galaxies, clusters of galaxies, and the observable sample of the universe itself.

PROJECT PERSONNEL:

The two principal investigators are D. G. B. Edelen of the Mathematics Department (Theoretical studies) and A. G. Wilson of the Planetary Sciences Department (Observational studies). In addition, the consulting services of T. Y. Thomas, and G. de Vaucouleurs have been employed in the past year. Supporting work in RAND has been contributed by Oliver Gross.

ACTIVITY AND RESULTS:

A. Observational

1) The initial Edelen theoretical prediction concerning discretization in galaxian diameters has been quantitatively confirmed in two samples of data: (1) EO galaxies in six clusters as measured on 200-inch plates, (2) EO bright galaxies from G. and A. de Vaucouleurs' new catalogue.

2) Two additional discretization parameters have been discovered, one probably related to ages of galaxies; the other, a discretization in redshifts. Patterns in discretization sequences have led to the formulation of several new empirical relations between cosmological observables which unlike many current cosmological results are purely empirical, independent of all theoretical models.

3) A new absolute constant has emerged from the diameter and redshift data which is numerically close in value to the fine-structure constant of quantum physics. The existence of the fine-structure constant in cosmology has been predicted by Eddington and Jordan. Its observational discovery, if definitively identified with the new empirical constant discovered in the current work, would constitute a proof of Mach's Principle and bridge some of the gaps between relativistic and quantum physics.

4) Some curious properties of the discretization sequences lead to the possibility of an entirely new construct for describing certain classes of physical phenomena. Already some of these properties have successfully predicted the numerical values of the mean cluster redshifts of all published cluster data. The possibility exists that certain phenomena currently stochastically described may be accounted for equally well by discretization "packets."

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1) After the prediction of discretization in the diameters of EO galaxies, the analysis was extended to the entire E-series and the number of necessary assumptions was reduced. Additional predictions were made as to the inclination angles of the axes of ellipticals and the distribution of the semi-major axes as a function of the true eccentricity.

2) The general theory was extended by considering what are termed conformally related metric spaces. This allows us to predict the actual shapes (morphology) of ellipticals. The results of these predictions will be put to the test by de Vaucouleurs.

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4) Preliminary investigations on the properties of invariant averaging operators have given very promising results. Since such operators are crucial for extensions of relativity theory to observations, the results will be developed and written for external publication.

5) T. Y. Thomas has derived the epocal discretization as well as the radial discretization by considering the galaxies to be imbedded in an overall cosmology in which the mean mass density and radius of the universe appear as a ratio which assumes only discrete values.

FUTURE WORK:

The direction which this work will take should not be predicted. (The unfinished portions of observational and theoretical results to date will be given the first priority.) However, certain problems such as the following clearly must be covered.

Diameters of elliptical galaxies, morphology of elliptical galaxies (theory), fine structures in galaxian morphology, galaxian masses, densities and luminosities, the Hubble Effect and discretization algebras, a discretized cosmology (observations), a discretized cosmology (theory), local and global discretization, cluster structure (the Coma and Virgo Clusters), statistical problems of discrete null functions, epistemological aspects of discretization, and invariant averaging operators.
Edelen, Kocher, Wilson — (1 2/3). (In addition, Thomas, de Vaucouleurs, and Page will be brought in as consultants.)

February 4, 196-

Dear Professor Thomas,

There are two new results which I feel may modify the derivations given in your paper and which you may wish to incorporate.

The first concerns the relationships of mass, luminosity, and density which I promised to investigate. Provisionally, I find confirmation of your equation (10.3) for the bright EO sample. Specifically, what is found is the following:

If S represents the linear diameter of a galaxy
θ the observed angular diameter
$\underline{u}$ the synoptic redshift
$\bar{\rho}$ the mean mass denisty,

equation (10.3) states

$$\log S = \text{const.} + \tfrac{1}{2}\log n(n+1) - \tfrac{1}{2}\log \bar{\rho} \, .$$

Observed: $\log \theta - \log u = \text{const.} + \tfrac{1}{2}\log n(n+1) + \log m(m+1)$.

If u^{-1} is a measure of distance, then $\log \theta - \log u$ is consistently interpreted as $\log S$.

Comparison states that

$$\log \bar{\rho} = \text{const} - 2 \log m(m+1) \, .$$

This result may be theoretically derivable from your equations. The mass M will be given by

$$\log M = \text{const.} + 3(\log \theta - \log u) - 2 \log m(m+1)$$

or by

$$\log M = \text{const.} + \frac{3}{2} \log n(n+1) + \log m(m+1) .$$

The luminosity of a galaxy is given by

$$\log L = \text{const.} - 0.4(m_c + 5 \log u)$$

where m_c is a corrected apparent magnitude. Hence, we predict

$$\log M - \log L = \text{const.} + 0.4(m_c + 5\log u) + 3(\log\theta - \log u) - 2 \log m(m+1) .$$

When $0.4(m_c + 5\log u)$ is plotted against
$3(\log\theta - \log u) - 2 \log m(m+1)$

the galaxies fall into two narrow bands with slope approximately equal to (−1). This seems like a good confirmation of (10.3) with $\bar{\rho}$ proportional to $[m(m+1)]^{-2}$.

The second matter involves equation (1.1). In this empirical result it may be erroneous to interpret the left member as log R. It was assumed because of the appearance of a $\frac{1}{2} \log n(n+1)$ discretization in the right member that the left member must be dimensionally R. However a new $\frac{1}{2} \log n(n+1)$ type of discretization has been found in redshifts which has nothing to do with the size discretization. It is therefore open to question whether the dimensional argument is valid since discretization of the type $\frac{1}{2} \log n(n+1)$ may not be uniquely associated with size.

I feel therefore that equation (1.2), based on the assumption that $\log \theta - \frac{2}{3} \log \overline{u} = \log R$, may now be open to question.

A. G. Wilson

P.S. I failed to mention that the ratio of mass to luminosity is currently believed to be constant for a given type of galaxy, which explains why the (−1) slope seems like a good confirmation of (10.3).

It would be most interesting if an $[m(m+1)]^{-2}$ discretization for density follows from your equations.

From the ASTRONOMICAL JOURNAL
69, No. 2, 1964, March—No. 1317
Printed in U. S. A.

p 153

Discretization in E0 Field Galaxies. Albert G. Wilson, *The RAND Corporation.*—The Edelen discretization prediction (Edelen, D. G. B., *Astron. J.* **68**, 535, 1963) states that the linear diameters S_n of E0 galaxies are given by a relation of the form

$$S_n = S_0 \xi^{-1/2}[n(n+1)]^{+1/2},$$

where S_0 is a constant and ξ is an energy parameter with $n=1, 2, 3 \cdots$. This prediction is tested against the sample of all E0 galaxies for which diameters and redshifts are given in G. and A. de Vaucouleurs' *Reference Catalogue of Bright Galaxies* (University of Texas Press, in press). The sample, consisting of 31 galaxies divides itself into five different "ξ classes." Within each class (ξ being constant) a relationship of the type

$$1+\log D_n - \tfrac{4}{9}\log u - \tfrac{1}{2}\log n(n+1) = C_\nu \qquad (1)$$

is observed. In Eq. (1), D_n is the catalogue angular diameter, $u=(1+Z)/Z$ is the synoptic redshift of the galaxy whose diameter is D_n, and C_ν is a constant for all members of the class. The two largest classes have 10 and 11 members with the mean spread (difference between the minimum and maximum values of C_ν) in each class being 0.02. Attempts to generate similar fits based on random numbers to within a spread of 0.02 failed in 100 tests on the RAND 7090 computer.

The close fit to the Edelen eigensequence suggests that Eq. (1) determines the principal component of the redshifts. Residual components representing peculiar velocities are small as deduced by G. de Vaucouleurs (*Astron. J.* **63**, 253, 1958) and Neyman and Scott (*Astron. J.* **66**, 148, 1961).

It is further found that the constants C_ν belonging to each class are related by an equation of the form

$$C_\nu + \tfrac{1}{3}\log \nu(\nu+1) = \text{a constant.} \qquad (2)$$

The mean color of the galaxies of each class is correlated with the class index ν, suggesting that the parameter ν represents an evolutionary or age parameter. If this interpretation is correct, the cube-root discretization relation of Eq. (2) allows the concept of discrete ages and, hence, of *discrete epochs of creation* for E0 galaxies. Enlarged samples should affirm or deny this interpretation's consistency and also its applicability to other morphological types.

DISCRETIZED STRUCTURE IN THE DISTRIBUTION OF CLUSTERS OF GALAXIES

A. G. Wilson

To PNAS April 1964
Communicated by T. Y. Thomas

DISCRETIZED STRUCTURE IN THE DISTRIBUTION OF CLUSTERS OF GALAXIES*

Albert G. Wilson
The RAND Corporation, Santa Monica, California
(Communicated by T. Y. Thomas, April, 1964)

From the basic Einstein equations positing the equivalence of the geometrical and physical tensors,

$$R_{AB} - 1/2\, R\, g_{AB} = \kappa\, T_{AB} \quad ,$$

Edelen[1,2] and Thomas[3] have independently predicted discretization in the geometrical sizes of galaxies. Both theories suggest that the diameters or major axes of the galaxies should be, under certain conditions, proportional to a sequence of eigen numbers of the form $[n(n+1)]^{1/2}$, where n is a positive integer. This theoretically predicted size discretization has been observationally confirmed by Wilson[4,5] for samples of field and cluster galaxies in the case of ellipticals of small eccentricity.

From general Machian-type arguments, one might surmise that the discretized geometry observed in galaxian structure is also manifested in large-scale cosmological structure. The present paper reports an observed structural relation in the mean redshifts of clusters of galaxies which apparently confirms this surmise of some form of discretized regularity on a cosmic scale and which, together with the observed discretization in galaxian sizes, may be interpreted as observational evidence for Mach's principle.

*This research was sponsored by The RAND Corporation.

Theoretical cosmologists customarily assume isotropy and homogeneity in constructing their models, ignoring the granularities which are observed to be present in the large-scale distribution of matter. While smoothing may be a justifiable simplification for ~~the~~ zero order cosmological models, there is danger of a large loss of information when observers habitually design and interpret their observations on such models. Furthermore, the offspring of the union of a structured world and a smoothed model may be epistemologically illegitimate. This point is well expressed by Neyman:[6]

~~by Neyman: (6)~~ "The contrast between the domain of current observations of individual galaxies and their clusters on the one hand, and the theory dealing with the smoothed-out substratum on the other, is the more striking because every effort to verify empirically the conclusions of the theory must deal with observations of objects whose very existence this theory ignores. Thus, in order to effect a verification it is necessary to adopt a number of ad hoc hypotheses and, as a result, the conclusions are open to the question." Hence, it is most important to examine the observations for any structure hitherto ignored in a mental climate conditioned to smoothing.

The granularities in the cosmic distribution of matter principally manifest themselves as galaxian clusters found in large numbers throughout the observable universe. Their distributions have been discussed by several observers, who have concentrated on counts and cluster sizes, whose statistics are unlikely to show any consistent quantifiable structure like that observed in the galaxian size discretization. Investigations of discretized structure require observables of higher precision.

The most precise observable is the redshift. Furthermore, it is particularly useful for our purposes because the total errors in redshifts are not directly dependent on their sizes, and their relative errors are consequently small except for nearby objects. This has made possible the results of observational cosmology out to great distances. The principal errors and uncertainties in observational cosmology reside not in redshifts, but in magnitudes and diameters. For a structure investigation, we need not be concerned with scale or dimensional calibrations requiring these latter observables; by intercomparing redshifts, we can use their full undiluted precision. When the redshifts for many individual galaxies in a cluster have been observed, the <u>mean cluster redshifts</u> should provide a highly precise measure of relative cluster distribution in depth and, hence, a promising tool for detecting any systematic structure in the granularity.

Though cluster mean redshifts are precise, the sample is unfortunately small. To date, galaxian redshifts have been published for only about 30 clusters. A few of these clusters, such as Virgo and Coma, have 50 or more observed redshifts, but most of the more distant clusters contain only one or two.

Table 1 collects the generally available observational data, reducing them to cluster mean redshifts. The five left-hand columns before the vertical double line are: 1) the cluster designation in terms of its 1950.0 position; 2) the common name for the cluster; 3) the number of individual redshifts entering into the calculation of the mean; 4) the mean velocity for the cluster,

$$\bar{V} = \frac{c}{n} \sum_{i=1}^{n} z_i \quad ,$$

Table 1

MEAN CLUSTER REDSHIFTS OBSERVED AND CALCULATED

Cluster	Name	Number of Redshifts	$\overline{V}$	log $\overline{u}_o$	M	N	P	P - log $\overline{u}_o$	References
	Virgo	73	1136	2.4234	3	2	2.4225	-0.0009	
0316+4121	Perseus	5	5433	1.7499	7	2	1.7534	-0.0035	
0123-0137	"NGC 541"	43	5439	1.7494	7	2	1.7534	0.0040	(8)
1257+2812	COMA	50	6432	1.6780	4	4	1.6779	-0.0001	(9)
1627+3937	Abell 2199	19	9028	1.5344	9	2	1.5474	0.0130	(10)
1603+1755	HERCULES	15	10775	1.4597	10	2	1.4603	0.0006	(11)
2308+0720	PEGASUS II	3	12821	1.3874	11	2	1.3811	-0.0063	
2322+1425		2	13187	1.3757	11	2	1.3811	0.0054	
1145+5559	U.M. I	4	15269	1.3149	12	2	1.3086	-0.0063	
0106-1536	Haufen A	2	15872	1.3012	12	2	1.3086	0.0074	
1024+1039	LEO	1	19489	1.2147	7	4	1.2306	0.0159	
1239+1853		2	21533	1.1741	14	2	1.1795	0.0054	
1520+2754	CORBOR	8	21651	1.1719	14	2	1.1795	0.0076	
0705+3506	GEMINI	2	23366	1.1360	15	2	1.1215	-0.0145	
0348+0613		1	25644	1.1038	15	2	1.1215	0.0177	
1513+0433	Shane Cloud	1	28333	1.0640	16	2	1.0671	0.0031	
1431+3146	Boötes	2	39367	0.9356	10	4	0.9375	0.0019	
1055+5702	U.M. II	2	40860	0.9213	19	2	0.9219	0.0006	
2253+2341	Abel 1413		42870	0.9031	7	6	0.9083	0.0052	(12)
1153+2341		2	42784	0.9037	7	6	0.9083	0.0046	
1534+3149		3	45951	0.8767	20	2	0.8785	0.0018	
0025+2223		2	47836	0.8616	11	4	0.8583	-0.0033	
1228+1050		2	49514	0.8487	9	5	0.8485	-0.0002	
0138+1840		1	51908	0.8312	21	2	0.8371	0.0059	
1309-0105		1	52362	0.8280	15	3	0.8205	-0.0075	
1304+3110	Coma B	1	54917	0.8104	15	3	0.8205	0.0101	
0925+2044		1	57498	0.7937	22	2	0.7975	0.0038	
1264+4422		1	59382	0.7819	12	4	0.7858	0.0039	
0855+0321	HYDRA	3	60860	0.7730	16	3	0.7661	0.0069	

$$P = 4.2799 - \log M(M+1) - \log N(N+1)$$

$$\log \overline{u}_o = \log [(C+\overline{V})/\overline{V}]$$

where $z_i = (\delta\lambda/\lambda)_i$ is the spectral displacement; and 5) the logarithm of the quantity $\bar{u}_o = (c + \bar{V})/\bar{V}$. The source of all redshifts in Table 1 is the Humason-Mayall-Sandage Catalogue,[7] except when indicated by a reference number in the far right-hand column.

The precision with which the mean redshift for any cluster is known depends on the errors in the individual redshifts, the number of redshifts observed in the cluster, and the dispersion of redshifts within the cluster. If we may take the redshifts published in the Humason-Mayall-Sandage Catalogue as typical, then we may assume in accordance with their estimates that each measured velocity is within 175 km/sec of the true value, and over half are within 35 km/sec of the true value. Assuming the extreme bound of 175 km/sec for each redshift, the <u>relative</u> errors in the individual redshifts should be less than 3%, except for the Virgo cluster. The relative errors in the more distant clusters become very small, and the values should be correct to the third decimal place in z.

The largest source of uncertainty in the <u>mean</u> redshift for a cluster is the velocity dispersion. Good means and dispersions have been obtained only for nearby clusters with large samples of observed redshifts, the values for distant clusters remaining uncertain. If the faint distant clusters containing observed samples of only a few redshifts are assumed to have the same velocity dispersions as the nearby clusters, a large uncertainty results from assuming that the mean of the observed sample (often only one galaxy) may be taken for the mean of the cluster. Further enhancing the uncertainty are the indications that velocity dispersions of the same order for near and distant clusters may not be assumed. Neyman and Scott[13] report an apparent increase in dispersion with cluster distance -- an increase that may be real or may be due to a selectivity effect that arises in assigning cluster membership by redshift similarities. Whether the dispersions in distant clusters equal or exceed those in nearby clusters, these dispersions along with the smallness of the observed samples result in large uncertainties. For example, the mean redshift with highest uncertainty in Table 1 is probably the Leo Cluster, the nearest cluster based on a single observation. If we assume a velocity dispersion of 1000 km/sec as a reasonable upper bound based on the value for several well-observed clusters, the relative error in the mean is about 5%, which is likely to be marginal for structural investigations.

Perhaps, however, relative errors of this size may be much too large. There is a mitigating physical phenomenon that may greatly reduce the uncertainty in a mean redshift based on a small sample. This is the observed fact that many clusters are centered on one or two very bright galaxies, which are presumably near the cluster's center of gravity with redshifts close to the value of the cluster mean redshift. Observational selectivity naturally favors these galaxies through ease of observation, and this results in the greater than statistical likelihood of an isolated observation being close to the cluster mean. If we assume that the redshifts of the giant central galaxies equal the cluster mean, then the relative errors for distant clusters drop to less than 1%, which is again within the margin of precision for discretization investigations. This assumption will be evaluated a posteriori when comparison is made between the observed mean redshifts and the structurally derived mean redshifts.

Comparisons of galaxian diameter discretization sequences in six clusters -- based on the data of Ref. (4) -- led to a prediction that the cluster mean redshifts must be discretized. This prediction was found to be valid, not only for the original six clusters, but for all clusters for which redshift data was available. An empirical relation was found between mean redshifts, $\bar{u}_o$, and discretization integers M and N of the form

$$\text{(1)} \qquad \log \bar{u}_o = Ш - \log M(M+1) - \log N(N+1) \quad ,$$

where $Ш$ is a constant whose approximate value is 4.2799, and M and N are positive integers.

The comparisons between the observed mean redshift $\bar{u}_o$ and the right member of Eq. (1), which we shall designate by P(M,N) and call the structural mean redshift, are given in the right-hand side of Table 1. The columns to the right of the vertical double line are, consecutively: M, N, P, and the residuals, P - log $\bar{u}_o$. The values of log $\bar{u}_o$ and of the residuals are often given to more significant places than are consistent with the precision of the observations. This is done to avoid contamination of discretization by possible round-off effects. The residuals range from less than 0.0030 (lowest meaningful value) to 0.0177, corresponding to a discrepancy of 4%. Except for the cluster Abell 2199, the residual is always bounded by the value of the uncertainty in the observed mean redshift, estimated from an assumed 1000 km/sec velocity dispersion for each cluster. To one side of Abell 2199 are four galaxies whose velocities are markedly higher and whose membership in the cluster has been questioned by Minkowski.[10] If these four are rejected, the value of log $\bar{u}_o$ for the remaining 15 galaxies is 1.5483 with a residual of -0.0009 -- well below the statistically estimated uncertainty in the observed mean. Note that the residual generally decreases as the size of the observed sample increases. For those clusters with large numbers of observed redshifts (such as Virgo and Coma), the residual becomes minute.

Thus we have a remarkable agreement between observed mean redshifts and structural mean redshifts, based on a numerical equation containing one arbitrary constant. But while it is valid for all clusters with published redshift, it nonetheless is established only over a very small sample. Here caution dictates that we determine whether pure chance might not also produce agreements of this order with Eq. (1). Certainly, if ω is taken very large, we could find large integer values of M and N that would give values of P(M,N) fitting any set of numbers to within any prescribed residual. All the integer values in Table 1, however, are small, or belong to sequences beginning with small numbers, or fit consistantly with derived sequences. Note also that P(M,N) may assume the same value for different combinations of M and N; consequently, the identification of M and N is not always unique. Alternative assignments of M and N are therefore possible for six clusters in Table 1. We can distribute clusters as listed in Table 1 into "N" classes, as follows:

18 of the 29 clusters to the class N 2, three to N 3, five to N 4, one to N 5, and two to N 6. Considered singly, classes with two or three members have no significant statistical existence. Their existence may be inferred only from structural consistency with classes for which a sufficiently large sample of members is available -- at least until the sample size in each class is larger.

In order to examine the probability that the distributions in Table 1 are reproducible by chance, let us consider the most stringent case: The class N 2 containing 18 clusters from a sample of 29 which fit a <u>one</u>-parameter sequence involving one arbitrary constant.

No standard tests are available for the statistical significance of fits to discretized null functions. We can, however, derive a good estimate of the probability that the observed fits are fortuitous by comparing the number of fits occurring between sets of random numbers and the theoretical values, with the number of fits occurring between the observed data and the theoretical values. In such tests, the ranges and density distribution in the random samples must be the same as in the observational sample, and the allowed theoretical values must be the same for both. Table 2 gives the results of one set of tests comparing the fits of the random samples and the observations to the values of $\wp$ for N2. A discussion of the problem of discretized null functions and the results of more extensive tests will be published elsewhere.

Table 2

RANDOM AND OBSERVATIONAL FITS TO THEORY

δ	0.02	0.015	0.01	0.005
N_o	18	17	16	9
$\langle N_r \rangle$	17.5	13.1	9.5	4.3
σ_r	2.1	2.8	2.5	1.4
p	85%	15%	0.7%	0.1%

The first row gives the values of δ, the maximum residual. The second row, N_o, gives the number of fits (within δ) between the observations and the theoretical values,

$$P(M,N) = 3.5017 - \log M(M+1) \quad ,$$

in the sample of 29 clusters. N_r is the mean number of fits to the null function (less than or equal to δ) occurring in each set of 29 random numbers: the third row, $\langle N_r \rangle$, the value of N_r averaged over all the sets, is the most probable number of fits to the random sample. The last row, σ_r, gives the dispersion in N_r. The probability, p, that the fit between the data and theory is the result of chance can be estimated from N_o, $\langle N_r \rangle$, and σ_r by means of the probability integral. The values of p are given in the last row.

Note how strongly the statistical significance of fits to the discretized sequence depends on δ. For low precision (0.02), there is no statistical significance. However, all but two of the clusters fitting the N 2 sequence have a residual of less than 0.01, corresponding to a discrepancy of 2 1/2%. The likelihood that this precision of fit occurs by chance is 0.7%. The consistency between the size of the residuals, P - log $\bar{u}_o$, and the estimated errors in the cluster mean redshifts lends further confirmation to the validity of Eq. (1) and to the assumption that the redshift of the bright central galaxy in a cluster is approximately equal to the mean redshift of the cluster. More difficult to attribute to chance are the nine clusters whose residuals are less than 0.005. These, for the most part, are clusters with larger numbers of individual observations contributing to the mean. The hypothesis seems unlikely, then, that chance agreement has produced the observed residuals. Unless contradicted by definative tests based on larger samples, Eq. (1) seems to represent a real relation governing the distribution of cluster mean redshifts.

If so, a further prediction should now be possible. If all matter is indeed concentrated in clusters or at nodal points whose redshifts are given by Eq. (1), and if the velocity dispersion about each node is bounded, then whenever z is sufficiently large, the relative error in replacing the mean nodal redshift by the redshift of _any_ single observable object in the node should be small. In particular, the redshifts of high-velocity radio sources (including the quasi-stellar sources) should also fit Eq. (1). This prediction is confirmed by the data of Table 3, in which $\bar{u}_o$ is taken as the function $(1 + z)/z$ of the single redshift observation.

Table 3

OBJECTS WITH LARGE REDSHIFTS

Object	z	log $\bar{u}_o$	M	N	P	P - log $\bar{u}_o$	Reference
3C 273	0.158	0.8651	11	4	0.8583	-0.0068	(14)
3C 48	0.367	0.5711	29	2	0.5622	0.0089	(15)
3C 47	0.425	0.5254	30	2	0.5332	0.0078	(17)
3C 295	0.461	0.5009	31	2	0.5052	0.0043	(16)
3C 147	0.545	0.4525	33	2	0.4518	-0.0007	(17)

As M becomes large, the logarithm of the interval between successive nodal values approaches zero. In Table 3 the intervals are small, but the residuals always remain less than one quarter of the interval. Note that most of the objects again belong to the N 2 sequence.

Also of considerable interest is the value of the dimensionless constant appearing in Eq. (1). This equation may be written in the form

$$\frac{\bar{z}}{1+\bar{z}} = 10^{-\omega} \quad M(M+1) \cdot N(N+1) \quad ,$$

which is to say that the redshifts to the centers of concentrations of matter, corrected for light travel time, may be mapped into a two-parameter grid whose basic structural constant is

$$10^{-4.2771} = (137.6)^{-2} \quad ,$$

where 4.2771 is the value of ω which minimizes residuals.

correct in the first three digits. The numerical coincidence between the dimensionless structural constant of the redshift discretization grid and the fine-structure constant is an unsuspected feature which a posteriori lends considerable additional confidence to the validity of Eq. (1) and strongly supports the possibility of Machian relationships between the structure and distribution of variously sized aggregates throughout the universe.

It should now be noted that a possible alternative interpretation of Eq. (1) exists. Rather than representing cosmic phenomena, this equation may express some property implicit in the observed spectra. It bears some resemblance to Balmer's formula governing the wave lengths in the hydrogen spectrum, consisting as it does of two integer parameters and a constant; but it is the occurrence of a number approximately equal to the fine-structure constant which gives rise to the possible spectral origin of the relation. It is conceivable, for example, that rather than doppler effects some discretization relation governing allowable values of $\delta\lambda/\lambda$ results in the observed spectral redshifts. Such an interpretation of Eq. (1) appears rather unlikely, however, since mean redshifts in the form $(1 + \overline{z})/\overline{z}$, and not individual redshifts, occur in the left member of Eq. (1). The existence of a non-doppler component of the redshift, however cannot be ruled out.

If, then, Eq. (1) indeed represents cosmic distributions, it states that the allowable coordinate values which may be assumed by the time and radius coordinates of matter concentrations as seen by an observer located at one of these concentrations may be mapped into a canonical bi-discretization grid developable with positive integers from a universal constant. At the moment, we lack a model allowing a physical interpretation of this observation. The Robertson-Walker line element, which provides a metric adequate for homogeneous models (including both evolutionary and continuous creation models), is not adaptable to this observed discretized structure.

The observation of galactic and cosmic discretization is only beginning. Consequently, we cannot now specify all of the discretization structures that cosmological theories will eventually have to take into account. It can be said, though, that models based on assumptions contradicting the <u>existence</u> of granularity will presumably be of little use in accounting for observed <u>structure</u> in the cosmic distribution of matter. Homogeneous models may nevertheless be of immediate use in providing a first-order description, from which models with structural inhomogeneities may be derived by perturbations.

Whenever a chasm opens between theory and observation, epistemologically either the ~~epistemological status of~~ the theory becomes ~~that of~~ an antiquary like Ptolemy's Epicycles, or ~~while~~ ~~the epistemological status of~~ the observation remains ~~that of~~ a curiosity like Bode's Law.

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ST. II

*DISCRETIZED STRUCTURE IN THE DISTRIBUTION OF CLUSTERS OF GALAXIES**

BY ALBERT G. WILSON

THE RAND CORPORATION, SANTA MONICA, CALIFORNIA

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① From the basic Einstein equations positing the equivalence of the geometrical and physical tensors,

$$R_{AB} - {}^1/_2\, Rg_{AB} = \kappa T_{AB},$$

Edelen[1,2] and Thomas[3] have independently predicted discretization in the geometrical sizes of galaxies. ~~Both~~ Theory suggests that the diameters or major axes of the galaxies should be, under certain conditions, proportional to a sequence of eigen numbers of the form $[n(n+1)]^{1/2}$, where n is a positive integer. This theoretically predicted size discretization has been observationally confirmed by Wilson[4,5] for samples of field and cluster galaxies in the case of ellipticals of small eccentricity.

The evidence for discretized distributions in sizes of galaxies suggests the possibility that there exist further regularities in the large-scale distributions of matter which have not been suspected. The present paper reports an observed regularity in the mean redshifts of clusters of galaxies which appears unlikely for randomly distributed clusters. There is at present no cosmological theory which predicts such regularities. This is not surprising, however, since most current cosmological models assume that the observed granularities in the distribution of matter — galaxies and clusters of galaxies — may be approximated by a uniform, smooth distribution. No theory which ignores the granularities can be held accountable for structure in the granularities. The epistemological difficulties which arise from smoothing assumptions have been well expressed by Neyman:[6] "The contrast between the domain of current observations of individual galaxies and their clusters on the one hand, and the theory dealing with the smoothed-out substratum on the other, is the more striking because every effort to verify empirically the conclusions of the theory must deal with observations of objects whose very existence this theory ignores. Thus, in order to effect a verification it is necessary to adopt a number of ad hoc hypotheses and, as a result, the conclusions are open to question."

Furthermore, when observations are designed and interpreted with specific theoretical models, there is a tendency to ignore information which does not bear directly on questions posed in the verification of the model. Accordingly, the design and interpretation of observations around cosmological models which assume a smoothed-out distribution of matter, tends to de-emphasize the investigation of possible regularities in the distribution of the observed granularities. Most published discussions concerning the distributions of galaxies and clusters have emphasized the stochastic aspects of counts, magnitudes and sizes.

In investigations of galaxy size discretization, the angular diameters of galaxies in five clusters (from the data of ref. 4) were combined with mean cluster redshifts, $\overline{V}$, to obtain linear diameters by using a velocity distance relation of the form

$$H \, \overline{u}_o d = 1$$

where H is a constant, $\overline{u}_o = (c + \overline{V})/\overline{V}$, c is the velocity of light and d is the distance. Comparisons of the resulting diameter discretization sequences suggested a relation between mean cluster redshifts of the form

$$\log \bar{u}_o + \log N(N + 1) = \text{constant} \qquad (1)$$

where N is a positive integer. Equation (1) was tested against the 28 clusters for which published redshifts are available. It was found that there exists an integer, N, such that the mean cluster redshifts of 20 of the 28 clusters fit equation (1) to within the estimated observational error (these are those clusters listed in Table 1 for which the parameter M = 2). Of the 8 remaining clusters, an integer N was found such that 7 fit equation (1) with a different constant in the right member (those clusters for which M = 4 in Table 1). Finally, it was found that all values of the constant in the right member of equation (1) can be expressed as

$$\omega - \log M(M + 1)$$

where ω is a constant and M takes on values 2, 4, and 6. A single empirical formula expressing the mean cluster redshifts in terms of two integer parameters, M and N, and one arbitrary constant, ω, can thus be written as

$$\log \bar{u}_o = \omega - \log M(M + 1) - \log N(N + 1) \qquad (2)$$

where the constant, ω, has a least square value of 4.2765 for the 28 clusters.

(7) Table 1 collects the generally available observational data, reducing them to cluster mean redshifts. The first five columns are: (1) the cluster designation in

replace table 1 with new table 1 attached.

TABLE 1

MEAN CLUSTER REDSHIFTS OBSERVED AND CALCULATED

Cluster	Name	No. of redshifts	$\bar{V}$	log $\bar{u}_0$	M	N	P	P − log $\bar{u}_0$
	Virgo	73	1136	2.4234	3	2	2.4225	−0.0009
0316 + 4121	Perseus	5	5433	1.7499	7	2	1.7534	−0.0035
0123 − 0137	"NGC 541"[8]	43	5439	1.7494	7	2	1.7534	0.0040
1257 + 2812	COMA[9]	50	6432	1.6780	4	4	1.6779	−0.0001
1627 + 3937	Abell 2199[10]	19	9028	1.5344	9	2	1.5474	0.0130
1603 + 1755	HERCULES[11]	15	10775	1.4597	10	2	1.4603	0.0006
2308 + 0720	PEGASUS II	3	12821	1.3874	11	2	1.3811	−0.0063
2322 + 1425		2	13187	1.3757	11	2	1.3811	0.0054
1145 + 5559	U.M. I	4	15269	1.3149	12	2	1.3086	−0.0063
0106 − 1536	Haufen A	2	15872	1.3012	12	2	1.3086	0.0074
1024 + 1039	LEO	1	19489	1.2147	7	4	1.2306	0.0159
1239 + 1853		2	21533	1.1741	14	2	1.1795	0.0054
1520 + 2754	CORBOR	8	21651	1.1719	14	2	1.1795	0.0076
0705 + 3506	GEMINI	2	23366	1.1360	15	2	1.1215	−0.0145
0348 + 0613		1	25644	1.1038	15	2	1.1215	0.0177
1513 + 0433	Shane Cloud	1	28333	1.0640	16	2	1.0671	0.0031
1431 + 3146	Boötes	2	39367	0.9356	10	4	0.9375	0.0019
1055 + 5702	U.M. II	2	40860	0.9213	19	2	0.9219	0.0006
2253 + 2341	Abel 1413[12]		42870	0.9031	7	6	0.9083	0.0052
1153 + 2341		2	42784	0.9037	7	6	0.9083	0.0046
1534 + ~~3149~~ 3749		3	45951	0.8767	20	2	0.8785	0.0018
0025 + 2223		2	47836	0.8616	11	4	0.8583	−0.0033
1228 + 1050		2	49514	0.8487	9	5	0.8485	−0.0002
0138 + 1840		1	51908	0.8312	21	2	0.8371	0.0059
1309 − 0105		1	52362	0.8280	15	3	0.8205	−0.0075
1304 + 3110	Coma B	1	54917	0.8104	15	3	0.8205	0.0101
0925 + 2044		1	57498	0.7937	22	2	0.7975	0.0038
1264 + 4422		1	59382	0.7819	12	4	0.7858	0.0039
0855 + 0321	HYDRA	3	60860	0.7730	16	3	0.7661	0.0069

$P = 4.2799 - \log M(M+1) - \log N(N+1)$; $\log \bar{u}_0 = \log [(C + \bar{V})/\bar{V}]$.

terms of its 1950.0 position; (2) the common name for the cluster; (3) the number of individual redshifts available for ~~entering into~~ the calculation of the mean; (4) the mean velocity for the cluster,

$$\bar{V} = \frac{c}{n} \sum_{i=1}^{n} z_i,$$

where $z_i = (\delta\lambda/\lambda)_i$ is the measured spectral displacement; and (5) the logarithm of the quantity $\bar{u}_0 = (c + \bar{V})/\bar{V}$. The source of all redshifts in Table 1 is the Humason-Mayall-Sandage Catalogue,[7] except when indicated by a reference number in the far right-hand ~~second~~ column.

(2) The comparisons between the observed mean redshift $\bar{u}_0$ and the right member of equation (X), which we shall designate by $P(M,N)$ and call the *structural mean redshift*, are given in the right-hand side of Table 1. Columns 6–9 are, consecutively: N, M, ~~M, N~~, P, and the residuals, $P - \log \bar{u}_0$. The values of $\log \bar{u}_0$ and of the residuals are usually ~~often~~ given to more significant places than are consistent with the precision of the observations. This is done to avoid contamination of discretization by possible round-off effects. The residuals range from less than 0.0030 (lowest meaningful value) to 0.0187 ~~0.0177~~, corresponding to a discrepancy of 4 per cent. ~~Except for the cluster Abell 2199, the residual is always bounded by the value of the uncertainty in the observed mean redshift, estimated from an assumed 1000 km/sec velocity dispersion for each cluster. To one side of Abell 2199 are four galaxies whose velocities are markedly higher and whose membership in the cluster has been questioned by Minkowski.[10] If these four are rejected, the value of log $\bar{u}_0$ for the remaining 15 galaxies is 1.5483 with a residual of −0.0009—well below the statistically estimated uncertainty in the observed mean. Note that the residual generally decreases as the size of the observed sample increases. For those clusters with large numbers of observed redshifts (such as Virgo and Coma), the residual becomes minute.~~

Table 1

MEAN CLUSTER REDSHIFTS OBSERVED AND CALCULATED

Cluster	Name	Number of Redshifts	$\overline{V}$	$\log \overline{u}_o$	N	M	P	$P - \log \overline{u}_o$	References
	Virgo	73	1136	2.4234	3	2	2.4191	-0.0043	
0316+4121	Perseus	7	5435	1.7494	7	2	1.7500	+0.0006	
0123-0137	"NGC 541"	43	5439	1.7494	7	2	1.7500	+0.0006	(8)
1257+2812	Coma	50	6432	1.6780	4	4	1.6745	-0.0035	(9)
1627+3937	Abell 2199	19	9028	1.5344	9	2	1.5440	0.0096	(10)
1603+1755	Hercules	15	10775	1.4597	10	2	1.4569	-0.0028	(11)
2308+0720	Pegasus II	3	12821	1.3874	11	2	1.3777	-0.0097	
2322+1425		2	13187	1.3757	11	2	1.3777	0.0020	
1145+5559	Ursa Major I	4	15269	1.3149	12	2	1.3052	-0.0097	
0106-1536	Haufen A	2	15872	1.3012	12	2	1.3052	0.0040	
1024+1039	Leo	1	19489	1.2147	7	4	1.2272	0.0125	
1239+1853		2	21533	1.1741	14	2	1.1761	0.0020	
1520+2754	Corona Borealis	8	21651	1.1719	14	2	1.1761	0.0042	
0705+3506	Gemini	2	23366	1.1360	15	2	1.1181	-0.0179	
0348+0613		1	25644	1.1038	15	2	1.1181	0.0143	
1513+0433	"Shane Cloud"	1	28333	1.0640	16	2	1.0637	-0.0003	
1431+3146	Boötes	2	39367	0.9356	10	4	0.9341	-0.0015	
1055+5702	Ursa Major II	2	40860	0.9213	19	2	0.9185	-0.0028	
1153+2341	Abell 1413	2	42784	0.9037	7	6	0.9049	0.0012	
1534+3749		3	45951	0.8767	20	2	0.8751	-0.0016	
0025+2223		2	47836	0.8616	11	4	0.8549	-0.0067	
1228+1050		2	49514	0.8487	11	4	0.8549	+0.0062	
0138+1840		1	51908	0.8312	21	2	0.8337	0.0025	
1309-0105		1	52362	0.8280	21	2	0.8337	+0.0057	
1304+3110	Coma B	1	54917	0.8104	21	2	0.8337	-0.0163	
0925+2044		1	57498	0.7937	22	2	0.7941	0.0004	
1253+4422		1	59382	0.7819	12	4	0.7824	0.0005	
0855+0321	Hydra	3	60860	0.7730	12	4	0.7824	+0.0094	

$$P = 4.2765 - \log M(M+1) - \log N(N+1)$$

$$\log \overline{u}_o = \log [(c+\overline{V})/\overline{V}]$$

Insert this Table 1 on manuscript page 4

It is apparent that if ω is taken very large, we could find large integer values of M and N that would give values of P(M, N) fitting the set of observations to within any prescribed residual. It is the low values of N that are the most stringent statistically. Accordingly, the integers, N, in Table 1 are selected as the smallest integers which provide as many distinct values over the range of log $\overline{U}_o$ (i.e. 2.4234 to 0.7730) as there are distinct cluster redshifts. Note also that P(M, N) may assume the same value for different combinations of M and N, with the result that the identification of M and N is not always unique.

While the sequences for M = 2 and M = 4 each contain enough members to establish the statistical significance of equation (1) for two different constants, there is no justification in the present sample of redshifts, other than mathematical convenience, for the writing of equation (2) with two integral parameters, M and N, and for assigning the unusually rich cluster Abell 1413 to a tentative M = 6 sequence. If this is done, however, taking the exponential of both sides of equation (2) leads to the form,

$$\frac{\overline{z}}{1+\overline{z}} = \frac{M(M+1) \cdot N(N+1)}{(137.5)^2}$$

where again $\overline{z}$ is the dimensionless quantity $(\overline{V}/c)$. The numerical value assumed by the fundamental constant, ω , affords us a curious numerical coincidence with the fine structure constant, which may or may not be meaningful but should be noted.

Before investigating the statistical significance of the agreement between the observed and structural mean cluster redshifts, it is necessary to evaluate the estimated errors in the observed values, $\bar{u}_o$. The uncertainty in $\bar{V}$ or $\bar{u}_o$ depends on: 1) the errors in the individual redshifts, 2) the velocity dispersion in the cluster, and 3) the size of the sample of measured redshifts in the cluster together with the extent to which the sample is contaminated by including redshifts of non-member galaxies.

Compared with other cosmological observables, the errors in redshifts are small. The total errors in redshifts are more or less independent of their sizes, and their relative errors are consequently small except for nearby objects. If we may take the redshifts published in the Humason - Mayall- Sandage Catalogue as typical, then we may assume in accordance with their estimates that each measured velocity is within 175 km/sec of the true value, and over half are within 35 km/sec of the true value. Assuming the extreme bound of 175 km/sec for each redshift, the <u>relative</u> error in the individual redshifts is less than 3 percent (except for the nearby Virgo cluster). The relative error in the more distant clusters becomes very small and the values are correct to the third decimal place in z. The dispersion σ_i, for individual redshifts will be taken as 35 km/sec.

A much larger source of uncertainty in the mean redshifts of clusters is the velocity dispersion. The values of the means, $\overline{V}$, and the dispersions, σ_V, for all clusters for which three or more redshift measurements have been obtained are given in Table 2. The contribution of σ_i to the error in $\overline{V}$ is negligible compared with σ_V for all clusters except the Hydra cluster (where the effect of σ_i has been included). The remaining columns in Table 2 give the relative error, $du/u = \sigma_V/\overline{V}(1 + \overline{z})\sqrt{n}$; the logarithmic error, $\delta_u = \log(u+du)-\log(u)$; and the observed residuals, ρ, from Table 1.

INSERT Table 2

For the clusters in Table 2, the magnitudes of the estimated errors of mean cluster redshifts δ_u, are of the same order as the residuals, ρ, with the mean δ_u equal to 0.010 and the mean $|\rho|$ equal to 0.005. This agreement is consistant with the interpretation that the structural redshifts, P, predicted by equation (2) are expected values of the means and that the residuals, ρ, are attributable to errors of observation.

However, of the remaining clusters for which there are only one or two measured redshifts, the mean $|\rho|$ of 0.006 is lower than expected. If we assume that the velocity dispersion of the remaining clusters is equal to the mean velocity dispersion, 720 km/sec, of the clusters in Table 2, then the mean value of δ_u for these remaining clusters is 0.008.

Table 2

ESTIMATED ERRORS FOR MEAN CLUSTER REDSHIFTS

Cluster	Name	n	$\overline{V}$	σ_V	$\sigma_V/\sqrt{n}$	$\frac{du}{u}$	δ_u	ρ
	Virgo	73	1136	643	76	0.067	0.028	- .0043
0316+4121	Perseus	7	5435	715	270	0.049	0.021	.0006
0123-0137	"NGC 541"	43	5439	450	69	0.012	0.005	.0006
1257+2812	Coma	50	6432	1745	246	0.038	0.016	- .0035
1627+3937	Abell 2199	19	9028	864	198	0.021	0.009	.0096
1603+1755	Hercules	15	10775	652	169	0.015	0.007	- .0028
2308+0720	Pegasus II	3	12821	662	383	0.029	0.012	- .0097
1145+5559	Ursa Major I	4	15269	358	179	0.011	0.005	- .0097
1520+2754	Corona Borealis	8	21651	1294	457	0.020	0.008	.0042
1534+3749		3	45951	408	236	0.005	0.002	- .0016
0855+0321	Hydra	3	60860	137	80	0.001	0.0004	.0094

Note: Table 2 includes only those clusters with 3 or more observed redshifts.

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The fact that the mean δ_u of the second set of clusters is less than that of the better observed clusters of Table 2 is due in part to the greater distances of the second set. But the low δ_u's may have another explanation. It is an observed fact that many clusters are centered on one or two galaxies which are appreciably brighter and larger than other cluster members. It is seen in Table 3 that for nearby well observed

INSERT Table 3

clusters, the values of the redshifts of the brightest galaxy, V_B, are very close to the cluster mean $\bar{V}$. If this is true for the more distant clusters, then the observational selectivity of the brightest galaxies in clusters generates a bias which gives better mean redshifts from small samples than is statistically expected.

The effects of contamination of the cluster sample with non-cluster members on the value $\bar{V}$ may become large as fainter clusters are included[12]. This serves to increase the size of the residuals in the well observed sample. However, in the present paper, no selection criterion has been used for selecting or rejecting individual redshifts to determine the $\bar{V}$'s of Table 1. All published redshifts have been used with equal weight.

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Table 3

COMPARISON OF CLUSTER MEAN REDSHIFT WITH BRIGHTEST GALAXY REDSHIFT

Cluster	Name	Brightest Galaxy	V_B	$\lvert\bar{V}-V_B\rvert$
	Virgo	NGC 4486	1171	35
0316+4121	Perseus	NGC 1275	5293	142
0123-0137	"NGC 541"	NGC 547	5472	33
0257+2812	Coma	NGC 4889	6428	4
1627+3937	Abell 2199	NGC 6166	9082	54

In order to examine the probability that the agreement between the observed and structural mean redshifts is not reproducible by chance, we shall use as a statistic the number of fits to within a prescribed δ occurring between test samples and the discrete values defined by the right member of equation (2). The test samples to be compared are 1) the observed values of the redshifts and 2) a set of random values drawn from a suitable density distribution defined over the same range as the observed sample, $0.7700 \leq \log \bar{u}_o \leq 2.4300$.

If y is the number of clusters with redshift $\bar{u}_o$, then the density function derived from the envelope of the observed sample is $\log y = 0.76 - 2 \log \bar{u}_o$ over the above range. This function is adopted for the density distribution of the random values.

Insert Table 4

The results of two Monte Carlo experiments programmed for the RAND 7044 computer are given in Table 4. In Table 4a, the observed sample of 28 redshifts is compared with 25 sets of 28 random values selected from the above density distribution. The comparison in Table 4 is between observational and random data fits to equation (1) written as

$$P = A - \log N(N+1)$$

where A is independently selected for the 28 observed redshifts and for each of the 25 sets of 28 random redshift values. In both the observational and each random case, the machine program selects that value of A which gives the maximum number of fits within the prescribed δ. For the observed redshifts, the machine-selected-A equals 3.4976. (This is the same as A equal to $\omega - \log M(M+1)$ in equation (2) with M set equal to 2 and $\omega = 4.2758$.)

Table 4

COMPARISON OF OBSERVATIONAL AND RANDOM FITS

	δ	N(obs)	$\overline{N}_R$	$N_R$max	σ_R	t	Probability
a.	0.020	26	24.56	27	1.24	1.17	0.242
	0.010	19	16.40	18	0.98	2.65	0.008
	0.005	14	11.28	14	1.04	2.62	0.009
	0.003	10	8.36	11	1.05	1.56	0.119
b.	0.020	28	27.44	28	0.50	1.13	0.258
	0.010	24	20.44	23	1.13	3.14	0.002
	0.005	17	14.48	16	0.98	2.56	0.010
	0.003	12	10.80	12	0.85	1.41	0.159

The first column in Table 4 gives the prescribed δ's defining the allowed margins of fit, $P \pm \delta$. The second column, N(obs), gives the number of fits (within δ) to ~~between~~ the 28 observed redshifts, ~~$\bar{u}/\lambda_0$,~~ using the above equation with $A = 3.4976$. N_R is the number of fits (within δ) occuring in any one sample of 28 random redshift values. Since there are 25 sets of computer generated N_R's, the third column of Table 4 lists $\bar{N}_R$, the most frequent value of N_R occuring in all 25 sets for a prescribed δ. $N_R max$ is the largest number of fits in the 25 sets and σ_R in the dispersion in N_R. In the sixth column, t is the standardized variable, $[N(obs) - \bar{N}_R]/\sigma_R$. The last column gives the probability, derived ~~devised~~ from t using the standard error integral, that N(obs) occurs by chance.

It is interesting that the largest value of t occurs at the value of δ which equals the estimated observational error, δ_u. This is expected if the values of N_R derive from a unimodal or monatonic density distribution and the values of N(obs) derive from a multi-modal density distribution consisting of a set of error functions whose means are located at the discrete values P(2,N) and whose standard deviations are equal to δ_u. In this situation, the size of N_R will decrease uniformly with decreasing δ, i.e. ~~W~~while the size of N(obs) will be relatively insensitive to δ until ~~while~~ $\delta = \delta_u$. For $\delta < \delta_u$, N(obs) will decrease sharply. This results in the difference, $N_R - N_{(obs)}$, having a maximum near $\delta = \delta_u$.

Table 4b is the same as 4a except that the comparison of the observed redshifts and random redshifts is for the M = 2 and M = 4 sequences taken together. Tables 4a and 4b both show that for the significant value of $\delta = \delta_u = 0.010$, the probability that N(obs) is due to chance is less than one in one hundred.

Table 5

COMPARISON OF OBSERVATIONAL AND RANDOM FITS TO EQUATION 2a

δ	N(obs)	$\overline{N}_R$	$N_R max$	σ_R	t	Probability
0.020	23	18.40	21	2.32	1.99	0.047
0.010	20	11.76	16	2.45	3.36	0.001
0.005	13	6.92	11	2.30	2.65	0.008

Of additional significance is the test summarized in Table 5. We recall that equations (1) and (2) were originally derived from relations between mean redshifts and diameters in five clusters (Coma, UMI, Corona Borealis, Boötes and UMII). The original equation (2), based on these five clusters was

$$\log \bar{u}_o = 4.2792 - \log N(N+1) - \log M(M+1) \tag{2a}$$

if equation (2a) is adopted as the complete hypothesis, including the value of the constant, ш, and is tested against the remaining 23 clusters, we get the results reported in Table 5.

insert Table 5

no ¶ Here we compare the number of fits in the remaining 23 clusters to the values of equation (2a) with the number of fits occurring in 25 sets of 23 random redshift values. In this test the value of the constant in each set of random values is not floating to maximize the number of fits, but is held fixed at the predicted value, ш = 4.2792. The most significant level of fit again occurs at $\delta = \delta_u$ with the probability of the number of observed fits occurring by chance equal to one in one thousand.

A conservative assessment of the statistical results of Tables 4 and 5 allows the hypothesis of regularized structure in the distribution of clusters, as expressed by equation 2, at least equal admissability with the hypothesis of random distribution. (The values of t actually suggest that greater weight be given to the hypothesis of discretized structure.) However, in the absence of theoretical justification for equation (2), it cannot be assumed that it is the best representation for the discretized structure which seems to exist. Several arbitrary expressions could be tested for possible better fits, but a more direct approach is to develop a cosmological theory allowing for inhomogeneities in the distribution of matter.

With empirical refinements and theoretical justification left for the future, the salient point of this paper is the inference for the existence of regularized structure in the large scale distrubution of matter. If we may assume for the present that equation (2) does represent the cosmic distribution of clusters, it follows that the allowable values for the temporal and radial coordinates of matter concentrations, as seen by an observer located at one of these concentrations, may be mapped into a bi-discretized net developable in integral steps from a single constant.

Following this interpretation, if the clusters are located on shells with discrete radii centered on a given observer, the cosmological principle requires that the clusters also be distributed on shells with discrete radii centered on all other observers. (Observers are here postulated to be located at cluster centers.) It follows that a discretized cluster distribution compatible with the cosmological principle must exhibit structure in angular distribution as seen by any observer. This is amenable to observational verification. In fact, the statistical evidence for second order clustering and sub-clustering, which is open to certain objections when interpreted as physical clustering, may be properly interpreted as the optical clustering expected with discretized distributions.

While the possibility of regularized structure on a cosmic scale poses several difficult questions and will require many additional observations, if confirmed, it will provide new techniques for cosmological investigations.

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*DISCRETIZED STRUCTURE IN THE DISTRIBUTION OF CLUSTERS OF GALAXIES**

By Albert G. Wilson

THE RAND CORPORATION, SANTA MONICA, CALIFORNIA

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From the basic Einstein equations positing the equivalence of the geometrical and physical tensors,

$$R_{AB} - {}^1/_2 R g_{AB} = \kappa T_{AB},$$

Edelen[1, 2] and Thomas[3] have independently predicted discretization in the geometrical sizes of galaxies. Theory suggests that the diameters or major axes of the galaxies should be, under certain conditions, proportional to a sequence of eigen numbers of the form $[n(n + 1)]^{1/2}$, where n is a positive integer. This theoretically predicted size discretization has been observationally confirmed by Wilson[4, 5] for samples of field and cluster galaxies in the case of ellipticals of small eccentricity.

The evidence for discretized distributions in sizes of galaxies suggests the possibility that there exist further regularities in the large-scale distributions of matter which have not been suspected. The present paper reports an observed regularity in the mean red shifts of clusters of galaxies which appears unlikely for randomly distributed clusters. There is at present no cosmological theory which predicts such regularities. This is not surprising, however, since most current cosmological models assume that the observed granularities in the distribution of matter—galaxies and clusters of galaxies—may be approximated by a uniform, smooth distribution. No theory which ignores the granularities can be held accountable for structure in the granularities. The epistemological difficulties which arise from smoothing assumptions have been well expressed by Neyman:[6] "The contrast between the domain of current observations of individual galaxies and their clusters on the one hand, and the theory dealing with the smoothed-out substratum on the other, is the more striking because every effort to verify empirically the conclusions of the theory must deal with observations of objects whose very existence this theory ignores. Thus, in order to effect a verification it is necessary to adopt a number of *ad hoc* hypotheses and, as a result, the conclusions are open to question."

Furthermore, when observations are designed and interpreted with specific theoretical models, there is a tendency to ignore information which does not bear directly on questions posed in the verification of the model. Accordingly, the design and interpretation of observations around cosmological models which assume a smoothed-out distribution of matter, tend to de-emphasize the investigation of possible regularities in the distribution of the observed granularities. Most published discussions concerning the distributions of galaxies and clusters have emphasized only the stochastic aspects of counts, magnitudes, and sizes.

In investigations of galaxy size discretization, the angular diameters of galaxies in five clusters (from the data of ref. 4) were combined with mean cluster red shifts, $\bar{V}$, to obtain linear diameters by using a velocity distance relation of the form

$$H\bar{u}_0 d = 1,$$

where H is a constant, $\bar{u}_0 = (c + \bar{V})/\bar{V}$, c is the velocity of light, and d is the distance. Comparisons of the resulting diameter discretization sequences suggested a relation between mean cluster red shifts of the form

$$\log \bar{u}_0 + \log N(N + 1) = \text{constant}, \tag{1}$$

where N is a positive integer. Equation (1) was tested against the 28 clusters for which published red shifts are available. It was found that there exist integers, N, such that the mean cluster red shifts of 20 of the 28 clusters fit equation (1) to within the estimated observational error (these are those clusters listed in Table 1 for which the parameter $M = 2$). Of the 8 remaining clusters, integers N were found such that 7 fit equation (1) with a different constant in the right member (those clusters for which $M = 4$ in Table 1). Finally, it was found that all values of the constant in the right member of equation (1) can be expressed as

$$\sqcup - \log M(M + 1),$$

where $\sqcup$ is a constant, and M takes on values 2, 4, and 6. A single empirical formula expressing the mean cluster red shifts in terms of two integer parameters, M and N, and one arbitrary constant, $\sqcup$, can thus be written as

$$\log \bar{u}_0 = \sqcup - \log M(M + 1) - \log N(N + 1), \tag{2}$$

where the constant, $\sqcup$, has a least square value of 4.2765 for the 28 clusters.

Table 1 collects the generally available observational data, reducing them to cluster mean red shifts. The first five columns are: (1) the cluster designation in terms of its 1950.0 position; (2) the common name for the cluster; (3) the number of individual red shifts available for the calculation of the mean; (4) the mean velocity for the cluster,

$$\bar{V} = \frac{c}{n} \sum_{i=1}^{n} z_i,$$

where $z_i = (\delta\lambda/\lambda)_i$ is the measured spectral displacement; and (5) the logarithm of the quantity $\bar{u}_0 = (c + \bar{V})/\bar{V}$. The source of all red shifts in Table 1 is the Humason-Mayall-Sandage Catalogue,[7] except when indicated by a reference number in the second column.

The comparisons between the observed mean red shift $\bar{u}_0$ and the right member of equation (2), which we shall designate by $P(M,N)$ and call the *structural mean red shift*, are given in the right-hand side of Table 1. Columns 6–9 are, consecutively:

TABLE 1

MEAN CLUSTER RED SHIFTS OBSERVED AND CALCULATED

Cluster	Name	No. of red shifts	$\bar{V}$	$\log \bar{u}_0$	N	M	P	$P - \log \bar{u}_0$
	Virgo	73	1136	2.4234	3	2	2.4191	−0.0043
0316 + 4121	Perseus	7	5435	1.7494	7	2	1.7500	0.0006
0123 − 0137	"NCG 541" [8]	43	5439	1.7494	7	2	1.7500	0.0006
1257 + 2812	Coma[9]	50	6432	1.6780	4	4	1.6745	−0.0035
1627 + 3937	Abell 2199[10]	19	9028	1.5344	9	2	1.5440	0.0096
1603 + 1755	Hercules[11]	15	10775	1.4597	10	2	1.4569	−0.0028
2308 + 0720	Pegasus II	3	12821	1.3874	11	2	1.3777	−0.0097
2322 + 1425		2	13187	1.3757	11	2	1.3777	0.0020
1145 + 5559	Ursa Major I	4	15269	1.3149	12	2	1.3052	−0.0097
0106 − 1536	Haufen A	2	15872	1.3012	12	2	1.3052	0.0040
1024 + 1039	Leo	1	19489	1.2147	7	4	1.2272	0.0125
1239 + 1853		2	21533	1.1741	14	2	1.1761	0.0020
1520 + 2754	Corona Borealis	8	21651	1.1719	14	2	1.1761	0.0042
0705 + 3506	Gemini	2	23366	1.1360	15	2	1.1181	−0.0179
0348 + 0613		1	25644	1.1038	15	2	1.1181	0.0143
1513 + 0433	"Shane Cloud"	1	28333	1.0640	16	2	1.0637	−0.0003
1431 + 3146	Boötes	2	39367	0.9356	10	4	0.9341	−0.0015
1055 + 5702	Ursa Major II	2	40860	0.9213	19	2	0.9185	−0.0028
1153 + 2341	Abell 1413	2	42784	0.9037	7	6	0.9049	0.0012
1534 + 3749		3	45951	0.8767	20	2	0.8751	−0.0016
0025 + 2223		2	47836	0.8616	11	4	0.8549	−0.0067
1228 + 1050		2	49514	0.8487	11	4	0.8549	0.0062
0138 + 1840		1	51908	0.8312	21	2	0.8337	0.0025
1309 − 0105		1	52362	0.8280	21	2	0.8337	0.0057
1304 + 3110	Coma B	1	54917	0.8104	21	2	0.8337	−0.0163
0925 + 2044		1	57498	0.7937	22	2	0.7941	0.0004
1253 + 4422		1	59382	0.7819	12	4	0.7824	0.0005
0855 + 0321	Hydra	3	60860	0.7730	12	4	0.7824	0.0094

$P = 4.2765 - \log M(M + 1) - \log N(N + 1)$; $\log \bar{u}_0 = \log [(c + \bar{V})/\bar{V}]$.

N, M, P, and the residuals, $P - \log \bar{u}_0$. The values of $\log \bar{u}_0$ and of the residuals are usually given to more significant places than are consistent with the precision of the observations. This is done to avoid contamination of discretization by possible round-off effects. The residuals range from less than 0.003 (lowest meaningful value) to 0.018, corresponding to a discrepancy of 4 per cent.

It is apparent that if ⊔ is taken very large, we could find large integer values of M and N that would give values of $P(M,N)$ fitting the set of observations to within any prescribed residual. It is the low values of N that are the most stringent statistically. Accordingly, the integers, N, in Table 1 are selected as the smallest integers which provide as many distinct values over the range of $\log \bar{u}_0$ (i.e., 2.4234 to 0.7730) as there are distinct cluster red shifts. Note also that $P(M,N)$ may assume the same value for different combinations of M and N, with the result that the identification of M and N is not always unique.

While the sequences for $M = 2$ and $M = 4$ each contain enough members to establish the statistical significance of equation (1) for two different constants, there is no justification in the present sample of red shifts, other than mathematical convenience, for the writing of equation (2) with two integral parameters, M and N, and for assigning the unusually rich cluster Abell 1413 to a tentative $M = 6$ sequence. If this is done, however, taking the exponential of both sides of equation (2) leads to the form,

$$\frac{\bar{z}}{1 + \bar{z}} = \frac{M(M + 1) \cdot N(N + 1)}{(137.5)^2},$$

where again $\bar{z}$ is the dimensionless quantity $(\bar{V}/c)$. The numerical value assumed by the fundamental constant, ⊔⊔, affords us a curious numerical coincidence with the fine structure constant, which may or may not be meaningful but should be noted.

Before investigating the statistical significance of the agreement between the observed and structural mean cluster red shifts, it is necessary to evaluate the estimated errors in the observed values, $\bar{u}_0$. The uncertainty in $\bar{V}$ or $\bar{u}_0$ depends on: (1) the errors in the individual red shifts, (2) the velocity dispersion in the cluster, and (3) the size of the sample of measured red shifts in the cluster, together with the extent to which the sample is contaminated by including red shifts of nonmember galaxies.

Compared with other cosmological observables, the errors in red shifts are small. The total errors in red shifts are more or less independent of their sizes, and their relative errors are consequently small except for nearby objects. If we may take the red shifts published in the Humason-Mayall-Sandage Catalogue as typical, then we may assume in accordance with their estimates that each measured velocity is within 175 km/sec of the true value, and over half are within 35 km/sec of the true

TABLE 2

ESTIMATED ERRORS FOR MEAN CLUSTER RED SHIFTS

Cluster	Name	n	$\bar{V}$	σ_V	$\sigma_V\sqrt{n}$	du/u	δ_u	ρ
	Virgo	73	1136	643	76	0.067	0.028	−0.0043
0316 + 4121	Perseus	7	5435	715	270	0.049	0.021	0.0006
0123 − 0137	"NGC 541"	43	5439	450	69	0.012	0.005	0.0006
1257 + 2812	Coma	50	6432	1745	246	0.038	0.016	−0.0035
1627 + 3937	Abell 2199	19	9028	864	198	0.021	0.009	0.0096
1603 + 1755	Hercules	15	10775	652	169	0.015	0.007	−0.0028
2308 + 0720	Pegasus II	3	12821	662	383	0.029	0.012	−0.0097
1145 + 5559	Ursa Major I	4	15269	358	179	0.011	0.005	−0.0097
1520 + 2754	Corona Borealis	8	21651	1294	457	0.020	0.008	0.0042
1534 + 3749		3	45951	408	236	0.005	0.002	−0.0016
0855 + 0321	Hydra	3	60860	137	80	0.001	0.0004	0.0094

Note: Table 2 includes only those clusters with 3 or more observed red shifts.

value. Assuming the extreme bound of 175 km/sec for each red shift, the *relative* error in the individual red shifts is less than 3 per cent (except for the nearby Virgo cluster). The relative error in the more distant clusters becomes very small, and the values are correct to the third decimal place in z. The dispersion σ_i, for individual red shifts, will be taken as 35 km/sec.

A much larger source of uncertainty in the *mean* red shifts of clusters is the velocity dispersion. The values of the means, $\bar{V}$, and the dispersions, σ_V, for all clusters for which three or more red shift measurements have been obtained are given in Table 2. The contribution of σ_i to the error in $\bar{V}$ is negligible compared with σ_V for all clusters except the Hydra cluster (where the effect of σ_i has been included). The remaining columns in Table 2 give the relative error, $du/u = \sigma_V/\bar{V}(1 + \bar{z})\sqrt{n}$; the logarithmic error, $\delta_u = \log (u + du) - \log (u)$; and the observed residuals, ρ, from Table 1.

For the clusters in Table 2, the magnitudes of the estimated errors of mean cluster red shifts δ_u, are of the same order as the residuals, ρ, with the mean δ_u equal to 0.010 and the mean $|\rho|$ equal to 0.005. This agreement is consistent with the interpretation that the structural red shifts, P, predicted by equation (2), are expected values of the means and that the residuals, ρ, are attributable to errors of observation.

However, of the remaining clusters for which there are only one or two measured red shifts, the $|\rho|$ of 0.006 is lower than expected. If we assume that the velocity dispersion of the remaining clusters is equal to the mean velocity dispersion, 720 km/sec, of the clusters in Table 2, then the mean value of δ_u for these remaining clusters is 0.008.

The fact that the mean δ_u of the second set of clusters is less than that of the better observed clusters of Table 2 is due in part to the greater distances of the second set. But the low δ_u's may have another explanation. It is an observed fact that many clusters are centered on one or two galaxies which are appreciably brighter and larger than other cluster members. It is seen in Table 3 that for nearby well-observed clusters, the values of the red shifts of the brightest galaxy, V_B, are very close to the cluster mean $\bar{V}$. If this is true for the more distant clusters, then the observational selectivity of the brightest galaxies in clusters generates a bias which gives better mean red shifts from small samples than is statistically expected.

The effects of contamination of the cluster sample with noncluster members on the value $\bar{V}$ may become large as fainter clusters are included.[12] This serves to increase the size of the residuals in the well-observed sample. However, in the present paper, no selection criterion has been used for selecting or rejecting individual red shifts to determine the $\bar{V}$'s of Table 1. All published red shifts have been used with equal weight.

In order to examine the probability that the agreement between the observed and structural mean red shifts is not reproducible by chance, we shall use as a statistic the number of fits to within a prescribed δ occurring between test samples and the discrete values defined by the right member of equation (2). The test samples to be compared are (1) the observed values of the red shifts, and (2) a set of random values drawn from a suitable density distribution defined over the same range as the observed sample, $0.7700 \leq \log \bar{u}_0 \leq 2.4300$.

If y is the number of clusters with red shift $\bar{u}_0$, then the density function derived from the envelope of the observed sample is $\log y = 0.76 - 2 \log \bar{u}_0$ over the above range. This function is adopted for the density distribution of the random values.

The results of two Monte Carlo experiments programed for the RAND 7044 computer are given in Table 4. In Table 4*a*, the observed sample of 28 red shifts is compared with 25 sets of 28 random values selected from the above density distribution. The comparison in Table 4 is between observational and random data fits to equation (1) written as

$$P = A - \log N(N + 1),$$

where A is independently selected for the observed red shifts and for each of the

TABLE 3

Comparison of Cluster Mean Red Shift with Brightest Galaxy Red Shift

Cluster	Name	Brightest galaxy	V_B	$\lvert\bar{V} - V_B\rvert$
	Virgo	NGC 4486	1171	35
0316 + 4121	Perseus	NGC 1275	5293	142
0123 − 0137	"NGC 541"	NGC 547	5472	33
0257 + 2812	Coma	NGC 4889	6428	4
1627 + 3937	Abell 2199	NGC 6166	9082	54

TABLE 4

COMPARISON OF OBSERVATIONAL AND RANDOM FITS

	δ	N(obs)	$\bar{N}_R$	$N_R$max	σ_R	t	Probability
(a)	0.020	26	24.56	27	1.24	1.17	0.242
	0.010	19	16.40	18	0.98	2.65	0.008
	0.005	14	11.28	14	1.04	2.62	0.009
	0.003	10	8.36	11	1.05	1.56	0.119
(b)	0.020	28	27.44	28	0.50	1.13	0.258
	0.010	24	20.44	23	1.13	3.14	0.002
	0.005	17	14.48	16	0.98	2.56	0.010
	0.003	12	10.80	12	0.85	1.41	0.159

25 sets of 28 random red shift values. In the observational and in each random case, the machine program selects that value of A which gives the maximum number of fits within the prescribed δ. For the observed red shifts, the machine-selected A equals 3.4976. (This is the same as A equal to $\sqcup - \log M(M + 1)$ in equation (2) with M set equal to 2 and $\sqcup = 4.2758$.)

The first column in Table 4 gives the prescribed δ's defining the allowed margins of fit, $P \pm \delta$. The second column, N(obs), gives the number of fits (within δ) to the 28 observed red shifts, using the above equation with $A = 3.4976$. N_R is the number of fits (within δ) occurring in any one sample of 28 random red shift values. There are 25 sets of computer-generated N_R's. The third column of Table 4 lists $\bar{N}_R$, the mean of the values of N_R occurring in all 25 sets for a prescribed δ. $N_R$max is the largest number of fits in the 25 sets, and σ_R is the dispersion in N_R. In the sixth column, t is the standardized variable, $[N(\text{obs}) - \bar{N}_R]/\sigma_R$. The last column gives an estimated probability, derived from the standard error integral, assuming a normal distribution for N_R.

It is interesting that the largest value of t occurs at the value of δ which equals the estimated observational error, δ_u. This is expected if the values of N_R derive from a unimodal or monatonic density distribution and the values of N(obs) derive from a multimodal density distribution consisting of a set of error functions whose means are located at the discrete values $P(2,N)$ and whose standard deviations are equal to δ_u. In this situation, the size of N_R will decrease uniformly with decreasing δ, while the size of N(obs) will be relatively insensitive to δ until $\delta = \delta_u$. For $\delta < \delta_u$, N(obs) will decrease sharply. This results in the difference, $N_R - N_{(\text{obs})}$, having a maximum near $\delta = \delta_u$.

Table 4*b* is the same as 4*a* except that the comparison of the observed red shifts and random red shifts is for the $M = 2$ and $M = 4$ sequences taken together. Tables 4*a* and 4*b* both show that for the significant value of $\delta = \delta_u = 0.010$, the probability is minimum.

The test of statistical significance is summarized in Table 5. We recall that equations (1) and (2) were originally derived from relations between mean red shifts and diameters in five clusters (Coma, UMI, Corona Borealis, Boötes, and UMII). The original equation (2), based on these five clusters, was

$$\log \bar{u}_0 = 4.2792 - \log N(N + 1) - \log M(M + 1); \qquad (2a)$$

if equation (2*a*) is adopted as the *complete* hypothesis, including the value of the constant, $\sqcup$, and is tested against the remaining 23 clusters, we get the results reported in Table 5. Here we compare the number of fits in the *remaining* 23 clusters to the values of equation (2*a*) with the number of fits occurring in 25 sets of *23*

TABLE 5

COMPARISON OF OBSERVATIONAL AND RANDOM FITS TO EQUATION (2a)

δ	N(obs)	$\bar{N}_R$	$N_R$max	σ_R	t	Probability
0.020	23	18.40	21	2.32	1.99	0.047
0.010	20	11.76	16	2.45	3.36	0.001
0.005	13	6.92	11	2.30	2.65	0.008

random red shift values. In this test the value of the constant in each set of random values is not floating to maximize the number of fits, but is held *fixed* at the predicted value, ⊔ = 4.2792. The most significant level of fit again occurs at $\delta = \delta_u$ with the probability of the number of observed fits being accounted for by a chance mechanism being 1 in 1,000.

A conservative assessment of the statistical results of Tables 4 and 5 allows the hypothesis of regularized structure in the distribution of clusters, as expressed by equation 2, at least equal admissibility with the hypothesis of random distribution. (The values of t actually suggest that greater weight be given to the hypothesis of discretized structure.) However, in the absence of theoretical justification for equation (2), it cannot be assumed that it is the best representation for the discretized structure which seems to exist. Several arbitrary expressions could be tested for possible better fits, but a more direct approach is to develop a cosmological theory allowing for inhomogeneities in the distribution of matter.

With empirical refinements and theoretical justification left for the future, the salient point of this paper is the inference for the existence of regularized structure in the large-scale distribution of matter. If we may assume for the present that equation (2) does represent the cosmic distribution of clusters, it follows that the allowable values for the temporal and radial coordinates of matter concentrations, as seen by an observer located at one of these concentrations, may be mapped into a bidiscretized net developable in integral steps from a single constant.

Following this interpretation, if the clusters are located on shells with discrete radii centered on a given observer, the cosmological principle requires that the clusters also be distributed on shells with discrete radii centered on all other observers. (Observers are here postulated to be located at cluster centers.) It follows that a discretized cluster distribution compatible with the cosmological principle must exhibit structure in angular distribution as seen by any observer. This is amenable to observational verification. In fact, the statistical evidence for second-order clustering and subclustering, which is open to certain objections when interpreted as *physical* clustering, may be properly interpreted as the *optical* clustering expected with discretized distributions.

While the possibility of regularized structure on a cosmic scale poses several difficult questions and will require many additional observations, if confirmed, it will provide new techniques for cosmological investigations.

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*ORGANIZATION IN A GAMETOPHYTE CALLUS OF LYCOPODIUM AND ITS MORPHOGENETIC IMPLICATIONS**

BY AUGUSTUS E. DE MAGGIO†

COLLEGE OF PHARMACY, RUTGERS, THE STATE UNIVERSITY, NEWARK, NEW JERSEY

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The study of organization in plants has been stimulated in recent years primarily as a result of refinements in the techniques of tissue and organ culture. It is now well established that vegetative cells of higher plants can be grown *in vitro* and propagated as relatively homogeneous, undifferentiated parenchyma or callus tissue. Recent findings suggest that these cells, even though maintained in an undifferentiated state for several years, still retain the potentiality for differentiation of vascular tissue, roots, shoots, and even whole plants. The well-documented studies of Skoog and Miller,[1] Steward and co-workers,[2-4] Reinert,[5] Halperin and Wetherell,[6] and others[7] have demonstrated that mature diploid cells can be induced to develop into plantlings representative of the species. One recognizes, therefore, that totipotency may be inherent in any vegetative cell of the plant and can be expressed to varying degrees when stimulated by the appropriate conditions of nutrient culture.

The potentiality for multiple pathways of cellular expression is equally characteristic of the vascular cryptogams although fewer studies have been carried out on these groups of plants. The regular alternation of a haploid gametophyte generation with a diploid sporophyte generation (both can be maintained and grown indefinitely in sterile nutrient culture) is an advantageous feature of these plants for morphogenetic experimentation. Studies of apogamy and apospory indicate that vegetative cells of the gametophyte can be induced to develop into sporophytic structures,[8-10] and also that vegetative cells of sporophytic origin can give rise to gametophytes.[11-13] It would appear that the totipotency of cells in both haploid and diploid plants is retained and can be evoked by manipulating the environmental conditions of growth. The implication from these and related studies[14, 15] is that the genetic complement of plants in either generation imposes

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Should it be spasmodic universe? U563

A DECISION as to which of the current cosmological theories is the most plausible is almost certain to rest upon more detailed observations of the distant galaxies. Put simply, if the so-called "steady-state" theory of the universe is correct and the universe, though continuously expanding, maintains a constant density by the spontaneous creation of matter, then space everywhere should contain a mixture of galaxies of all ages. Conversely, if the universe has evolved from one primordial "big bang", the most distant galaxies, from which the light or radio waves take hundreds of millions of years to reach us, should appear to be the youngest in age. Looking outwards into space we would see younger and younger stages of the evolutionary process.

Last year an American physicist, D. G. B. Edelen, of the Rand Corporation, presented a theory to the annual meeting of the American Astronomical Society, backed up by some preliminary experimental observations made by another researcher from Rand, suggesting a third type of galactic arrangement. His ideas have now been incorporated into a tentative new cosmology derived by T. Y. Thomas, of Indiana University (*Proceedings of the National Academy of Sciences*, Vol. 51, No. 5, p. 718).

Essentially what Edelen suggested was that, rather than having a completely variable and continuous range of diameters, the sizes of the extragalactic nebulae might fall into discrete groups, rather as if they were "quantised". The mathematical reasoning that led to the idea was, naturally, fairly abstruse, but measurements made by A. G. Wilson, of Rand, on photographs of certain galaxies obtained with the Mount Palomar 200-inch telescope, taken in conjunction with their ranges deduced from the "red shifts" in their light, indicated that there may be some foundation for the hypothesis (*Astronomical Journal*, Vol. 68, p. 547).

The way in which galaxies probably evolve is from a simple spherical form, through an elliptical one, to a mature form having characteristic spiral arms like our own Milky Way. Wilson studied the younger elliptical ones, classified as the Eo galaxies, occurring in the clusters of Coma, Corona Borealis, Ursa Major and Bootes. They seemed to have diameters that could be grouped according to a simple mathematical relationship. Clearly, if his results are substantiated, this galactic structure is incompatible with the theory of continuous creation of matter in the universe; equally it does not fit the idea that the universe has been steadily evolving from some catastrophic event.

The cosmological theory of Thomas is completely at variance with the new gravitational theory of Hoyle and Narlikar (see *New Scientist*, Vol. 22, p. 730) in that it assumes fields of force rather than "action at a distance" to be important. It starts from the belief that originally the universe contained no "ponderable matter". The accidental creation of the first material particle by some disturbance of the fields in this placid universe—like a ripple on a smooth pond—led to further disturbances and the creation of more particles. In time they built up and aggregated to form one large body. The ultimate explosion of this body then began the expanding phase of the universe as astronomers are now familiar with it.

However, the violence of the first episode set up extremely intense gravitational waves which, Thomas supposes, lapped backwards and forwards across the "pond" of space—causing, each time they returned to the site of their origin, an "epoch" in which the conditions were right for the creation of a fresh batch of galaxies. His theory is thus an evolutionary one but also stipulates that spontaneous creation of matter occurs—though in his conception it is discontinuous rather than continuous creation as in the steady-state theories.

H II regions (66) gives a theoretical interpretation of the correlation between diameters of the largest H II regions and the morphological type of the galaxy. He concludes that the age of the galaxies is approximately the same but that their evolutionary rates depend on the morphological type. J. L. Sérsic and R. Sisteró are investigating the pulsational stability of a plasma in an external axisymmetrical magnetic field under the action of a concentric gravitational field arising from another mass distribution. This may have application to matter in elliptical galaxies.

According to S. van der Bergh (67) studies of the metal abundances of stars in the Galaxy lead to the following conclusions. (*a*) The heavy element enrichment of the interstellar medium was well advanced at the termination of the halo phase of stellar evolution. (*b*) The rate of heavy element formation in the Galaxy has declined more rapidly than the rate of star formation. (*c*) The enrichment of the interstellar medium in heavy elements has been negligible during the last $4{\cdot}5 \cdot 10^9$ years.

galaxy

A. G. Wilson and D. Edelen have conducted work at the Rand Corporation on relativistic discretization of cluster diameters (Preliminary results were reported at the meeting of the American Astronomical Society in July 1963). Edelen has shown that the Einstein theory of general relativity, when used in conjunction with the epistological equivalents of certain well known properties of galaxies, predicts a relation between the galaxian semi-major axis r and the eccentricity ϵ (or ellipticity) of the form $r(n, m, \epsilon) = \text{const.}$, where n is a positive integer and $0 \leq m < n$. In the particular case $\epsilon = 0$, the relation between r and n takes the form,

$$r^2\xi = n(n + 1)$$

independent of m, where ξ is a physical parameter corresponding to the jump in the total energy density across the surface of the world tube representing the galaxy, as seen by an observer moving along an intrinsic time line of the surface. If ξ is constant, or of limited variation, it follows that the diameters of Eo galaxies should exhibit discretization of size.

The earlier data of Wilson (68) which first suggested discretization among globular galaxies have been re-assessed and combined with new measures. The present observational confirmation rests on (1) Wilson's angular diameters of Eo galaxies in six clusters re-measured on 200-inch plates. (2) The diameters of all Eo galaxies in the new Reference Catalogue of de Vaucouleurs, (3) the fine structure in Abell's (69) luminosity function of the Coma cluster. The diameter redshift relation for cluster galaxies confirms the Hubble law and reveals the hitherto unsuspected relation that the redshifts of all clusters so far published obey the empirical relationship $\mu\, n(n + 1) = K_\sigma$, where $\mu = \frac{1+z}{z}$, n is a positive integer, and K_σ is a limited set of discrete constants related to the parameter ξ of Edelen's discretization function.

Cosmology

In conclusion, some investigations in which cosmological theory has been applied to observational problems may be briefly mentioned. Sandage (70) has examined the possibility of using observations made with the 200-inch Hale telescope to distinguish among the members of the sub-group of general relativity models of the universe characterized by a zero cosmical constant and also to contrast these models with the steady-state model. He points out that the observations of redshift versus apparent magnitude are the most promising for the purpose. Sandage (71) has also investigated the effect on the redshift versus apparent magnitude relation of a postulated evolutionary change in the absolute magnitudes of the brightest members of clusters of galaxies. Significant changes in the value of the acceleration factor are obtained if the evolutionary change of absolute luminosity amounts to $0^m{\cdot}4$ or $0^m{\cdot}3$ per billion years. Observational evidence for such changes is difficult to find though estimates of colour variations due to evolution have been said to amount to $0^m{\cdot}03$ per billion years and there are indications that a residual Stebbins-Whitford effect may be present in the spectra of galaxies (72). Sandage (73) and McVittie (74) have also examined the possibility of detecting changes in the redshift, the

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PROGRAM FOR OBSERVING REDSHIFTS IN NEAR-BY CLUSTERS

I. Past observational programs of redshifts in clusters have been directed toward collecting data needed in the investigation of two problems: 1) The law of redshifts to the greatest distances observationally feasible, and 2) the dynamics of individual clusters. These programs to date have accumulated more than 250 published redshifts of individual galaxies in clusters which are distributed among 29 different clusters. It is interesting that the selectivity effect of these two programs has resulted in measurements of redshifts in only six of the 27 nearest clusters (distance classes 0, 1, and 2) listed in the Abell Catalogue. Recent investigations (Wilson PNAS Sept. 1964) suggest that there may exist "preferred values" for cluster redshifts implying some sort of structure in the distribution of the clusters. There is also preliminary evidence that many clusters are centered on a single giant elliptical whose redshift is very close to the mean redshift of the brightest galaxies of the cluster.

If the latter hypothesis is correct, a check of the structure hypothesis may be made from the redshifts of the brightest one or two galaxies in a cluster. As an initial step, therefore in testing the discretum - redshift hypothesis it will be useful to measure the redshifts of the brightest galaxies in the 21 near-by clusters of Abell's list for which no measurements have yet been made.

Since the brightest galaxies in these clusters are all in the neighborhood of 13th magnitude, the B spectrograph at the Newtonian of the 100-inch would be an observationally efficient instrument for the purpose. (The brightness of these objects also assures that suitable finding charts can be made from 48-inch Schmidt plates.)

A possible difficulty may be encountered in interference with H and K by L. A. sky lines for values around 0.02 of $\delta\lambda/\lambda$. Large eastern hour angles then may be required.

II. The current program of galaxy diameters using the 48-inch Schmidt depends on redshifts for reductions to linear diameters. The present indications of discretization in diameters must be tested against additional samples of cluster galaxies. For this purpose redshifts in near-by clusters are required. In addition to galaxies in the clusters listed for part I, redshifts of about 15 bright individual ellipticals will be needed.

It is estimated that, with a margin for cloudy nights, that the 60 to 90 required redshifts can be obtained in one years' time with the B spectrograph with 3 full nights per dark run. In the event this time is not available, the nature of the requirement is such that any fraction of this time would still prove very useful.

A. G. wilson

NEAR BY CLUSTERS

Abell No.	Richness Class	Distance Class	$\bar{V}$/km/sec
194*	0	1	5439
262*	0	1	
347*	0	1	
400	1	1	
407*	0	2	
426*	2	0	5433
539*	1	2	
548*	1	1	
569*	0	1	
576*	1	2	
779*	0	1	
1060*	1	0	
1185	1	2	
1213	1	2	8731
1228	1	1	
1314*	0	1	
1367	2	1	
1656	2	1	6432
1736*	0	2	
2147	1	1	
2151	2	1	10,775
2152	1	1	
2162*	0	1	
2197	1	1	
2199	2	1	9028
2634*	1	1	
2666*	0	0	

Nov. 13, 1964

AMERICAN ASTRONOMICAL SOCIETY

ABSTRACT OF PAPER

(submitted for presentation at the forthcoming meeting)

The abstract must be submitted by the deadline in the preliminary announcement. A copy will be furnished by the Secretary to the Astronomical Journal for publication, and as voted by the Council will carry a $25 publication charge to be billed to your institution.

The abstract may include key references (parenthesized in the text) but should not exceed 300 words; also unusual symbols and complex mathematical formulas should be avoided. Please submit clean double-spaced copy, suitable for the printer, with a carbon copy on plain white paper. Revisions for the A. J. copy will be accepted by the editors until 5 days after the close of the meeting. *These revisions must be sent direct to the A. J. and not to the Secretary.*

Content of abstracts should be as specific as possible, giving concise conclusions and numerical results. Abstracts which do little more than verbosely repeat the title will not be accepted.

The Abstract of a paper by a scientist who is not a member of the Society must be countersigned by a member and must be accompanied by an application for membership in the Society by the author.

Abstracts should be submitted as far as possible in advance of the announced deadline to G. C. McVittie, Secretary, American Astronomical Society, University of Illinois Observatory, Urbana, Illinois.

ABSTRACT-AMERICAN ASTRONOMICAL SOCIETY

Type DOUBLE SPACE on this form, READY FOR THE PRINTER
Not more than 300 words
Please include CARBON COPY on plain white paper

ON SUPER-ORGANIZATIONS AMONG CLUSTERS OF GALAXIES
(Title of Paper)

A. G. Wilson
(Author)

The RAND Corporation
(Institution)

1700 Main Street, Santa Monica, California
(Address)

(Address to which publication charge bill should be sent, if different from above)

(Please continue on additional sheet if necessary, and please staple.)

The mean redshifts of clusters of galaxies do not appear to be distributed randomly, but rather show a tendency to be distributed in accordance with functionally related discrete values (Wilson, A. G., Proc. Nat. Acad. Sci. Vol. 52, 1964). This may be interpreted as implying that clusters are located on a set of shells which possess a definite relation between successive radii. The cosmological principle requires that all equivalent observers (observers located at clusters) should view the clusters as similarly distributed. (For present purposes we may be associated with the Virgo Cluster.) Structured <u>radial</u> distribution of clusters observed by equivalent observers requires structured <u>angular</u> distribution of clusters observed by the equivalent observers. Hence if the regularity in radial distribution of clusters is real, angular structure in the distribution of clusters should also be in evidence.

The large numbers of clusters observed in all unobscured directions in the sky renders statistically meaningless any patterns selected ab initio on the basis of angular distribution criteria. This difficulty may be avoided by invoking an independent

selectivity factor. A study was made of the clusters in Abell's catalogue (Abell, G. O., Ap. J. Suppl. 3, No. 31, 1958) selected on the basis of membership in the richest classes (4 and 5). Though widely separated, these clusters have angular positions consistent with structured rather than random distribution (the details to be published elsewhere). In addition, the same distribution properties observed for the richest clusters obtain in the subset of the rich nearby clusters. These non-random angular distribution patterns lend confirmation to the hypothesis of the existence of some sort of super-organization of which the clusters of galaxies are members.

In view of the same difficulties which arise in explaining super or second order clusters as dynamic systems (Zwicky, F., Pub. Ast. Soc. Pac. 69, 518, 1957), it is completely unsupportable to postulate the existence of a dynamic system with a diameter of the order of 10^9 parsecs, the value consistent with the distances and angular separation of the clusters involved. Consequently, if the apparent super-organizations to which these clusters belong are real, they must originate through physical communication processes other than those presently recognized.

Paper read before Am. Ast. Soc. - Dec. 31, 1964
Montreal Meeting

December 28, 1964

ON SUPER-ORGANIZATIONS AMONG CLUSTERS OF GALAXIES

Albert Wilson

A 848

The measurement of redshifts of galaxies beyond the local supercluster has been primarily to determine the law of redshifts and secondarily to study the dynamics of rich regular clusters such as Coma and Abell 194. The selectivity affects of these two programs have resulted in the measurements being distributed among a very small sample of less than 30 clusters.

This sample is far too small to allow any definitive conclusions on the distributions of clusters in distance, but an analysis of this limited sample which was reported last September (1) showed that the distribution of the mean cluster redshifts might not be random. The data were fit by a discrete distribution function to a level which allows the hypothesis of regularity in the distribution of clusters with distance.

Redshifts of galaxies in even the near by clusters can be observed in only the largest telescopes, so it may be a good many years before a satisfactory sample of mean cluster redshifts may be amassed. There are, however, other ways to test the hypothesis of regularized distribution of clusters. One of these, as described in the abstract, is to investigate the possibility of statistically significant patterns in the angular distribution of clusters.

The angular position data of cluster centers may be used to generate "quasi-redshifts." If we assume redshifts to be proportional to distance and space to be approximately Euclidean for z's up to 20% we may use the law of cosines to determine the distance between two clusters whose redshifts are known. In this way the values of the redshifts of a set of clusters as measured by observers located in other members of the set may be derived.

The sample of observed cluster redshifts has been augmented with some recently reported redshifts of radio dources [2,3] which in many cases are identifiable with giant ellipticals. It is both interesting and useful that these galaxies in the few cases which may be checked have redshifts near the value of the cluster mean. The augmented sample of 45 objects -- clusters and radio sources -- can be used to generate almost one thousand quasi-redshifts, which, as far as the assumptions of proportionality and flatness of space are valid, represent the distances between the 45 clusters.

If the clusters are distributed at random, the histogram of quasi-redshifts should present a noise-like spectrum. If there exists a single structure not distorted by the Euclidean approximation, the spectrum should consist of a set of sharp resonances or peaks. Actually, the spectrum consists of several singular peaks which emerge from a set of lower values which we can call noise. Slide 1 shows a portion of the histogram from 4000 to 8000 km/sec. The resolution is 100 km/sec. The questions to be investigated is whether the high peaks are random fluctuations.

The second slide illustrates the situation for a typical resonance, the one centered at 10281 km/sec. The quasi-redshifts corresponding to the separations of the two objects listed at the left are given in kilometers/second. The fifth value from the top is an actual observation. Half the spread, S, is near in value to the mean error of an individual observation.

The probability of success in a single trial is $p = S/r$, where the range r is taken over $M \pm 1000$ km/sec to assure that the mean density used is the <u>local</u> mean density. The total population of this range is n. The probability of c successes in n trials is determined in the usual way for binomial distributions.

The score 3.68σ corresponds to a probability of 2 parts in 10^4 that the observed number of successes can be obtained by chance from a uniform probability density over the range r.

Slide 3 gives a table summarizing the statistics for ten of the resonances which occur. M is the midvalue, S the spread, c the number of redshifts in the resonance, x the expected number of successes, and $\frac{c-x}{\sigma}$ the measure of statistical significance. All resonances with scores of $\frac{c-x}{\sigma} > 3$ (probability 3 parts in 10^3) will be taken as not likely to be due to random fluctuations. Corresponding probabilities are given in the right column.

It should be noted that the resonance at 5461 km/sec agrees with the observed mean redshifts of 5435 km/sec(from 7 galaxies) of the Perseus cluster and 5439 km/sec (from 43 galaxies) of the A194 cluster. Five quasi-redshifts also have this value. It is also interesting that four observed redshifts occur in the interval 9024 to 9080 km/sec and five quasi-redshifts fall in the same interval ± 25 km/sec. It might be suspected that peaks in quasi-redshifts are results of proliferation of this peak in observed redshifts, but a check of the objects associated with each peak shows this factor is not a contribution.

The midvalues of the resonances are themselves not randomly distributed but bear simple ratios to one another. The resonances out to 19,000 km/sec may be readily expressed by the numbers.

$$\sqrt{10},\ 3\sqrt{3},\ 2\sqrt{7},\ 6,\ 3\sqrt{5},\ 2\sqrt{14},\ \sqrt{11},\ 6\sqrt{2},\ 3\sqrt{13}$$

times a scale factor of 1720 km/sec. The residuals are of the order of 30 km/sec. Beyond 20,000 km/sec the fits to simple ratios disappear. The interesting thing about these numbers is, that with the exception of $2\sqrt{14}$, these are the ratios of the distances between centers of ~~uniform~~

~~most~~ closely packed uniform spheres arranged in a cuboctahedral pattern.

This suggests that the observed angles of separation of the clusters may also conform to the angles expected for ~~most~~ closely packed spheres. Statistically significant fits are found for the sample (selected on the basis of possession of redshifts) in the case of some of the very rich clusters, particularly Abell 1689 and 2065.

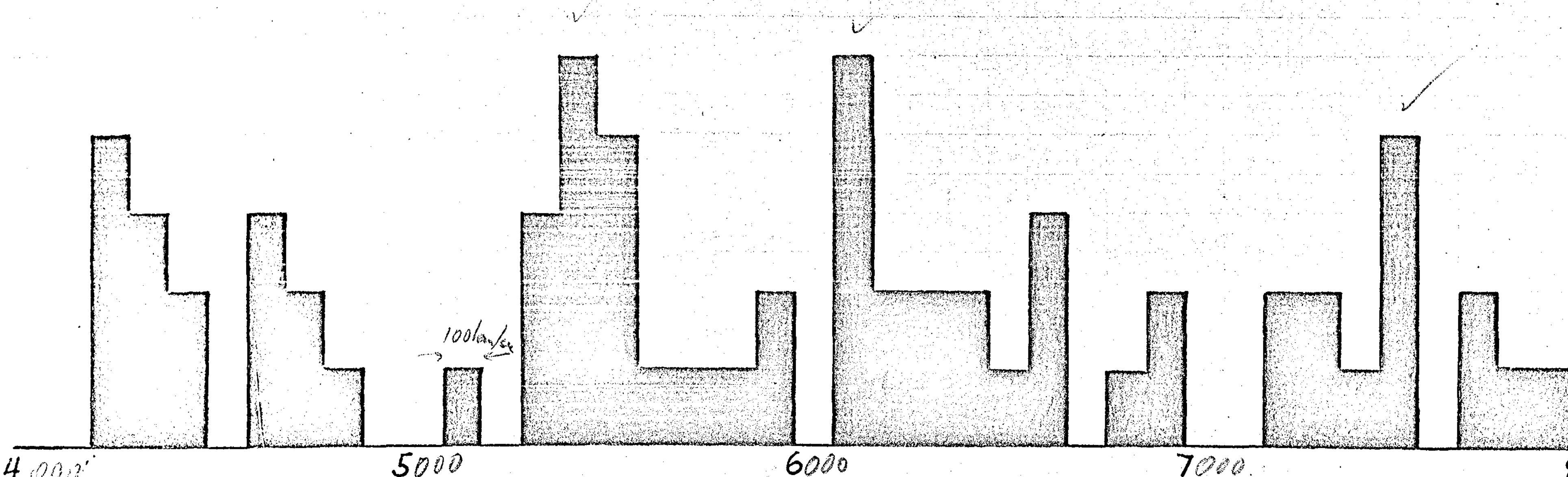
Further the number of fits to $\pm$ 35 km/sec between quasi-redshifts for a given cluster and the cuboctahedron ratio sequence is found to depend on the richness class of the cluster. The percent of successes various from 0.34 for radio sources (not identified with clusters) through 0.36 for Abell richness class 0, 0.47 for richness class 1, to 0.55 for richness class 2.

These several representations suggesting structure pose some fundamental cosmogonic questions.

In view of the same difficulties which arise in explaining super or second order clusters as dynamic systems (Zwicky, F. Pub. Ast. Soc. Pac. 69, 518, 1957), it is completely unsupportable to postulate the existence of a dynamic system with a diameter of the order of 10^9 parsecs, the value consistent with the distances and angular separation of the clusters involved. Consequently, if the apparent super-organizations to which these clusters belong are real, they must either originate through physical communication processes other than those presently recognized or be the result of structure which existed in an early stage of the universe when matter was highly compact. We may speculate that in a universe which expands from a "primeval atom" according to one of the evolutional models, the structure observed at present may be the vestige of a crystal or molecule-like structure which obtained during a brief time during the earliest stages of the expansion. At this time certain aggregates of matter may have been positioned according to one of the configurations available to ~~most~~ closely packed spheres. A uniform dialation would have preserved the general features of this arrangement with regard to relative angular and linear distribution.

The existence of a structured distribution affords a test for evolutionary vs. other cosmological models. If the hypothesis of structure being the result of a stage through which the universe passed during its expansion is valid, then the world map (corrected for light travel time) should possess a higher level of statistical significance than the world picture (not corrected for light travel time). That is, when the distribution of the peaks beyond v = 20,000 km/sec are compared, with a sequence such as the cuboctahedral, the peaks of redshifts of the form V(1 + z) whould possess better fits than peaks in the V histogram. If, on the other hand, the resonances depend on some communication processes proceeding with the velocity of light, the reverse would be expected. ~~In either case, no hypothesis of structure can be considered satisfactory until the noise in the histogram is accounted for.~~

The large residual "noise" in the quasi-redshift histogram must be accounted for before any hypothesis of structure can be given serious weight. It may be due to two or more co-existing structures, it may be due to non-linearity of redshifts, the non-flatness of space, or to combinations of these causes. ~~Or~~ It may yet be due to the random distribution of the clusters -- a distribution, however, with some very unlikely random fluctuations.



QUASI-REDSHIFT HISTOGRAM

Slide 1

RESONANCE IN QUASI-REDSHIFTS

OBJECTS			Km/Sec
A1656	--	3C66	10314
3C442	--	3C353	10300
A426	--	A1213	10271
A1377	--	A1656	10269
Local	--	ML518	10253 (observed)
3C270	--	3C66	10252

r = range = 2000
n = number of trials = 59
c = number of successes = 7

$\sigma = 1.38$

midvalue 10281
S = spread = 66
p = S/r = .033
x = np = 1.95

$$\frac{c - x}{\sigma} = 3.68$$

Slide 2

$\frac{\pi}{4} dc = 1718.17$

$\times\sqrt{2} = 2429.86$

$\times\sqrt{3} = 2975.96$

$\times 2 = 3436.34$

$\times\sqrt{5} = 3841.94$

$\times\sqrt{6} = 4208.64$ 12 %

$\times\sqrt{7} = 4545.85$

$\times\sqrt{8} = 4859.72$

$\times 3 = 5154.51$

$\times\sqrt{10} = 5433.33 \longleftarrow$ (2)

$\times\sqrt{11} = 5698.53$

$\times\sqrt{12} = 5951.92$ 68

$\times\sqrt{13} = 6194.95$

$\times\sqrt{14} = 6428.80 \longleftarrow$ (1)

$\times\sqrt{28} = 9091.70$

$\times 6 = 10309.02 \longleftarrow$

RESONANCES IN QUASI-REDSHIFTS

M	c	S	x	$\frac{c-x}{\sigma}$	P
5461 $\sqrt{5}$	7	85	1.41	4.72	<.0001
$\sqrt{3}$ 9018	6	48	1.42	3.88	.0001
10281 $3\sqrt{2}$	7	66	1.95	3.68	.0002
$\sqrt{8}$ 11535	6	68	1.49	3.73	.0002
$2\sqrt{2}$ 14677	6	85	1.98	2.86	.004
$\sqrt{13}$ 18672	9	152	3.48	3.00	.003
21566	10	175	2.61	4.62	<.0001
$2\sqrt{6}$ 25343	5	83	1.24	3.38	.0007
42519	8	102	1.89	4.56	<.0001
43152	5	86	1.59	2.78	.005

× 1720 km/sec

The lcm is 1720 km/sec

Note that $\frac{\pi}{4} \alpha c = 1718.17$

SLIDE 3.

References:

(1) A. G. Wilson, Discretized Structure in the Distribution of Clusters of Galaxies -- Proc. Nat. Acad. Sci., Vol. 52, No. 3, pp. 847-854. Sept. 1964.

(2) P. Maltby, J. A. Mathews, and A. T. Moffet, Brightness Distribution in Discrete Radio Sources IV, A Discussion of 24 Identified Sources. Ap. J., Vol. 137, p. 153, January 1963.

(3) M. Schmidt, Ap. J.

(4) F. Zwicky, Non-Uniformities in the Apparent Distribution of Clusters of Galaxies, Proc. Ast. Soc. of the Pac., Vol. 69, No. 411, p. 518-529, December 1957.

ON SUPER-ORGANIZATION AMONG CLUSTERS OF GALAXIES

A. G. Wilson

The mean redshifts of clusters of galaxies do not appear to be distributed randomly. The small sample of available mean redshifts is consistent with the hypothesis that the clusters are located on a set of concentric shells which possess a definite relation between successive radii (Wilson, A. G., Proc. Nat. Acad. Sci., Vol. 52, 1964). If this distribution is real, the cosmological principle requires that apparent cluster distribution should be on concentric shells for all equivalent observers, (i.e., observers located in or near a cluster). The actual spatial locations of clusters must then be at the intersections of the several sets of cluster-centered concentric shells. This requires structure in the angular distribution of cluster centers as seen by equivalent observers.

The investigation of regular structure in the distribution of clusters may be investigated further by combining the angular positions of clusters with the mean redshifts to generate additional "quasi-red-shifts" by triangulation. If a linear redshift-distance relation and Euclidean space are assumed, the quasi-redshifts may be derived by the ordinary laws of cosines. If these assumptions are valid and if the spatial distribution of clusters is regular the frequency distribution of quasi-redshifts should be a set of discrete peaks ~~of~~ or resonances which represent the allowable separations between clusters.

The observed histogram of 1000 quasi-redshifts shows a set of peaks distributed among a "noise" background. Statistical tests show that over fifteen of these resonances are not likely to be random fluctuations, (observed occurrence minus expected occurrence $> 3\sigma$). It may be inferred that at least a subset of clusters manifests structured distribution. The noise may be due to breakdown of the Euclidean and linear-redshift approximations, to the co-existence of two or more independent organizations, and/or to actual random distributions.

It is further found that the ratios of the values at which some of the resonances occur ~~at~~ are $\sqrt{3}$, 2, $\sqrt{5}$, $\sqrt{8}$, $\sqrt{13}$ suggesting the distance ratios which obtain for closely packed spheres.

The unlikelihood of the occurrence of these peaks and ratios in distances between clusters distributed in a random uniform manner suggests either that some form of super-organization exists among the clusters or that we are observing the vestiges of a structure whose angular and linear ratios have been preserved under a uniform and isotropic expansion from a time when the universe was in a highly compact stage. The latter hypothesis if physically consistent, would be corroborative of an oscillatory or other evolutionary model.

Alternatives to the vestige-hypothesis must account for an organization extending over 10^9 parsecs, the value bounding the separations of the clusters involved. It is difficult to explain such an extended organization without the introduction of physical communication processes not at present recognized.

A.J. vol 70, No. 2. No. 1327, March 1965

particular, the high disk temperature at 1.35 cm places an upper limit on the amount of water vapor in the lower atmosphere of the planet. Various types of atmospheric constituents are suggested to explain the spectrum.

In September 1964, a series of observations of Jupiter, Saturn, and Mercury were made at 1.53 cm with the 85 ft reflector of the Radio Astronomy Laboratory at Berkeley. Saturn, which was close to opposition at the time, was found to have a disk temperature close to that of Jupiter: $T_S/T_J=0.94\pm0.15$. The implication is that the emission from Saturn at this wavelength probably arises from layers of saturated ammonia below the visible cloud surface of the planet. Several days observations of Mercury were averaged to obtain an adequate signal to noise ratio. A disk temperature about three times that of Jupiter was obtained: $T_M/T_J=3.0\pm0.75$. If one takes the temperature of Jupiter as 155°K at this wavelength, one obtains a temperature of 465°±115°K for Mercury. During the period of observations, the average illumination of the disk was 25%. Assuming a subsolar temperature of 620°K and a pole darkening proportional to $\cos^{\frac{1}{4}}\theta$, one finds that the contribution from the illuminated part of the disk amounted to about 100°K. If one assumes that the surface properties of Mercury are similar to those of the moon, one can explain the large dark side contribution to the disk temperature by postulating internal radioactive heat sources which produce a flux of heat at the surface of Mercury some 30–40 times that which is observed in the case of the earth.

This work has been sponsored by the Office of Naval Research under contracts NONR 222(66) and NONR 222(54).

On Super-Organization Among Clusters of Galaxies. A. G. Wilson, *The Rand Corporation.*—The mean redshifts of clusters of galaxies do not appear to be distributed randomly. The small sample of available mean redshifts is consistent with the hypothesis that the clusters are located on a set of concentric shells which possess a definite relation between successive radii (Wilson, A. G., *Proc. Nat. Acad. Sci.,* 52, 1964). If this distribution is real, the cosmological principle requires that apparent cluster distribution should be on concentric shells for all equivalent observers (i.e., observers located in or near a cluster). The actual spatial locations of clusters must then be at the intersections of the several sets of cluster-centered concentric shells. This requires structure in the *angular* distribution of cluster centers as seen by equivalent observers.

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It is further found that the ratios of the values at which some of the resonances occur are $3^{\frac{1}{2}}$, 2, $5^{\frac{1}{2}}$, $8^{\frac{1}{2}}$, $13^{\frac{1}{2}}$, suggesting the distance ratios which obtain for closely packed spheres.

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Alternatives to the vestige-hypothesis must account for an organization extended over 10^9 parsecs, the value bounding the separations of the clusters involved. It is difficult to explain such an extended organization without the introduction of physical communication processes not at present recognized.

Measurement of the Flux Density of Cas A at 4080 Mc/sec. R. W. Wilson, A. A. Penzias, and D. C. Hogg, *Bell Telephone Laboratories.*—The 20-ft-aperture horn reflector at the Crawford Hill Laboratory,, Holmdel, New Jersey, has been used to measure the flux density of Cas A at 4080 Mc/sec. Some forty drift curve observations, each approximately forty minutes in duration, were made on five nights in September and October of 1964. An argon noise tube was used for comparison in each observation. The night to night variation in the results was less than ±0.3%.

The equivalent effective temperature contributed

Reprinted from
THE ASTRONOMICAL JOURNAL VOLUME 70, NUMBER 2 MARCH 1965 p. 150

Abstracts of Papers Presented at the 117th Meeting of the American Astronomical Society, held 28–31 December 1964 at Montreal, Canada

On Super-Organization Among Clusters of Galaxies. A. G. Wilson, *The Rand Corporation.*—The mean redshifts of clusters of galaxies do not appear to be distributed randomly. The small sample of available mean redshifts is consistent with the hypothesis that the clusters are located on a set of concentric shells which possess a definite relation between successive radii (Wilson, A. G., *Proc. Nat. Acad. Sci.,* **52,** 1964). If this distribution is real, the cosmological principle requires that apparent cluster distribution should be on concentric shells for all equivalent observers (i.e., observers located in or near a cluster). The actual spatial locations of clusters must then be at the intersections of the several sets of cluster-centered concentric shells. This requires structure in the *angular* distribution of cluster centers as seen by equivalent observers.

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D No. D-13189-PR-RC

Assigned to: Engr.

A. G. Wilson

Project No. 2000-3/4
7000-1/4

Contract No. AF 49(638)-700
RSR

Task Order No.

RAND DOCUMENT

ANNUAL PROGRESS REPORT OF THE DEPARTMENT OF GEOPHYSICS AND ASTRONOMY

Staff, Department of Geophysics and Astronomy

January 14, 1965

PREFACE

Accounts of 1964 activities in the Department of Geophysics and Astronomy, together with estimates of future work, are listed by RAND project number.

A list of 1964 publications is included at the end of this document.

(2052)

RPN 2052: RADIOACTIVE FALLOUT

DESCRIPTION:

The purpose of Project 2052 is to improve the techniques of predicting radioactive fallout hazard and keep abreast of what is being done in the field elsewhere.

PROJECT PERSONNEL:

R. R. Rapp, R. E. Huschke, L. C. Kern, C. Leovy, E. Rodriguez.

ACTIVITY AND RESULTS:

The major activity of the past year has been an attempt to incorporate a fallout-prediction scheme into an existing simple machine method for predicting the winds. Leovy, with the assistance of Kern and Rodriguez has programmed a simple two-layer quasi-geostrophic wind prediction scheme. Rapp with the assistance of Rodriguez has introduced a particle-tracing program into the wind-prediction routine. These routines have been written and debugged. A method for determining the fraction of the nuclear device at any location, given the particle positions, has been devised by Rapp and Huschke and will eventually be added to the program. They are at present preparing to put the weather situations chosen for QUICK COUNT into this model.

Rapp has served as an advisor to the DASA fallout review panel, and has consulted with fallout people from Lawrence Radiation Laboratory and Planning Research Corporation. These contacts have been mutually beneficial.

(2052)

FUTURE WORK:

This year should see the completion of RAND's most advanced fallout model. With the joining of a wind prediction scheme to the basic concepts of fallout models developed long ago at RAND, it is hoped that the last deficiencies in the atmospheric transport phase of the fallout problem can be removed. Future gains will be expected from a study of the rise and development of the atomic cloud and better knowledge of the nature of the radioactive particles. (Huschke, Rapp--1/2)

COMMITMENTS:

Although some major policy changes have relieved the pressure of RAND's commitments to NORAD, the Air Weather Service still has a major problem in providing fallout warning, and this department hopes to assist them in updating their methods.

RPN 2054: GENERAL GEOPHYSICS

DESCRIPTION:

The project conducts basic and applied research in atmospheric physics, meteorology, planetary and space physics, and earth physics. It consists of both a series of continuing studies and various short-term efforts to meet specific Air Force requests, and provides aid on geophysical problems to other departments.

PROJECT PERSONNEL:

Staff: D. Deirmendjian, R. H. Ball, S. M. Greenfield, R. E. Huschke, A. B. Kahle, H. K. Kallmann-Bijl, W. W. Kellogg, J. W. Kern, L. C. Kern, G. E. Kocher, M. Portl, R. R. Rapp, G. F. Schilling, E. H. Vestine, A. G. Wilson.

Consultants: N. Divine, J. R. Jokipii, A. H. Marcus, Y. Mintz.

ACTIVITY AND RESULTS:

Within the spirit of what this project constantly attempts to do (i.e., provide the background knowledge and information in geophysics enabling us to advance the state of the art and thus better provide a consulting service to the rest of RAND), the following diverse activities have gone on:

L. C. Kern, M. Portl, and R. R. Rapp have continued the investigation of the utility of falling spheres in measuring density at high altitudes. A computer program that calculates the velocity of these spheres through various model atmospheres was used to advise the weather support unit at the Pacific Missile Range on an experiment, which was to determine whether the density

(2054)

fluctuation calculated from the observed rates of falling spheres was real or a misinterpretation due to the manner in which the spheres fall.

Mintz continued his study of methods of improving the numerical-modeling techniques for the cyclone-scale motions in our atmosphere.

Greenfield and Davis continued their investigations of balloons for planetary exploration. Davis made a study of the effect of balloon fabric stresses, based on the conclusions reported in R-421-JPL concerning the use of balloons in the study of the atmosphere of Mars. This work, as well as the principal results of the previous study (with extensions), was reported at the AFCRL Scientific Balloon Symposium held October 19--22 at Portsmouth, New Hampshire, in two joint papers by Greenfield and Davis.

Divine took a computer program he had developed and adapted it to analyze radiative heating in the Martian atmosphere so that it can be used to provide inputs to Mintz' models of the general circulation of Mars.

Marcus completed several reports on the accretion of particles in interplanetary space and the statistics of lunar craters.

Jokipii considered effects of the impact of the solar-proton wind upon the lunar atmosphere. The effect apparently both augments and depletes the gaseous envelope about the moon, and limits the density of the lunar atmosphere to a small value.

Schilling's work was concentrated on problems of the earth's atmosphere that affect aerospace planning and operations. In this connection he showed that by the application of standard meteorological principles one could determine the limited atmospheric regions where certain minor constituents like water should or should not appear. Applying his work to water vapor, he showed that his technique did agree with the formation of noctilucent clouds in the region of the atmosphere where they normally appear. In collaboration with L. C. Kern, he developed a computer program for the construction of model atmospheres. He also proposed the possibility that observations of lunar eclipses might be utilized to infer the height of the mesopause as a function of latitude. He continued his work with NORAD on the problem of predicting atmospheric densities.

Vestine, Ball, and J. W. Kern continued their work on the geomagnetic field. Observational results of surveys of the geomagnetic field have been collected and analyzed. Changes with time in the general configuration of the geomagnetic field have been examined. The general field configuration both above and below the surface of the earth, as affected by internal forces, is being considered since the findings are of assistance or interest in connection with the navigation of aircraft and in studies of the Van Allen radiation belts. Assistance was rendered in planning the present World Magnetic

Survey at both the national and international level. Some results of these studies were discussed informally with representatives of the Strategic Air Command.

Kallmann-Bijl continued her work on the determination of basic physical properties of the upper atmosphere. She is, as of the year's end, on leave of absence as a Research Professor at Utrecht University, Holland.

Kellogg finished his study on the pollution of the upper atmosphere due to contaminants produced by rocket exhausts (RM-3961-PR). The Memorandum was submitted to and published in the Space Science Reviews. It was his conclusion that dangerous pollutant levels would not be reached without an increase of several orders of magnitude in our present rate of rocket launching.

Wilson and Kocher continued their work on Project FLOSS HILDE for NORAD. Experiments were continued with the 48-inch and 18-inch Schmidt telescopes on Palomar Mountain to determine the optimum optical-photographic systems for observation of deep space probes. Kocher was added to the observational team and was trained in the use of the Schmidt telescopes. Observational exercises were conducted whenever space probes were available to photographic observations. The list of successful observations of probes was increased in 1964 with two photographs of Zond II, the Soviet Mars probe. These photographs were made by Kocher on December 1st, with the 48-inch Schmidt.

(2054)

In addition to the above, a considerable effort was expended in consulting with various other departments in RAND. Most of such consulting is very transitory and at most consists of several man-days of effort at any one time. Typical of such efforts is the following submission by Huschke: (1) Provided advice and data on effect of clouds on visual post-attack damage surveillance; (2) participated in planning sessions for study of hard-site post-attack maintenance problems; (3) participated in planning sessions for tactical air-warfare study; (4) evaluated Herman Roth's system of "climatic diagrams".

FUTURE WORK:

Observational and laboratory work will be continued on Project FLOSS HILDE toward providing NORAD with guidance on the best techniques for optical observation of deep space probe. (Wilson, Kocher--3/4)

Work will continue on the study of balloon physics in two specific directions. First will be the attempt to extend prior work on possible balloon uses to include a consideration of using them on Venus and Jupiter as well as Mars. Second is the completion of a study already started describing a new variable-altitude balloon system recently conceived. (Davis, Greenfield--3/4)

Work will be done on studies of the possibility of making eclipse observations from manned spacecraft, and the usefulness of such observations in determining upper atmosphere characteristics. (Schilling, Moore--2)

In a continuation of the work in geomagnetism, aspects of the motion of a charged particle in the geomagnetic field will be considered and studies of geomagnetic secular changes will be performed. Assistance as required in the planning and coordination of the present World Magnetic Survey will be continued, as will work on the impact of the solar-proton wind on the lunar atmosphere. (Vestine, Ball--1/2)

Observational and theoretical studies in cosmology are planned in the coming year. These studies stem from the observational findings under RSR-7086 that suggest structure in the distribution of large-scale aggregates of matter. Wilson has begun a program on the 100-inch telescope at Mount Wilson to measure the redshifts of brightest galaxies in nearby clusters to check the structure hypothesis. Part of the work will be to develop cosmological models that do not assume the uniformity principle, but allow for the existence of discrete distributions of clusters and other aggregates. It is also planned by Edelen to investigate the epistemological properties of cosmological observables in a manner somewhat analogous to that developed under quantum mechanics. (Wilson, Kocher--1 1/4)

Specific meteorological and climatological studies will be undertaken and consultant service will be provided to aid in conducting RAND's multitude of studies of tactical warfare. (Batten, Huschke--1/2)

This project will continue to provide information, guidance and data as needed to personnel of other departments. (All project personnel--1/2)

(2054)

COMMITMENTS:

Vestine has agreed to present an invited lecture covering purely scientific observational aspects of the geomagnetic field at the forthcoming NATO Conference on The Magnetic Fields of Celestial Bodies, at Newcastle, England, in early April.

Schilling (jointly with C. Gazley, Jr. and L. N. Rowell) has promised completion of a report for NORAD presenting the principal results of the satellite decay and impact studies.

(2162)

RPN 2162: ATMOSPHERIC RADIATION

DESCRIPTION:

Theoretical studies of the scattering, polarization and absorption produced by electromagnetic radiation in the visible, infrared, and microwave range, falling on finite particles and atmospheres.

PROJECT PERSONNEL:

Staff: D. Deirmendjian, J. L. Carlstedt.

Consultants: T. W. Mullikin, Z. Sekera.

ACTIVITY AND RESULTS:

The results reported in R-422-PR on the scattering and absorption properties of hydrometeors illuminated by microwaves in the λ1-mm to λ10-cm range were used to investigate the atmosphere of Venus. It was shown that, assuming the existence of water substance in this atmosphere, the observed microwave radiations from the planet could be explained by an atmosphere of moderate surface pressure of the order of 2 to 3 atmospheres, rather than the tens or hundreds of atmospheres needed if water is excluded (see RM-4060-PR and ICARUS, 3, 109, 1964). The existence of water vapor and ice particles seems to be borne out by J. Strong's recent infrared spectrometry with balloon-borne instruments.

A shorter version of R-422-PR was delivered at the World Conference on Radio Meteorology and prepared for publication in Radio Science (see P-2914-1).

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Active participation in the Leningrad Radiation Symposium (August 1964) resulted in establishing contacts and evaluating the Soviet scientific effort in this field (see D-12907-PR).

The ability of pulsed laser-radar systems to detect and measure high atmospheric dust layers was briefly investigated and found to be limited to detection only. This result is reported in a letter to the editor of J. Geophys. Res. (to be published early 1965) and discussed in the Aerosol Conference organized by the Army Electronics Laboratory, Air Force Cambridge Research Laboratories and New York University (October 1965).

Sekera and Mullikin collaborated in perfecting a machine computation program that will yield certain functions and the actual intensities and polarizations produced by Rayleigh scattering atmospheres of large optical depth.

FUTURE WORK:

The theoretical investigation of the scattering characteristics of various types of particle aggregates will continue with a view to a compilation of the results under one cover. Numerical results of the Rayleigh atmosphere problem will be made available. Further studies in radiative transfer will continue with the assistance of consultants. (D. Deirmendjian, T. W. Mullikin, Z. Sekera--1 1/2)

RPN 2250: CLOUD PHYSICS AND WEATHER CONTROL

DESCRIPTION:

The long-range objective is to investigate the possibilities of weather control. The short-range goal is to take one step in this direction by understanding both the fundamental, microscopic interactions of individual droplets and ice crystals in clouds, and the mechanism by which the droplets coalesce to produce rain and snow.

PROJECT PERSONNEL:

Staff: M. H. Davis, J. O. Fletcher, S. M. Greenfield, C. L. Olson.

Consultants: A. H. Marcus, Y. Mintz.

ACTIVITY AND RESULTS:

A shortened version of RM-3860-PR, which gives the electrostatic theory of charged-droplet interaction, appeared in the Quarterly Journal of Mechanics and Applied Mathematics. Based on this work, calculations were carried out that indicate that electrostatic forces may be of considerable importance in the early stages of the precipitation process. Preliminary results were reported at the Cloud Physics meeting of the American Meteorological Society held in Chicago, March 24--26, 1964. (See P-2885.)

Olson and Davis examined the mathematical basis for the hydrodynamical equations used for cloud-droplet trajectory calculations. Olson succeeded in reformulating the equations in a way that promises to predict hydrodynamic forces for small cloud-droplet separations with considerably more accuracy. A report on this work was awaiting publication at the year's end.

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Marcus began an examination of the statistical problem of how droplet distributions interact and develop.

By numerical integration of the equations of atmospheric motion, a preliminary study was made by Mintz of the effect of removing the Arctic Sea ice on the energy budget and the circulation of the atmosphere over the globe.

Fletcher systematically investigated the heat budget and thermal processes of the Arctic region. The objective was to relate these processes to the general circulation of the atmosphere and to climatic change, in both the short term and the long term.

FUTURE WORK:

Computations on the effect of electrostatic forces on small droplet collisions, using Hocking's hydrodynamics, are nearly complete and will be reported soon. The collaborative work of Mintz and Fletcher on the Arctic heat balance and its effect on global atmospheric circulation will continue. (Davis, Fletcher, M. Warshaw--2)

(2342)

RPN 2342: STRUCTURE AND DYNAMICS OF THE 20-120 km ATMOSPHERE

DESCRIPTION:

The purpose of this project is to improve the description and understanding of the atmosphere between 20 and 120 km. Specifically the project includes studies of (1) the time and space distribution of wind, temperature, pressure and density; (2) the energy balance; and (3) the relationship between atmospheric motions and the photochemical and radioactive processes occurring in the upper atmosphere.

PROJECT PERSONNEL: E. S. Batten, C. B. Leovy, R. R. Rapp.

ACTIVITY AND RESULTS:

Work charged to RPN-2054 at the beginning of last year was continued and expanded under this new project.

Batten continued work on the seasonal and latitudinal variations of winds and temperatures in the upper stratosphere. In RM-4144-PR the seasonal changes in the circulation and temperature patterns are described.

In the last year new rocket observations were obtained from the tropics. These observations revealed some unexpected features of the mesosphere. Rapp and Batten evaluated the soundings and instrumentation to determine the validity of the measurements. The observations suggest changes in the current concept of the mesospheric temperature structure.

A study to determine the extent to which observed features of the temperature structure of the upper atmosphere can be explained by radiative processes was started by Leovy. Such knowledge is a prerequisite for the understanding of atmospheric motions. To date,

(2342)

a computer program for computing infrared fluxes and heating rates due to the 9.6-micron band of ozone has been written and checked out. Although the results have not been satisfactory, the method looks promising. It is felt that a refined model will give sufficiently accurate results.

Leovy also developed a model to determine the stability in a nonrotating, isothermal atmosphere influenced by the interrelations between chemical heating, absorption of solar radiation, and infrared radiative transfer. The results indicate that chemical heating may produce rather rapid growth in the amplitude of disturbances formed above 75 km and having time scales of a few hours or more. Large-amplitude irregularities observed in this region may be due to this mechanism. A possible relationship between atmospheric motions and components of the airglow is also suggested by the results.

FUTURE WORK:

Since the number of observations is increasing rapidly, computer methods of data processing will be developed. Up-to-date summaries of the data will be maintained and used to revise models of the atmospheric structure. (Batten, Rapp--3/4)

The model for calculating the 9.6-micron band's heating rate will be improved and applied to the atmospheric region below 75 km. An attempt will also be made to adapt the model to compute heating in other bands. The effects of chemical heating, absorption of solar radiation and infrared radiative transfer on the stability of specific types of atmospheric motion, such as the tides, will be investigated. (Leovy--1/2)

RPN 7065: SPECIAL RELATIVITY IN A UNIFIED FIELD THEORY

DESCRIPTION:

This project is an attempt to determine the conditions under which a gravitational field is invariant under a large-scale Lorentz transformation in a manner similar to the well-known invariance of the electromagnetic field.

PROJECT PERSONNEL:

R. L. Kirkwood.

ACTIVITY AND RESULTS:

The Lorentz invariance of Newtonian gravitational fields has been investigated previously (RM-3146-RC) for the case in which the relative velocity of the two coordinate systems is much less than the velocity of light. More recent work on this project has been an attempt to extend the earlier results to include more general gravitational fields and arbitrary relative velocities of the coordinate systems. It has been shown that the type of invariance investigated previously cannot be applied under these more general conditions, and a more general type of invariance has been defined and investigated in the hope of overcoming this difficulty.

FUTURE WORK:

It is hoped that the conditions for the existence of a large-scale Lorentz transformation will provide a new and improved system of gravitational field equations. (Kirkwood--1/2)

COMMITMENTS:

The results of the project will be reported as soon as meaningful conclusions are reached.

RSR 7086: GALACTIC SCALE DISCRETIZATION

This is the second and final year of this project, which is a combined theoretical and observational investigation of discretization phenomena among the large-scale aggregates of the universe, the galaxies and clusters of galaxies. (See D-11954-PR for additional description).

PROJECT PERSONNEL:

Staff: D. G. B. Edelen, A. G. Wilson, J. L. Carlstedt, L. C. Kern, G. E. Kocher, M. Portl.

Consultants: T. L. Page, T. Y. Thomas, G. de Vaucouleurs.

ACTIVITY AND RESULTS:

This two-year investigation has led to the discovery of several observational relations of possible basic importance to cosmology. The project has resulted in the development of a theoretical basis for the morphology of elliptical and SO-type galaxies, including their size discretization. Observations have been secured that corroborate several of the main features of the theory. In addition, observational evidence has been obtained suggesting the structured distribution of clusters of galaxies. No existing theoretical cosmologies account for such structure, however.

Lectures on discretization phenomena were given at UCLA, the University of Michigan, and at Hamburg by Edelen, who also summarized the work of RSR 7086 for the RAND Board of Trustees. Wilson presented invited lectures at the University of Texas, Indiana University, and UCLA. He also gave one of the Loyola Golden Jubilee Science Lectures entitled "Physical Views on the Origin of the

Universe." Kocher assisted Wilson with the development of high-contrast techniques for defining diameters of galaxies and assisted with the observational transfer program at Palomar Observatory.

A conference on "Discrete Parameters in Cosmology" was organized and held at RAND on June 29--30, 1964. The proceedings of this conference are in RM-4267-RC by Thornton Page.

FUTURE WORK:

The full report of the observational work on discretization is yet to appear in RM-3771-RC. The completion of this report must await closure of the 48" Schmidt transfer-plate net. The remaining plates cannot be taken before April 1965.

Project RSR 7086 is to terminate with the completion of the report on the observational transfer program. (It is consequently believed appropriate to list on the following pages the reports and presentations associated with this project.) It is planned to follow through on the findings of this project with theoretical and observational work under Project RAND. (A separate section under 2054 summarizes this proposal.)

REPORTS AND PRESENTATIONS: ASSOCIATED WITH PROJECT 7086

Edelen, D. G. B., "Galactic Scale Discretization," Astron. J., Vol. 68, No. 8, p. 535, October 1963 (Paper read before Amer. Astron. Soc., July 1963).

Edelen, D. G. B., "Relativistic Surface Dynamics of an Isolated World Tube of Perfect Fluid," Proc. Nat. Acad. Sci., Vol. 50, No. 3, p. 469, September 1963.

Edelen, D. G. B., Possible Galactic Scale Discretization, The RAND Corporation, RM-3941-RC, November 1963.

Edelen, D. G. B., "Deformations and Momentum-Energy Complexes," Arch. Rat. Mech. Anal., Vol. 16, p. 316, 1964

Edelen, D. G. B., Relativistic Galactic Morphology I: General Theory The RAND Corporation, RM-4017-RC, August 1964.

Edelen, D. G. B., Galactic Morphology and Scale, The RAND Corporation, P-3020, November 1964.

Edelen, D. G. B., and T. Y. Thomas, "Dynamics of Discontinuity Surfaces in General Relativity Theory," J. Math. Anal. and Appl., Vol. 7, No. 2, p. 247, 1963.

Page, Thornton (Comp.), Proceedings of the Conference on Discrete Parameters in Cosmology, The RAND Corporation, RM-4267-RC, August 1964.

Thomas, T. Y., "Outline Theory of the Universe," Proc. Nat. Acad. Sci., Vol. 51, No. 5, p. 718, May 1964.

Thomas, T. Y., "Radial Discretization in Spherical Galaxies," Proc. Nat. Acad. Sci., Vol. 52, No. 1, p. 1, July 1964.

Thomas, T. Y., Discretization in Galactic Structure and Cosmology, The RAND Corporation, RM-3990-RC, July 1964.

Wilson, A. G., "Tentative Observational Confirmation of Discretization in Galaxies," Astron. J., Vol. 68, No. 8, p. 547, October 1963 (Paper read before Amer. Astron. Soc., July 1963).

Wilson, A. G., "Discretization in EO Field Galaxies," Astron. J., Vol. 69, No. 2, p. 153, March 1964 (Paper read before Amer. Astron. Soc., December 1963).

Wilson, A. G., "Discretized Structure in the Distribution of Clusters of Galaxies," Proc. Nat. Acad. Sci., Vol. 52, No. 3, pp. 847-854, September 1964.

(7086)

Wilson, A. G., "On Super-Organizations Among Clusters of Galaxies," paper read before the American Astronomical Society, December 1964 (to be published).

Discretization is also cited in the following:

1. Carnegie Institution of Washington Year Book 62, p. 44, 1962-1963.

2. Transactions of the International Astronomical Union, XII General Assembly. Draft Report, p. 362, June 1964.

3. New Scientist, June 25, 1964.

RSR 7087: SOLAR ECLIPSE EXPERIMENTS

DESCRIPTION:

Completion of report on a photographic photometry experiment conducted aboard a DC-8 aircraft during the total solar eclipse of July 20, 1963.

PROJECT PERSONNEL: G. Kocher

ACTIVITY AND RESULTS:

Microdensitometer tracings were made of the negatives obtained during the eclipse flight. A Memorandum, RM-4226-RC, Eclipse Observations from a Jet Aircraft, presents the experimental results. Some conclusions reached as a result of this experiment are that (1) observations from an aircraft similar to that used are not only feasible, but more desirable than ground-based observations, for certain research programs, and (2) manual guiding of experimental apparatus is satisfactory for many eclipse experiments.

Kocher was invited to the July 20 meeting of the Society of Photographic Instrumentation Engineers (Los Angeles Chapter) to participate in a panel discussion of the instrumental aspects of the experiment.

FUTURE WORK:

This project is now completed and no further activity is contemplated under it.

PROJECT RAND PUBLICATIONS -- 1964

Batten, E. S., A Model of the Seasonal and Latitudinal Variation of Zonal Winds and Temperatures in the Stratosphere Above 30 km, The RAND Corporation, RM-4144-PR, June 1964.

Davis, M. H., Two Charged Spherical Conductors in a Uniform Electric Field: Forces and Field Strength, The RAND Corporation, RM-3860-PR, January 1964.

Davis, M. H., Electrostatic Forces and Cloud-Droplet Interaction, The RAND Corporation, P-2885, March 1964.

Deirmendjian, D., A Water-Cloud Interpretation of Venus' Microwave Continuum, The RAND Corporation, RM-4060-PR, April 1964.

Deirmendjian, D., Random Notes -- Trip to Leningrad, August 1964, The RAND Corporation, D(L)-12907-PR, October 1964.

Deirmendjian, D., Complete Scattering Parameters of Polydispersed Hydrometeors in the λ 0.1 to λ 10 cm Range, The RAND Corporation, P-2914-1, November 1964.

Ernst, Barbara (Comp.), A Bibliography of RAND Publications on Fallout, The RAND Corporation, RM-4276-PR, September 1964.

Fletcher, J. O., The Study of NATO Strategy (U), The RAND Corporation, D-12205-PR, March 10, 1964 (Secret).

Jokipii, J. R., The Distribution of Gases in the Proto-Planetary Nebula, The RAND Corporation, RM-3977-PR, May 1964.

Kellogg, W. W., Pollution of the Upper Atmosphere by Rockets, The RAND Corporation, RM-3961-PR, June 1964.

Kern, L. C., and G. F. Schilling, MODAT: A Computer Program for the Construction of Model Atmospheres, The RAND Corporation, RM-4204-PR, July 1964.

Klappert, M. T., and G. F. Schilling, Selected Hygrometric Tables for Low Temperatures and Pressures, The RAND Corporation, RM-4244-PR, August 1964.

Kocher, G. E., Eclipse Observations From a Jet Aircraft, The RAND Corporation, RM-4226-RC, July 1964.

Leovy, C., Simple Models of Thermally Driven Mesospheric Circulation, The RAND Corporation, P-2847, January 1964.

Page, Thornton (Comp.), Proceedings of the Conference on Discrete Parameters in Cosmology (June 29--30, 1964), The RAND Corporation, RM-4267-RC, September 1964.

Schilling, G. F., On the Limit of the Atmosphere and Space Sovereignty (U), The RAND Corporation, D-12047-PR, January 17, 1964 (Confidential).

Schilling, G. F., Forbidden Regions for the Formation of Clouds in a Planetary Atmosphere, The RAND Corporation, RM-4084-PR, April 1964.

Schilling, G. F., Theoretically Permissible Altitudes and Seasons for the Occurrence of Clouds Near the Mesopause, The RAND Corporation, P-2899, April 1964.

Schilling, G. F., Comments on "The Secular Increase of the World-Wide Fine Particle Pollution", The RAND Corporation, P-2910, May 1964.

Schilling, G. F., Latitudinal Variation of Mesopause Height Inferred From Eclipse Observations, The RAND Corporation, D-12473-PR, May 26, 1964.

Schilling, G. F., Atmospheres of the Planets, The RAND Corporation, P-2964, September 1964.

Schilling, G. F., Latitudinal Variation of Mesopause Height Inferred From Eclipse Observations, The RAND Corporation, RM-4321-PR, October 1964.

Schilling, G. F., On Predicting Upper Atmosphere Densities, The RAND Corporation, D-12943-PR, October 12, 1964.

Staff, Reports of Operations in the Department of Planetary Sciences for the Year 1963, The RAND Corporation, D-11954-PR, January 13, 1964.

For annual report of 1964 Jan 1965

RSR-7068: Galactic Scale Discretization

Description: This is the second and final year of this project which is a combined theoretical and observational investigation of discretization phenomena among the large-scale aggregates of the universe, the galaxies and dusters of galaxies. (See D-11954-PR for additional description).

Project Personnel:

The two principal investigators have been D.G.B. Edelén of the Mathematics Department and A. G. Wilson of the Department of Geophysics and Astronomy. In addition, the following personnel have participated in the project: George E. Kocher, Louise Kern and Marianne Portl of the Department of Geophysics and Astronomy and Jim Carlstedt of the Computer Sciences Department. The consulting services of T. L. Page, T. Y. Thomas, and G. de Vaucouleurs have been employed in the past year.

Activity and Results:

In brief, the project has resulted in the development of a theoretical basis for the morphology of elliptical and SO type galaxies, including their size discretization. Observations have been secured which corrobotate several of the main features of the theory. In additionaobservational evidence has been obtained suggesting the structured distribution of clusters of galaxies. No theoretical cosmologies exist at present which account for such structure, however.

The detailed results of the project are contained for the most part in the papers listed in the following Published Discretization Bibliography, which is complete through 1964.

The full report of the observational work on discretization is yet to appear in RM- 3771- RC. The completion of this report must await closure of the 48" Schmidt transfer-plate-net. The remaining plates cannot be taken before April 1965. The publication of this report will complete project RSR-7086.

In addition to the listed publications, several lectures on discretization phenomena were given during the year.

Edelen gave lectures at UCLA, the University of Michigan, and summarized the work of RSR 7068 for the RAND Board of Trustees. He presented a paper before Commission 28 of the I.A.U. in Hamburg.

Wilson presented invited lectures on observational evidence for discretization at the University of Texas, Indiana University, and UCLA He also gave one of the Loyola Golden Jubilee Science Lectures entitled "Physical Views on the Origin of the Universe" and contributed a paper to the meeting of the American Astronomical Society.

Kocher assisted Wilson with the development of high contrast techniques for defining diameters of galaxies and assisted with the observational transfer program at Palomar Observatory. He attended the I.A.U. meeting in Hamburg.

A Conference on "Discrete Parameters in Cosmology" was organized and held at RAND on June 29-30, 1964. The Proceedings of this conference are contained in RM-4267-RC by Thornton Page.

Future Work:

Project RSR-7068 is to be terminated ~~wih~~ with the completion of the observational transfer program and ~~RM- -RC~~. The results of this two-year investigation have led to the discovery of several observational relations of possible basic importance to cosmology. It is planned to follow through on these findings with theoretical and observational work under Project RAND. (A separate memo summarizes this proposal).

For annual report

Observational and theoretical studies in cosmology are planned in the coming year. These studies stem from the observational findings under RSR-7086 that suggest structure in the distribution of large-scale aggregates of matter. Wilson has begun a program on the 100-inch telescope at Mount Wilson to measure the redshifts of brightest galaxies in nearby clusters to check the structure hypothesis. Part of the work will be to develop cosmological models that do not assume the uniformity principle, but allow for the existence of discrete distributions of clusters and other aggregates. It is also planned by Edelen to investigate the epistemological properties of cosmological observables in a manner somewhat analogous to that developed under quantum mechanics. (Wilson, Kocher--1 1/4)

~~Specific meteorological and climatological studies will be undertaken and consultant service will be provided to aid in conducting RAND's multitude of studies of tactical warfare. (Batten, Huschke-- 1/2)~~

~~This project will continue to provide information, guidance and data as needed to personnel of other departments. (All project personnel -- 1/2)~~

RELATIVISTIC GALACTIC MORPHOLOGY I: GENERAL THEORY

For Report to Board of Trustees

The theory of Galactic Morphology described in this report could prove to be by far RAND's most significant contribution to science, for if substantiated, it could alter the body of current fundamental cosmological theory. The theory of morphology grew out of mathematical research involving Einstein's field equations of general relativity. Results of the theory provide for the first time an explanation of many of the optical morphological forms assumed by elliptical galaxies. The theory further predicts that the diameters of such galaxies are not continuously distributed, but occur in discrete sizes.

Indications of discrete distribution in sizes of globular galaxies had been observed and reported by another RAND staff member, astronomer Al Wilson, in 1950 using the Mt. Wilson 100-inch reflector. Further observations and measurements made since the theoretical prediction of discrete distribution, have led to provisional confirmation of the theory. Additional observations are currently being secured, in hopes of making a definitive test.

M E M O R A N D U M

TO: S. M. Greenfield MEMO NO: M-392

FROM: A. G. Wilson DATE: 1-18-65

SUBJECT: RESEARCH IN COSMOLOGY UNDER PROJECT RAND

COPIES TO: D.G.B. Edelen, T. E. Harris, R. L. Kirkwood, G. E. Kocher

Two years ago project RSR-7068, "Galactic Scale Discretization," was set up to investigate the indicated conformity of Edelen's theoretical prediction of discretization with 1950-1952 diameter observations by Wilson, upon which a prior report of observational evidence for "quantized sizes of galaxies" was based. Work conducted under RSR-7068 has led to provisional- but not definitive - observational confirmation of the discretization hypothesis. The results are published in papers listed in the attached bibliography or are to appear in RM-3771-RC, whose publication awaits the completion of the transfer-plate program on the 48-inch Schmidt.

In addition to the central research question of project RSR-7068 - the geometrical discretization of elliptical galaxies - several important by-products have developed. On the theoretical side these include a theory of the optical morphology of elliptical galaxies (RM-4017-RC) and of relatavistic averaging operators. On the observational side, these include galaxy-diameter redshift relations (A.J., March 1964) and a cluster redshift discretization (relation (PNAS, Sept. 1964). These by-product findings are themselves of extreme interest and of possible major cosmological significance. Accordingly, I would like to recommend that follow-up research be authorized to investigate the following matters:

1. The evidence for structure in distributions of clusters of galaxies and other aggregates.

    a. An observational program to observe the redshifts of bright galaxies in nearby clusters. (B-spectrograph, 100-inch telescope, Mt. Wilson Observatory).

    b. Study of redshifts, quasi-redshifts, and angular distributions of clusters, D-galaxies, and radio sources.

    c. Structure of clusters and galaxies.

2. Preparation of a "Handbook of Cosmological Models" giving the relations between various observables which would be expected to obtain in evolutionary, kinematic, and steady state models.

3. Development of "non-uniform" cosmological models.

4. Investigation of observables and the modifications they exhibit under various degrees of resolution.

5. Further development of invariant averaging operators.

6. Investigation of the metrizability of cosmic space.

7. Development and application of observational techniques supportive of the above.

Items 1) and 7) would be carried on by the Department of Geophysics and Astronomy. Items 3), 4), 5), and 6) would be carried on in the Department of Mathematics (Edelen and consultants). Item 2) would be shared. The principal investigators under this project would be Wilson and Edelen. Others participating would include Kocher and Kirkwood. Page would be continued as a consultant. It is estimated that Kocher would spend 1/4 time, Wilson 3/4 time, and Kirkwood consultative time.

A detailed budget estimate including computer and other supportive costs will be worked out contingent on the approval of this proposal.

The growing interest in relatavistic and cosmological studies by the Air Force (Wright Field Group), the recognition of the importance of new effects, new forces, and new fundamental relationships in astrophysics and physics by the military and NASA (see enclosure) all suggest this research proposal may legitimately be considered proper for Project RAND.

A. G. Wilson

AGW:cs
Encls.

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Dynamics of Discontinuity Surfaces in General Relativity Theory
Journal of Math. Anal. and Appl. vo. 7, No. 2, p. 247, 1963.

2. D. G. B. Edelen
Relativistic Surface Dynamics of an Isolated World Tube of Perfect Fluid, Proc. N.A.S. vol. 50, No. 3, p. 469, Sept. 1963.

3. D. G. B. Edelen
Galactic Scale Discretization
Astron. J. Vol. 68, No. 8, p. 535, Oct. 1963
(Paper read before Amer. Astron. Soc. July 1963)

4. A. G. Wilson
Tentative Observational Confirmation of Discretization in Galaxies
Astron. J. Vol. 68, No. 8, p. 547, Oct. 1963
(Paper read before Amer. Astron. Soc. July 1963)

5. D. G. B. Edelen
Possible Galactic Scale Discretization
RAND Corporation RM-3941-RC Nov. 1963

6. A. G. Wilson
Discretization in EO Field Galaxies
Astron. J. Vol. 69, No. 2, p. 153, March 1964
(Paper read before Amer. Astron. Soc. Dec. 1963)

7. T. Y. Thomas
Outline Theory of the Universe
Proc. N.A.S. Vol. 51, No. 5, p. 718, May 1964

8. T. Y. Thomas
Radial Discretization in Spherical Galaxies
Proc. N.A.S. Vol. 52, No. 1, p. 1, July 1964

9. T. Y. Thomas
Discretization in Galactic Structure and Cosmology
RAND Corporation RM-3990-RC July 1964

10. D. G. B. Edelen
Deformations and Momentum-Energy Complexes
Arch. Rat. Mech. Anal. 16, p. 316, 1964

11. D. G. B. Edelen
Relativistic Galactic Morphology - I; General Theory
RAND Corporation RM-4017-RC Aug. 1964

12. T. L. Page
Proceedings of the Conference on Discrete Parameters in Cosmology
RAND Corporation RM-4267-RC Aug. 1964

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RAND Corporation P-3020, November 1964

15. A. G. Wilson
On Super-Organizations Among Clusters of Galaxies
A. J.
(Paper read before American Astronomical Society, Dec. 1964)

Discretization is also cited in the following.

1. Carnegie Institution of Washington Year Book 62, p. 44, 1962-1963.

2. Transactions of the International Astronomical Union, XII General Assembly. Draft Report p. 362, June 1964.

3. New Scientist, June 25, 1964

4. The Blade - Toledo Ohio - Mar 5, 1965

16. T. Y. Thomas Proc. Nat. Acad. Sci. Vol 53, No. 2. Feb, 1965 pp 227-228
"Galactic discretization and the future of the universe"

Space Science Board to NASA
Nov 1964

3. Recommendations

Aware of the parallel criteria of scientific and intellectual importance and of significance to the national interest, the Board summarizes its recommendations on the primary national objectives in the field of space science for the 1971-1985 period as follows:

1. Exploration of the planets with particular emphasis on Mars

(a) This objective includes both physical and biological investigations, and especially the search for extraterrestrial life.

(b) The experimentation should be carried out largely by unmanned vehicles while the solution of difficult biomedical and bioengineering problems proceeds at a measured pace so that toward the end of this epoch (1985) we shall be ready for manned planetary exploration.

(c) Alternatives to the Mars and planetary exploration goal -- (i) extensive manned lunar exploration including lunar base construction and (ii) major manned orbiting space station and laboratory program -- are not viewed as the primary goal because they have less scientific significance though both have sufficient merit to warrant smaller programs.

2. An enhanced effort in basic astrophysical research aimed toward a better insight into the fundamental nature of matter and energy

Particular attention should be paid to observations in the far ultraviolet and long radio wavelengths and in the x-ray and gamma-ray wavelengths because fundamental relationships might be discovered between the physics of the very large (relativity) and the physics of the very small (elementary particles). Also attempts to observe gravitational radiation should be supported and encouraged.

3. Continuing pursuit of other physical and astronomical, and biological investigations on a broad scientific front using sounding rockets, earth satellites, space probes, lunar orbiters and lunar landers

4. Continuing development of technical applications of space technology in the fields of communication, meteorology, geodesy and navigation

(a) Such work should be concentrated on basic technological development and on engineering demonstrations, but

(b) routine operational use of space systems in these fields should generally not be undertaken by NASA; instead, it should be assigned to the appropriate operating agency of the government or, as feasible, to private corporations.

On Super-Organization Among Clusters of Galaxies. A. G. WILSON, *The Rand Corporation.*—The mean redshifts of clusters of galaxies do not appear to be distributed randomly. The small sample of available mean redshifts is consistent with the hypothesis that the clusters are located on a set of concentric shells which possess a definite relation between successive radii (Wilson, A. G., *Proc. Nat. Acad. Sci.,* **52,** 1964). If this distribution is real, the cosmological principle requires that apparent cluster distribution should be on concentric shells for all equivalent observers (i.e., observers located in or near a cluster). The actual spatial locations of clusters must then be at the intersections of the several sets of cluster-centered concentric shells. This requires structure in the *angular* distribution of cluster centers as seen by equivalent observers.

The investigation of regular structure in the distribution of clusters may be investigated further by combining the angular positions of clusters with the mean redshifts to generate additional "quasi-redshifts" by triangulation. If a linear redship-distance relation and Euclidean space are assumed, the quasi-redshifts may be derived by the ordinary ~~laws~~ law of cosines. If these assumptions are valid and if the spatial distribution of clusters is regular the frequency distribution of quasi-redshifts should be a set of discrete peaks or resonances which represent the allowable separations between clusters.

The ~~observed~~ frequency histograms of ~~1000~~ the quasi-redshifts show~~s~~ a set of peaks distributed among a "noise" background. Statistical tests show that over fifteen of these resonances are not likely to be random fluctuations, (observed occurrence minus expected occurrence$>3\sigma$). It may be inferred that at least a subset of clusters manifests structured distribution. The noise may be due to breakdown of the Euclidean and linear-redshift approximations, to the coexistence of two or more independent organizations, and/or to actual random distributions.

It is further found that the ratios of the values at which some of the resonances occur are $3^{\frac{1}{2}}$, 2, $5^{\frac{1}{2}}$, $8^{\frac{1}{2}}$, $13^{\frac{1}{2}}$, suggesting the distance ratios which obtain for closely packed spheres.

The unlikelihood of the occurrence of these peaks and ratios in distances between clusters distributed in a random uniform manner suggests either that some form of super-organization exists among the clusters or that we are observing the vestiges of a structure whose angular and linear ratios have been preserved under a uniform and isotropic expansion from a time when the universe was in a highly compact stage. The latter hypothesis if physically consistent, would be corroborative of an oscillatory or other evolutionary model.

Alternatives to the vestige-hypothesis must account for an organization extended over 10^9 parsecs, the value bounding the separations of the clusters involved. It is difficult to explain such an extended organization without the introduction of physical communication processes not at present recognized.

5 Rev. (7-55)

MEMORANDUM

TO: S. M. Greenfield — DATE: 3-3-65

FROM: A. G. Wilson — MEMO NO.: M-1454

SUBJECT: GOALS AND OUTLINE OF THE RESEARCH PROJECT ON COSMIC STRUCTURE

COPIES TO:

The observational work which was initiated under RSR-7086 did not result in definitive confirmation of the primary hypothesis being tested but did lead to other very interesting results which suggest the existence of unsuspected <u>regularities</u> in the distribution of the clusters of galaxies. The existence of regular structure on a cosmic scale has most important implications not only to cosmology but to physics. This is primarily because such structure furnishes clues to the properties of hypothesized cosmic scale repulsion forces. Because of the importance of the possible existence of structure, the Observatory Committee of the Mt. Wilson and Palomar Observatories, on the endorsement of Dr. Allan Sandage, has allotted me monthly time on the 100-inch telescope to measure the redshifts of objects in representative clusters to be used to investigate structured distribution.

The current research project on cosmic structure has two observational phases:

1. Measurement of redshifts of cluster objects with the B spectrograph of the 100" telescope (dispersion 350 Angstroms/mm).

2. Galaxy counts on 48-inch Schmidt plates for cluster size, population, and position.

The following specific problems are at present being analyzed:

1. The differences in the distribution of the clusters and the cosmic radio sources, including the unexplained anomalies in the radio source counts.

2. The resonances in the quasi-redshift and cluster angle distributions.

3. The dependence on structure on the $(1+z)^{\alpha}$ parameter. The possibility exists of differentiating between evolutionary and steady state models on the basis of the value of the index α.

4. Description of the redshift and angular regularities as complete as is possible with available data. The eventual goal is the synthesis of the exact inter-cluster structural relations.

Whereas the emphasis may shift from these specific questions to others as the data accumulates, the fundamental importance of exhibiting any and all forms of structure which may exist in aggregates of matter is unquestioned. The current recommendation of the Space Science Board to NASA (Nov. 1964) urges, "An enhanced effort in basic astrophysical research aimed toward a better insight into the fundamental nature of matter and energy." The relativity research group at Wright Air Force Base has repeatedly expressed interest in cosmological problems and recognizes the scientific significance in efforts in this area. It is also of interest in this connection to reference the remarks of Alexei Kosygin, then Vice Chairman of the Council of Ministers of the USSR, in his remarks to the Tenth General Assembly of the International Astronomical Union gathered in Moscow in August 1958. In talking of the goals of Soviet science, he said "Soviet astronomers are keenly searching for hitherto unknown cosmic forces and new fundamental concepts which might suddently emerge from research in astrophysics and space." (p. 12, P-1801, The RAND Corp., Sept. 1959).

The remote distances to the particular objects which are the subjects of this research investigation - the galaxies and clusters of galaxies - must not lead us to assume that knowledge of their structures and properties is ~~also~~ therefore of remote significance to the mainstream of our nation's position in science.

A. G. Wilson

AGW:cs

D No: D-14379-PR-RC

Assigned to: Eng

A. Wilson

Project No: 3000/7000

Contract No: AF 49(638)-1700

Task Order No:

RAND DOCUMENT

ANNUAL PROGRESS REPORT OF THE DEPARTMENT OF GEOPHYSICS AND ASTRONOMY

Staff, Department of Geophysics and Astronomy

14 January 1966

For RAND Use Only

PREFACE

Accounts of 1965 activities in the Department of Geophysics and Astronomy, together with estimates of future work, are listed by RAND project number.

A list of 1965 publications is included at the end of this document.

RPN 3380: ASTRONOMY

DESCRIPTION:

Project 3380 consists of basic research projects of an astronomical nature and of astronomical activities supporting current Air Force interests.

PROJECT PERSONNEL:

Staff: A. G. Wilson, G. E. Kocher, M. Portl; and S. Dole (Aero), R. Mobley (CSD).

Consultants: T. Page, G. de Vaucouleurs.

ACTIVITY AND RESULTS:

(1) Using the 48-inch and 18-inch Schmidt telescopes on Palomar Mountain, Wilson and Kocher have continued Project Flosshilde, the photographing of Soviet and U.S. deep-space probes to evaluate the effectiveness of optical techniques for the tracking of distant spacecraft. During 1965, six field exercies were held. Kocher succeeded in obtaining a series of four plates of Luna VII at a distance of 170,000 km on October 5. This project is coordinated through NORAD Headquarters at Ent Air Force Base.

(2) Of importance to projects such as Flosshilde is the development of automatic means of reducing plate positions and angular velocities of moving objects. Kocher, with the assistance of Mobley, adapted the Bell Telephone Laboratory Star Chart program for use with Rand's IBM 7044. This adaptation, which renders the program of general use, will be made available to some 25 other users on a tape-exchange basis. The program tabulates all catalogue stars and their positions to very high accuracy in a given field of view. It produces a graphic output, which allows ready identification from photographs.

(3) Wilson has continued studies of galactic morphology and cosmic structure. With the assistance of Portl, programs for analyzing the distributions of nearby galaxy clusters and radio sources have been developed. Redshifts, quasi-redshifts and angular separations have been derived and examined for evidence of structure. A method of viewing the spatial distribution of cosmic objects, as projected on any given plane, has been worked out using the SC4020 plotter. First results of the structure investigation were published in the Astronomical Journal,

Vol. 70, March 1965, p. 150.

(4) Wilson with Page (and G. C. Omer, a visiting professor at UCLA) studied the distribution of galaxies in the Coma Cluster through counts made on 48-inch Schmidt plates. The results were published in the Astronomical Journal, Vol. 70, August 1965, p. 440.

(5) Wilson received an appointment as guest investigator at the Mt. Wilson and Palomar Observatories for 1965. He initiated a series of observations of the spectra of the brightest galaxies in nearby clusters with the B-spectrograph on the 100-inch for purposes of studying the spatial distributions of clusters in the local metagalaxy. Twenty-seven plates were secured during the first year (1965).

(6) Wilson and Kocher, in cooperation with CSD, have helped develop designs for adapting the RAND Tablet for projection inputs. Hardware is now being contructed that will allow photo plates to be focused directly on the tablet and photo-data inputs to be made directly to the computer. Portl has programmed the 7044 to give digital outputs which characterize the graphic inputs. This program is of importance in establishing suitable techniques for the study of galactic morphology and astronomical photographs in general. (See Memo M-8069, Wilson and Kocher; and D-13599-PR, Wilson.)

(7) Dole has continued his computer simulation experiments on planetary formation through accretion of particulate matter within a cloud of dust and gas surrounding the newly-formed sun.

The present program consists of an attempt to test the validity of one version of this hypothesis through use of a computerized Monte Carlo technique. In the model being used, nuclei are "injected" into the cloud one at a time on elliptical orbits. The dimensions of the semi-major axis and the eccentricity of the orbit of each nucleus are generated by using random numbers. As nuclei orbit within the cloud, they grow by accretion and gradually sweep out dust-free annular lanes. If they grow larger than a specified critical mass, they can begin to accumulate gas from the cloud as well. If the orbit of a planet comes within a certain interaction distance from a planet formed earlier, the two bodies coalesce in accordance with a few simple rules to form a single, more massive planet, which may then continue to grow by accretion. The process of nucleus injection is continued until all the dust has been swept from the system. At this point the run is terminated and the machine output displays the masses and orbital parameters of the planets remaining in the final configuration.

Each planetary system produced by using a different random number sequence is unique. However, all systems so produced share the major regular features of our solar system. The orbital spacings have patterns of regularity suggestive of "Bode's law." The innermost planets are small rocky bodies; the midrange planets are large gassy bodies; the outermost planets are generally small.

The general pattern of planetary mass distribution is similar to that in our solar system with masses ranging from smaller than Mercury's to larger than Jupiter's; orbital eccentricities are of the same order as those found in our solar system. Typical systems contain 8 to 13 planets.

The effects of changing various input parameters have been studied and will continue to be studied in the coming year. Parameters that can be changed readily include: the density distribution in the original cloud (a function of distance from the center of mass); the critical mass (a function of periastron distance); the gas-to-dust ratio in the cloud; the growth pattern of gas giants; the planetary interaction radius (a function of mass); the orbital eccentricity probability function.

No publications relating to this program have yet been produced because it is desired to explore further the effects of varying input parameters and to obtain improved models. Preliminary results, however, indicate that the model based on the accretion hypothesis is compatible with the main features of the solar system.

FUTURE WORK:

(1) Project Flosshilde will be continued through 1966 in cooperation with NORAD, Mt. Wilson and Palomar Observatories, and JPL.

(2) Further development of the scope of Star Chart is planned by Kocher (1/4). The program will be made more useful by including parallax and radial velocity information on all catalogue stars for which these data are available. Also, orbital data on visual binaries,

NGC data, and Bright Galaxy Catalogue data will be included.

(3) Continuation of analysis of galaxy and cosmic structure with RAND computer-graphic equipment. Wilson (1/10)

(4) Dole plans continued study of the accretion hypothesis along lines elaborated under (7).

ADDITIONAL PUBLICATIONS: (not referenced above)

(1) Kocher: Observations of the December 18, 1964 Lunar Eclipse, D-13426-PR.

(2) Wilson and Kocher: Earth-Based, Moon-Based, and Space-Based Astronomical Observatories, D-13730-PR.

(3) Wilson: Physical Views on the Origin of the Universe, P-3247-PR.

(4) Wilson: Olbers' Paradox and Cosmology, P-3256-PR.

COMMITMENTS:

None.

RPN 7065: LORENTZ INVARIANCE IN A GRAVITATIONAL FIELD

DESCRIPTION:

The object of this project is to investigate the possibility of introducing a large-scale Lorentz transformation into the gravitational field so that the principle of special relativity can be applied to the entire gravitational field.

PROJECT PERSONNEL:

R. L. Kirkwood.

ACTIVITY AND RESULTS:

A detailed analysis has been made of the infinitesimal Lorentz transformation, in which the relative velocity of the two coordinate systems is much less than the velocity of light. The results suggest that the analysis should be carried out for an arbitrary relative velocity, and this analysis has been partially carried out and will be reported in a forthcoming document, "Lorentz Invariance in a Gravitational Field," by R. L. Kirkwood.

FUTURE WORK:

It was hoped that this analysis could be completed and augmented by a physical investigation of the relative rotation of the two systems of coordinates, but these will be delayed by the shortage of funds.

COMMITMENTS:

None.

PROJECT RAND PUBLICATIONS, 1965

R-444 "The Heat Budget of the Arctic Basin and Its Relation to Climate" J. O. Fletcher, RPN 3250

RM-4458 "Positive Stable Laws and the Mass Distribution of Planetesimals" A. H. Marcus, RPN 3054

RM-4480 "Weather Factors in Air-to-Surface Missile Operations" S. M. Greenfield, W. W. Kellogg, and R. R. Rapp, RPN 3300

RM-4557 "Eclipse Observations from Orbiting Spacecraft" R. C. Moore and G. F. Schilling, RPN 3054

RM-4619 "On the Prediction of Satellite Orbit-Decay and Impact" C. Gazley, Jr., L. N. Rowell, and G. F. Schilling, RPN 3054 (also 3006, 3227)

RM-4681 "A Stochastic Model of the Formation and Survival of Lunar Craters II. Approximate Distribution of Diameter of all Observable Craters" A. H. Marcus, RPN 3054

RM-4682 "A Stochastic Model of the Formation and Survival of Lunar Craters, III. Filling and Disappearance of Craters" A. H. Marcus, RPN 3054

RM-4715 "Elaboration of Hocking's Hydrodynamic Theory" C. L. Olson, RPN 3250

RM-4747 "Numerical Estimates of Luminance and Total Brightness of the Terrestrial Halo" R. C. Moore and G. F. Schilling, RPN 3054

RM-4818 "Photochemical Destabilization of Gravity Waves Near the Mesopause" C. B. Leovy, RPN 3342

RM-4846 "Multiple Scattering in Homogeneous Plane-Parallel Atmospheres" T. W. Mullikin, RPN 3162

P-3088 "The Coma Cluster of Galaxies, I: Size and Structure" G. C. Omer, Jr., T. L. Page, and A. G. Wilson, RPN 3380

P-3099 "The Effect of Electric Charges and Fields on the Collisions of Very Small Drops" M. H. Davis, RPN 3250

P-3105 "Some Economic Aspects of Weather Modification" R. R. Rapp, RPN 3250

P-3130 "Eclipse Observations from the Moon and Cislunar Space" G. F. Schilling and R. C. Moore, RPN 3054

P-3145 "The Difference Between Weather Forecasters and Weather Advisors" R. R. Rapp, RPN 3054

P-3208 "The Influence of the Arctic Pack Ice on Climate" J. O. Fletcher, RPN 3250

P-3232 "On the Apparent Brightness of the Earth's Halo" R. C. Moore and G. F. Schilling, RPN 3054

P-3234 "Predicting the State of the Upper Atmosphere" G. F. Schilling, RPN 3054

P-3245 "Comments on the Detection of Water and Ice Clouds on Venus" D. Deirmendjian, RPN 3162

P-3247 "Physical Views on the Origin of the Universe" A. G. Wilson, RPN 3380

P-3250 "Climate and the Heat Budget of the Central Arctic" J. O. Fletcher, RPN 3250

P-3256 "Olbers' Paradox and Cosmology" A. G. Wilson, RPN 3380

P-3270 "A Survey of Magnetic Storms" E. H. Vestine, RPN 3054

P-3282 "On the Scattering of Sunlight into Planetary Shadow Cones" R. C. Moore and G. F. Schilling, RPN 3054

D-13274 "The Effective Power of T in Planck's Radiation Law" M. H. Davis, RPN 3054

D-13420 "Photoelectronic Devices" G. E. Kocher, RPN 3054

D-13426 "Observations of the December 18, 1964 Lunar Eclipse" G. E. Kocher, RPN 3054

D-13599 "Digitized Data from Astronomical Photographs" A. G. Wilson, RPN 3380

D-13690 "Climatology for War Games" R. R. Rapp, RPN 3054

D-13730 "Earth-Based, Moon-Based, and Space-Based Astronomical Observatories" A. G. Wilson and G. E. Kocher, RPN 3380

D-13775 "Weight and Volume Estimates of Astronomical Equipment for an Observatory on the Moon" G. F. Schilling, R. C. Moore, and G. E. Kocher, RPN 3054

D-13809 "Some Thoughts on the Needs of a Military Weather-Satellite System" Y. H. Katz, RPN 3054

D-13970 "Highlights of Weather Around Hanoi, Djambi, Palembang, and Kupang" C. Schutz, RPN 3054

D-14012 "The Satellites of Mars -- What are they Like?" V. A. Bronshten (J. O. Fletcher), tr. R. Olenicoff, RPN 3054

D-14091 "Diameter Distribution of Martian Craters: Preliminary Analysis" A. H. Marcus, RPN 3054

D-14192 "Standard Unit of Illumination for Lunar and Planetary Research" R. C. Moore and G. F. Schilling, RPN 3054

D-14278 "Trip Notes--Notes on a So-called 'International Symposium on Electromagnetic Sensing of the Earth from Satellites' held in the Deauville Hotel, Miami Beach, Florida, November 22-24, 1965" D. Deirmendjian, RPN 3162

D(L)-14170 "Comments on General War Notebook" J. O. Fletcher, RPN 3000

D(L)-14171 "Trip Report: XVI Astronautical Congress, Athens, and International Symposium on Weather Prediction, Vienna, September 1965" G. F. Schilling, RPN 3054

S. M. Greenfield 1/19/66

A. G. Wilson M-463

PRESENT STATUS OF WORK ON "OBSERVATIONAL INVESTIGATIONS OF GALAXY MORPHOLOGY AND DIAMETER DISCRETIZATION"

To date, the diameters of "stellarized images"* of galaxies in eight different clusters have been measured by micrometric techniques. The distributions of the amajor axes of these galaxies show multi-modal distributions which are highly improbable if generated by random null mechanisms. Because of an uncertainty in the photometric calibrations of the plates it is not possible to combine the data into a single sample without introducing some unknown systematic errors. Some calibration plates have been taken and additional plates will be taken to estimate this error.

It is fair to say at this time that some sort of "discretization" appears to exist in the sizes of cluster galaxies. It cannot be asserted, however, that this distribution conforms to the Edelen discretization sequence. Sharper calibration is necessary before the $\sqrt{n(n+1)}$ hypothesis can be tested.

In addition to calibration studies, a new method of diameter measuring is being developed using computer graphics. It is hoped to get an independent set of values as soon as the projection mechanism for the RAND tablet is completed.

A. G. Wilson

AGW:cs

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*Stellarized galaxy images are obtained by successive printing on very high contrast emulsions. By means of two or three iterations it is possible to sharpen isophotic contours sufficiently to provide an operationally defined galaxy diameter which can be measured with very good internal consistency.

A.G. Wilson
BH

THE BLADE
TOLEDO, OHIO
D. 180,484 SUN. 187,134

MAR 5 1965

THE BLADE: TOLEDO, OHIO, FRIDAY, MARCH 5,

Epochs Of Creation Envisioned

Universe Likened To Sleeping And Waking Giant

Indiana Scientist Offers New Theory

By RAY BRUNER
Blade Science Editor

For a tremendous period of time our universe existed inert and unstructured, "devoid of ponderable matter."

Then suddenly, like a sleeping giant, it came awake. Vast gravitational waves occurred, analogous to shock waves in a gas. As these waves traveled through space, they eventually converged, and again spread out, only to converge again.

There have been at least three "cosmological epochs" in which this happened. The process may have begun 20 billion years ago, or perhaps farther back in time.

In each epoch, atoms of matter that make up the present galaxies, stars, and interstellar material, were created.

This process is continuing. Eventually, however, the universe must decay. It will return to a state resembling its inert and unstructured beginning.

This new theory of creation was presented today in the current Proceedings of the National Academy of Sciences by Dr. Tracy Y. Thomas, professor of mathematics, Indiana University. It is based on a substantial volume of data assembled by Dr. A. G. Wilson of the Rand Corp., Santa Monica, Calif.

The data are based on estimates of the size of galaxies such as our own Milky Way, and measurements of the red shift in spectroscopic analysis of light from galaxies that are traveling away from us at a rapid rate.

Using the data, Dr. Thomas applied field equations of Albert Einstein, to develop his theory. One of Dr. Thomas' purposes was to provide possible explanation of a number of current observations of astronomers which have not been adequately explained by present theories.

Chief present theories of the creation of the universe are the "big bang" theory, developed by Abbe Georges E. Lemaitre of the University of Louvain, Belgium, and Dr. George Gamow of the University of Colorado, and the theory of continuous creation, expounded by Dr. Thomas Gold, professor of astronomy, Cornell University; Dr. Fred Hoyle, University of Cambridge mathematician, and Dr. Herman Bondi, University of London.

Constant Process

According to the Gold-Hoyle-Bondi theory the material of the universe is continually being created and destroyed. This process evidently has gone on endlessly and will continue.

The Lemaitre-Gamow theory states that at one time the universe consisted of a great mass of plasma containing a tremendous volume of energy. Suddenly it exploded, with the formation of atoms and molecules of matter in the stars, interstellar gas and dust, planets, asteroids, and other objects in space.

The explosion is still in progress with galaxies speeding through space, with their speed indicated by the red shift in their spectra.

In their efforts to explain many current phenomena in the universe, scientists have not been completely satisfied with either the "big bang" or the "continuous creation" theory. For one thing the "big bang" theory would limit the age of the universe to about 6.5 billion years. There are many indications that the universe is much older.

Quasar Speculation

Astronomers have also run into difficulty in trying to use either of these theories to explain the occurrence of quasars, or recently observed quasistellar radio sources, which are immense sources of energy about 3 billion or more light years away.

Dr. Thomas suggested that the quasars may be in a region far out in space, where gravitational waves may be converging to produce new matter.

It is reasonable to expect that his theory, like the two others, will be a subject of criticism and controversy.

Criticism is customarily expected when a new theory such as his is published. The history of astronomy is replete with theories that have been born and have died. This is the way science advances.

In foreseeing an end to the universe, his theory, at least, in that respect, is different.

The bright flash of light in the sky which frightened many of the world's inhabitants from Iraq to Japan in A.D. 1006 has been identified today as a supernova—the giant explosion of a star.

Historic Sky Flash Termed A Supernova

In the current Astronomical Journel, Dr. Bernard Goldstein Yale University, said the supernova was only the fourth to have appeared during historic times in our own Milky Way galaxy.

1962-1963

found in visible and ultraviolet spectra.

Dr. George Wallerstein of the University of California has used both the 100- and 60-inch telescopes in a search for the presence of the lithium doublet at λ6708 in F stars. The line was not present in 5 stars taken at 6 A/mm or in 11 stars observed at 30 A/mm. Three plates at 6 A/mm were taken for the measurement of the ratio of Li^6/Li^7 in 111 Tauri, a star in whose spectrum Wallerstein had found lithium at the Lick Observatory.

In 1956 Wilson and M. K. Aly reported that λ Andromedae, a K0 IV star with exceptionally strong Ca II emission, probably showed the helium line at λ5876 in absorption. This has been confirmed by Wallerstein on spectrograms of higher dispersion (6 A/mm). Wilson and Wallerstein have examined 6 A/mm spectrograms of 48 stars of types G8–K2 for evidence of λ5876 in absorption. It is possibly present in 3 stars: β Ceti, π Cephei, and HR 6791. Spectra of the first two of these in the violet show that the Ca II emission is not strong.

Recently the investigations of Wilson have disclosed some extremely interesting correlations involving the H and K Ca II emission in late-type stars. So far, however, a quantitative interpretation of both the width and intensity of the emission in terms of a series of model stellar chromospheres has not been made. To do this, reliable profiles of the emission peaks must be obtained.

Dr. Ray Weymann of the University of California at Los Angeles has obtained spectrograms of suitable quality for reliable microphotometry at 4.5 A/mm of a number of stars, mostly luminosity class III giants in the K0–K3 range. Striking differences in the intensity and character of the central self-reversal are apparent among stars of nearly identical spectral class.

These spectrograms have been supplemented by some at 8 A/mm covering Hα and the Ca II infrared triplet. It is hoped that additional plates of stars showing much more intense emission can be obtained; in particular, manifestations of chromospheric activity in the Ca II infrared triplet are being sought.

Dr. Albert G. Wilson and Dr. George Abell, cooperating with the Jet Propulsion Laboratory and the Air Force, undertook to test the feasibility of "deep space" tracking by optical methods. The Palomar 48-inch schmidt was successfully used on August 28 and 29, 1962, to photograph at distances out to 600,000 km the Agena carrier rocket which injected the Mariner II Venus probe. (These photographs were made five weeks before the Soviet announcement of their "first" photograph of a space craft on an interplanetary mission.)

Dr. D. G. Edelen of the Rand Corporation, applying the Einstein field equations to galaxies considered as aggregates of granulated matter, has found that under certain very general conditions their ellipticities must be functions of the semimajor axes. The functional relationships are multibranched, predicting that the distribution functions of the major axes of galaxies should have discrete peaks distributed proportionally to the eigen sequence, $[n(n + 1)]^{1/2}$, where n is a positive integer. To test this prediction, Dr. A. G. Wilson of the Rand Corporation has resumed his earlier studies of galactic diameters (*Carnegie Year Books 49* and *50*), reexamining previous observational suggestions of discretization in the diameters of cluster galaxies. Using plates of clusters taken by Baade, Humason, and Sandage with the 200-inch, Wilson has constructed distribution functions of relative diameters of early ellipticals in the Coma and Corona Borealis clusters. The diameter distributions show asymmetric density maxima occurring at values whose ratios are consistent with the theoretical prediction to within the observational errors. The preliminary results indicate that larger samples must be studied before there can be definitive confirmation of the Edelen hypothesis.

1963-1964

Photoelectric U, B, V magnitudes in the range $V=10^{\mathrm{m}}$ to $17^{\mathrm{m}}5$ for stars in the Small Sagittarius Cloud, in NGC 6871 and 6883, and in M 37 have been obtained by Tammann, working as a guest investigator with the 60- and 100-inch. These stars will be used as standards for the Basel Observatory program of a three-color photometric investigation on 48-inch plates of star fields in different galactic latitudes and longitudes as well as of some galactic clusters. The 20-inch was used in combination with the RCA 7265 phototube to get the first photoelectric magnitudes in W. Becker's R, G, U system. This system is photographically defined, and the best approximation for photoelectric work was obtained with the filters Schott RG1, GG5+BG7, and UG2, respectively. A fourth color with the filters GR1+UG2 was measured only to correct for the red leak of the ultraviolet filter.

With the nebular spectrograph at the 60-inch, Tammann obtained some spectra of four bright cluster members of NGC 7790 as well as of some comparison stars. These spectra will be used to determine the radial velocity of the cluster, and with this to obtain an additional membership criterion for the double cepheid CE Cassiopeiae, the cepheid CF Cass, and the eclipsing binary QX Cass.

Radiometric and photometric mapping of the moon through a lunation was carried out with the 60-inch telescope by Dr. Richard W. Shorthill and Dr. John M. Saari of the Boeing Scientific Research Laboratories. For this work a focal-plane scanner was mounted at the Newtonian focus. With a separation between scan lines equal to the diameter of the scanning aperture (8″), a scan of the full moon requiring a raster of 240 lines could be made in less than 30 minutes. (Auxiliary photographs were obtained with the 60-foot solar tower and an 8-inch telescope mounted on the 60-inch.) The illuminated lunar disk was simultaneously mapped with an infrared detector (10 to 12 μ band pass) and a photomultiplier (4450 A peak response). The purpose of the program was to produce isothermal and isophotic contours that could be related to visible surface features. In six periods of observing between July and December 1963, mapping of the moon at 35 different phases was completed. Measurements could not be made during the December 30 lunar eclipse because of a haze.

As part of a program to determine the reddening and distance to certain CH stars, Dr. George Wallerstein of the University of California used the X spectrograph on the 60-inch to obtain classification spectrograms of selected A and B stars.

Dr. Ray Weymann of the Steward Observatory, University of Arizona, using the 100-inch telescope, obtained spectrograms at 4.5 A/mm showing emission components of the H and K lines in G, K, and M stars of all luminosity classes. Observations of the infrared triplet of Ca II at 6 A/mm failed to reveal any sign of analogous emission. Plates well suited for microphotometering the emission profiles of approximately 40 stars have now been obtained at the Mount Wilson and Lick Observatories. These data are now being reduced, and the relevant parameters describing the profiles will be published shortly.

A study is being made by Drs. G. O. Abell, G. E. Kocher, and A. G. Wilson of the Rand Corporation to determine the limiting magnitude of the schmidt telescopes when photographing moving objects under sky brightness conditions varying from dark to full moon. It is desired to find the optimum emulsion and filter combination for use in moonlight, as well as the limiting magnitude as a function of the rate of angular motion of the object.

A. G. Wilson and G. E. Kocher are continuing investigations of comparative galaxy diameters using, among other techniques, iterative reproductions on high-contrast emulsions. Transfer plates between selected galaxies and clusters of galaxies are being taken to secure comparable images. The principal purpose of the program is to investigate possible size discretization relations and to test the Edelen $\sqrt{n(n+1)}$ discretization hypothesis for elliptical galaxies.

Dr. R. v. d. R. Woolley of the Royal Greenwich Observatory took a number of direct photographs of NGC 6522 besides continuing determinations of radial velocity with the coudé spectrograph. The direct photographs were repeat plates of exposures made by the late Walter Baade. The purpose of taking the repeat plates was twofold: to attempt to determine the proper motion of the cluster, and to attempt to find at least some information about the velocity dispersion of the field stars at the center of the Galaxy. Two pairs of plates, one of each epoch, have been measured at Herstmonceux, and the results have been closely studied. The time interval is a little short for the purpose, and the object is inconveniently far to the south, but it is hoped to arrive at some tentative conclusions shortly.

STAFF AND ORGANIZATION

The retirement of Dr. Ira S. Bowen after 18 years as Director, to become Distinguished Service Staff Member, was noted in the introduction. Horace W. Babcock was appointed to succeed him as Director, effective July 1, 1964.

Other changes in the organization that have occurred during the year include the following:

Dr. Robert B. Leighton, a member of the Observatory Committee and Professor of Physics at the California Institute of Technology, was appointed to the staff of the Observatories as of July 1, 1963. For several years Dr. Leighton's chief research interests have been in the field of solar physics, and he is also concerned with many problems of astronomical instrumentation.

Dr. Edwin W. Dennison, whose special interests lie in electronic instrumentation, came from the Sacramento Peak Observatory to accept an appointment as Staff Member, effective September 1, 1963.

Dr. John B. Irwin took up his appointment as Staff Associate on June 6, 1964, with responsibility for site-testing operations in Chile.

Mrs. Mary F. Coffeen retired after five years as Librarian and many years as a research assistant. She first came to the Observatory as a computer in 1922.

Research Division

Staff Members

Halton C. Arp
Horace W. Babcock, *Associate Director*
William A. Baum
Ira S. Bowen, *Director*
Edwin W. Dennison
Armin J. Deutsch
Olin J. Eggen
Jesse L. Greenstein
Robert F. Howard
Robert P. Kraft
Robert B. Leighton
Guido Münch
J. Beverley Oke
Allan R. Sandage
Maarten Schmidt
Olin C. Wilson
Fritz Zwicky

Staff Members Engaged in Post-Retirement Studies

Harold D. Babcock
Alfred H. Joy

Senior Research Fellows

Leonard T. Searle[1]
Arne A. Wyller[2]

Carnegie Research Fellows

Leonard V. Kuhi
Hugo van Woerden
John B. Whiteoak

Research Fellows

Claude Arpigny[3]

[1] Resigned July 31, 1963.
[2] Resigned June 30, 1964.
[3] Resigned March 31, 1964.

1964-1965

distributions is important when the magnitudes of redshifted galaxies are to be corrected to zero velocity, and is relevant to theories concerning the formation of galaxies. Color–magnitude relations among members of clusters of galaxies will also be obtained. Furthermore, it is planned to supplement these data with spectra of moderate dispersion in order to study the stellar content of a few selected galaxies. Standards have been established and compared to absolutely calibrated stars, and a few galaxies have been measured. Preparations are also in progress for the narrow-band photography, in emission lines, of some small, dense, density-limited H II regions.

Dr. A. G. Wilson and Dr. G. E. Kocher of the Rand Corporation have continued their investigations of the diameters of galaxies in nearby clusters. A process of iterative photographic copying of cluster plates on high-contrast emulsions is used to "stellarize" the galaxy profiles. (This is in effect the inverse of the method adopted by Baum in his galaxy-image synthesizer.) The diameters of stellarized galaxy images may be measured with good internal consistency by conventional micrometric techniques. Provisional results show that the distributions of the sizes of major axes of elliptical galaxies are multimodal. While the distributions are suggestive of those theoretically predicted by Edelen, the present results cannot be sharpened to an observational confirmation of the theoretical $[n(n + 1)]^{1/2}$ diameter distribution.

Wilson has begun a program of securing redshifts of the brightest galaxies in the nearby previously unobserved clusters, using the B spectrograph or the 100-inch. Distances to these clusters are required not only for galaxy diameter studies, but also for investigation of the structure in the spatial distribution of clusters.

As part of the continuing Rand–University of California at Los Angeles study of the effectiveness of schmidt telescopes in observing distant space vehicles, Kocher succeeded in photographing the Soviet Mars probe ZOND II at a distance of 300,000 km, using the 48-inch telescope under conditions of good seeing and dark sky. Two 103a-O plates showing the faint trail of the spacecraft in Hydra were obtained on December 1, 1964.

The existing Palomar Sky Survey plates do not allow many galaxies to be differentiated from stars for objects fainter than about magnitude 18.5, a serious limitation in Dr. T. A. Matthews' program to identify the optical objects that produce radio sources. To improve the information content of the faint images and to study some of the brighter identifications, Matthews, of Caltech's Owens Valley Radio Observatory, has obtained plates of 65 identifications using the 48-inch schmidt telescope and fine-grained IIa-D and IIa-E plates. These show a significant improvement in the amount of information available over the 103a-O and 103a-E plates used in the Sky Survey, especially when the seeing is better than 2. Also, plates of 26 brighter identified sources have been taken with the 100-inch telescope, again using the fine-grained plates. This plate material has been invaluable in classifying the fainter identified radio galaxies in an extension of the work begun by Matthews, Morgan, and Schmidt. The results will be incorporated in a paper on the identification of the 3C revised sources, in collaboration with Wyndham, Fomalont, and Veron.

In examining the plates already taken, it was noted that 75 per cent of the radio galaxies show peculiarities, such as jets (7 per cent), diffuse plumes outside the galaxy (9 per cent), absorption features (10 per cent), structure in the nuclear region (17 per cent), structure in the envelope (9 per cent), nonsymmetry in the surface brightness or extent of the envelope (47 per cent). The percentage (in parentheses) of each group showing the peculiarity is only a lower limit since large-scale plates are not available for many of the objects.

spiral arms, disk, halo and intergalactic space of about 5×10^{-6}, 2×10^{-6}, 5×10^{-7} gauss is suggested.

S. Hayakawa and Y. Yamamoto (62) have computed the intensity and the energy spectrum of high energy γ-rays arising from the collisions of cosmic ray protons and thermal photons in the intergalactic space.

K. Ishida (63) finds negative correlation between H I gas and young stellar objects in the Magellanic clouds. Together with S. Aoki (64) he has studied collisions and relaxation time for gas clouds in galaxies as basis for a study of two evolutionary sequences, spherical and flat.

J. L. Sérsic (65) has discussed the time scale of the Universe, starting by defining the extragalactic scale of distances in terms of the absolute magnitude of the RR Lyrae stars and the corresponding time scale is compared with the nuclear scale given by the theory of stellar evolution according to computations by Hoyle. If the Hubble constant is $H = 116$ km sec^{-1} Mpc^{-1} and the absolute magnitude of the RR Lyrae stars is $M_B = 0{\cdot}00$, it is concluded that the initial abundance of hydrogen should be smaller than 0·66 in the Old Population I. A detailed study of H II regions (66) gives a theoretical interpretation of the correlation between diameters of the largest H II regions and the morphological type of the galaxy. He concludes that the age of the galaxies is approximately the same but that their evolutionary rates depend on the morphological type. J. L. Sérsic and R. Sisteró are investigating the pulsational stability of a plasma in an external axisymmetrical magnetic field under the action of a concentric gravitational field arising from another mass distribution. This may have application to matter in elliptical galaxies.

According to S. van den Bergh (67) studies of the metal abundances of stars in the Galaxy lead to the following conclusions. (*a*) The heavy element enrichment of the interstellar medium was well advanced at the termination of the halo phase of stellar evolution. (*b*) The rate of heavy element formation in the Galaxy has declined more rapidly than the rate of star formation. (*c*) The enrichment of the interstellar medium in heavy elements has been negligible during the last $4{\cdot}5 \cdot 10^9$ years.

A. G. Wilson and D. Edelen have conducted work at the Rand Corporation on relativistic discretization of diameters in clusters (Preliminary results were reported at the meeting of the American Astronomical Society in July 1963). Edelen has shown that the Einstein theory of general relativity, when used in conjunction with the epistological equivalents of certain well known properties of galaxies, predicts a relation between the galaxian semi-major axis r and the eccentricity ϵ (or ellipticity) of the form $r(n, m, \epsilon) = $ const., where n is a positive integer and $0 \leq m < n$. In the particular case $\epsilon = 0$, the relation between r and n takes the form,

$$r^2\xi = n(n + 1)$$

independent of m, where ξ is a physical parameter corresponding to the jump in the total energy density across the surface of the world tube representing the galaxy, as seen by an observer moving along an intrinsic time line of the surface. If ξ is constant, or of limited variation, it follows that the diameters of E0 galaxies should exhibit discretization of size.

The earlier data of Wilson (68) which first suggested discretization among globular galaxies have been re-assessed and combined with new measures. The present observational confirmation rests on (1) Wilson's angular diameters of E0 galaxies in six clusters re-measured on 200-inch plates. (2) The diameters of all E0 galaxies in the new Reference Catalogue of de Vaucouleurs, (3) the fine structure in Abell's (69) luminosity function of the Coma cluster. The diameter redshift relation for cluster galaxies confirms the Hubble law and reveals the hitherto unsuspected relation that the redshifts of all clusters so far published obey the empirical relationship $\mu\, n(n + 1) = K_\sigma$, where $\mu = \frac{1+z}{z}$, n is a positive integer, and K_σ is a limited set of discrete constants related to the parameter ξ of Edelen's discretization function.

LECTURE

THE UNIVERSITY OF TEXAS - DEPARTMENT OF ASTRONOMY

Colloquium No. 48

Thursday, April 2, 1964

4:00 p.m.

Physics Building

Room 448

Subject

OBSERVATIONAL INVESTIGATIONS OF GALACTIC

SCALE DISCRETIZATION PHENOMENA

DR. A. G. WILSON
Planetary Sciences Department
The RAND Corporation
Santa Monica, California

Same Lecture: Indiana University, April 7, 1964

Edelen: RAND BOARD: April 10, 1964

I. HISTORY

A. Prior to 1962 — Jump problems

B. r = n(n + 1) Oct. 1962

C. Wilson — Palamar — 1949-50

D. Comparison — Nov 1962

E. Nov 1962 to present

refine theory, refine observations, morphology, observational methods on diameters automated

II. Objects of Investigation

1 A. Spirals

2 B. Ellipticals — note composit images

3 C. EO Galaxies

4 D. Clusters

III. Theory

5 A. Geometry = Physics

B. Average

C. No knowledge of T_{AB}

6 D. $L = \rho(n, 0, \epsilon)$ linear diameter

7 E. Morphology

IV. Observations

A. Observables, angular diameters, red shifts, coulor, magnitude

8 B. Original form of EO— de V —associate n with each

9 C. First Reduction

10 D. Second reduction

11 E. coulor correlation

12 F. alternative presentation — number vs. observables

G. Cluster data

13 H. Red Shift Discretization — EPITACTIC COSMOGRAPHY

V. Implications

14 A. true vs observed ϵ

B. Theory of galactic morphology

C. Universal callebration scale

D. Eliminated previous cosmologies

E. crystal structure and epoches

F. Bodes Law

G. Test of general relativity and G = T.

CONFERENCE

INFORMAL CONFERENCE ON COSMIC DISCRETIZATION PHENOMENA

June 29 and 30, 1964
The RAND Corporation
1700 Main Street, Santa Monica

MONDAY, June 29:

9:30 a.m. — Relativistic Models of Elliptical Galaxies, by D. G. B. Edelen

General discussion: The morphology of elliptical galaxies, evolution of galaxies.

2.00 p.m. — Observational Evidence of Galactic Scale Discretization, by A. G. Wilson.

General discussion: Measurement of diameters, Law of Redshifts in the local metagalaxy and in clusters.

TUESDAY, June 30:

9:30 a.m. — General discussion: Statistical tests of discretization hypotheses.

2:00 p.m. — Open.

Announcements: Cocktails + Dinner — Time + Place

1) No. for dinner

2) Lunch – number

3) Entrance + meeting place on Tuesday

x7246
x553

PARTICIPANTS: DISCUSSION ON COSMIC DISCRETIZATION PHENOMENA

Outside Invitees:

Name	Notification	Program Invitation and Map	Acceptance
~~Neyman~~		✓	
Scott		✓	
Abell		✓	
Page		✓	
A. & G. de Vaucouleurs		✓	
T. ~~G.~~ Deming T. J. Deeming		✓	
Reaves			
~~Sandage (Gunn)~~		✓	
~~Schmidt~~			
~~Arp~~		✓	
~~Baum~~		✓	
~~Feynman~~		✓	
~~Zwicky~~		✓	
~~T. Y. Thomas & Mrs.~~		✓	

RAND Invitees:

Kirkwood
Genensky
Harris T
Madansky T
Carlstedt
~~Vara~~
Marcus T
Brad Efron T
Davis
Greenfield
Kocher
Edelen
Wilson ~~& Mrs.~~
D. Wilson

The RAND Corporation

1700 MAIN ST. • SANTA MONICA • CALIFORNIA 90406

18 June 1964

INFORMAL CONFERENCE ON COSMIC DISCRETIZATION PHENOMENA, June 29-30, 1964

Enclosed is a tentative agenda for the informal discussions on discretization phenomena. We hope very much you will be able to participate and want to invite you to present any of your own work which you feel is relevant. We are allowing two days so there will be ample time to discuss several matters of current mutual interest. We hope you will be able to join us full time, but if not you are most welcome at any portions of the discussion you find you can attend.

The enclosed map shows the location of The RAND Corporation. The entrance on Monday, June 29th, will be the North Lobby (marked with a red star). The South Lobby will be used on the 30th.

Dom Edelen Al Wilson

CONFERENCE ON DISCRETE PARAMETERS IN COSMOLOGY

June 29-30, 1964

The RAND Corporation

Thornton L. Page DRAFT

Those present:

George Abell, UCLA
T. J. Deeming, Texas
Thornton Page, Wesleyan, Chairman
Gibson Reaves, USC
Elizabeth Scott, Berkeley
A. de Vaucouleurs, Texas
G. de Vaucouleurs, Texas

Carlstedt, RAND
W. Davis
D. Edelen
B. Efron
S. Genensky
S. Greenfield
T. Harris
R. Kirkwood
G. Kocher
A. Marcus
A.G. Wilson
D. Wilson

Introductory remarks were made concerning the dichotomy between discrete and continuous theories starting with the atomists Democritus and Dalton, the quantizers Planck, Einstein and Bohr, and the continuous field theories of Newton, Maxwell and Einstein. It is contended that the continuous distribution of mass assumed for mathematical convenience in relativistic cosmology is inconsistent with the observed galaxies and clusters of galaxies.

Edelen then summarized his theoretical studies of stable aggregates of stars, gas clouds and dust based on the field equations of general relativity. Although these equations have not been solved for the general case of n-body motions, certain restrictions can be deduced based on the assumption of geometric stability; that is, if the aggregate (a galaxy) is to be identified and to retain its identity in time, it must be possible to define its boundary in 3-space by a difference between the energy-momentum tensor inside the galaxy and that outside. This implies a deviation function, , on the surface of a galaxy which measures how the vector field changes across the surface. In Newtonian mechanics and conventional relativistic mechanics of a continuous fluid to which the Schwarzschild solution applies,

= 0. Edelen's theory of granular mass density has the effect of introducing e^2 instead of 1 (unity) in equations for stable geometrical forms such as rotating spheroids, and must satisfy the Helmholtz equation

$$\Delta_2(\ \ ) = - r^2 \xi \qquad (1)$$

where Δ_2 is the Laplacian operator, r is the radius vector to the surface, and ξ is a function of the density change defining the surface and two constants of integration.

The solutions of equation (1) form <u>three classes</u>:

a) = 0

b) = C, a constant different form 0, and

c) variable, in which case,

$$r^2\xi = \rho^2(n,\varepsilon) \qquad (2)$$

$$= n(n+1) \text{ if } \varepsilon = 0 \text{ (\underline{spherical} surface)}$$

where n = 1,2,3, ... (any integer),

ε = eccentricity of a spheroidal surface.

That is, stable spherical galaxies formed of discrete masses (stars) can only occur with discrete radii, $r = n(n+1)/\xi$. For oblate spheroids of eccentricity ε 0, r is the equatorial radius, and $\rho(n,\varepsilon)$ is shown in Fig. 1. Other non-spheroidal forms are also possible; in fact the stable forms of case (c) are not exactly spheroidal; the allowed 3-space surface are given by

$$x^2 + y^2 + z^2/(1-\varepsilon)^2 = e^{2\psi} \qquad (3)$$

All stable forms of density fluctuations are summarized as follows:

Case		ξ	r	Form
(a)	0	(any values)		Continuous density distributions in perfect spheroids (no stars)
(b)	Const.	0	(any)	Continuous density distributions in any shape without definable boundaries
(c-1)	Variable	> 0	$\rho(n,0)/\sqrt{\xi}$	Near-spherical (ϵ=0), discrete sizes
(c-2)	Variable	> 0	$\rho(n,\epsilon)/\sqrt{\xi}$	Near-spheroidal, discrete sizes
(c-3)	Variable	$\neq 0$	(other)	Non-spheroidal forms of various sizes. (spirals and irregulars) including those with turbulent internal motions. Probably no ~~xxx~~ discretization in size.

The curves of Fig. 1 referring to cases (c-1) and (c-2) are used by Edelen to explain the upper limit on ellipticity, $E = 10(1 - \sqrt{1-\epsilon^2})$ 7, by the additional assumption that in the process of formation, rotational energy tends to be the minimum allowed for a given major diameter, 2r. It follows that the larger elliptical galaxies (larger n) should have smaller ϵ and smaller E. There is no prediction of the distribution of values of n.

A more detailed equilibrium theory of the surface layers in case (c-2) shows deviations from underlying spheroidal form depending on the integer n, the eccentricity ϵ, and the absolute value of (which depends on where the "edge" of a galaxy is -- that is, on what isophote is measured.) For n = 2,4,6...(even). The boundary layer is symmetrical about the equatorial plane and intersects the spheroid along n circles. Fig. 2 shows that with n = 4 the theoretical cross section resembles isophotes of NGC 3115, and (with more extreme values of) is similar to SO galaxies. With odd values of n, the predicted surface is asymmetrical about the equatorial plane. In the case n = 3, a saucer shape like that of IC 3973 is predicted. These results offer a means of confirming the existence of discrete sizes by observations of the morphology of elliptical galaxies. They may also provide the basis for an adequate theory of the variety of non-spheroidal shapes observed. Circular velocity (about the axis) of stars in the boundary layer can also be predicted to vary with radial distance from the axis as shown in Fig. 2, and has the appearance of velocity curves measured along major axes of spirals, but it does not represent internal motions.

A. G. Wilson described new measures of the angular diameters of 130 elliptical galaxies (types E0, E1 and E2) in 8 clusters. In order to improve precision, he made contact copies of Palomar 200-inch plates on high-contrast emulsion with carefully controlled exposure times and development. Kocher's microphotometer tracings of images on the original plates and on the copies showed that the rate of change of density with distance from the image cluster was increased by a factor of 10 near the edge. Each image size was measured 5 times with a micrometer along the major axis (and 5 more measures are being made along the minor axis). Deviations from the mean values of d show that the internal error of measurement is proportional to d; the range between largest deviations being 0.04d (the r.m.s. error of $\bar{d}$ being about 0.007d), corresponding to a range in the logarithm, $\Delta \lg d = 0.015$. Values of d ranged from 3.125 to 0.130 mm, corresponding to 34".4 to 1".4 (seconds of arc).

Star images on the same copy plates were measured at 0.020 mm, or 0".2 diameter (due to optical defects and photographic spread), implying an increment in d not removed as yet. Moreover, the diameters measured in different clusters on different plates may refer to slightly different isophotes, and Wilson is getting similar exposures of 4 different clusters on one plate to eliminate such errors. He had hoped that the steep surface-brightness gradient would prevent excessive errors in d from this source; from de Vaucouleurs' formula,

$$\lg I(r) = \text{const.} - 3.33(r/r_e)^{1/4}$$

an error of $0^m.2$, or 0.08 in lg I(r), corresponds to an error in d somewhat less than 0.1 d or Δlgd = 0.03. (Microphotometer tracings of images with magnitudes determined photoelectrically will determine ΔlgI and the ~~xxxxx~~ absolute value of I in magnitudes per square second of arc at the edges of all measured images.)

Wilson derives linear dimensions in 8 different clusters by multiplying measured angular diameters d_i by mean cluster redshifts $\bar{z}_j$, where $\bar{z}_j$ is the mean of all published redshifts for galaxies in the j^{th} cluster. That is, the linear diameter of the i^{th} galaxy in the j^{th} cluster is given by

$$\lg D_{ij} = \lg d_i + \lg \bar{z}_j + \lg(Sc/H) \qquad (5)$$

where S = scale factor = (206265)(11.06) parsecs/mm on Palomar 200-inch plates,

c = velocity of light = 3×10^5 km/sec

H = Hubble const. = 10^{-4} km/sec parsec

The error in $\bar{z}_j$ is less than 0.02 $\bar{z}_j$, or $\Delta\lg \bar{z}_j$ - 0.008, estimated from the dispersion in measured z for each cluster. Hence the relative errors in one cluster (after correction for optical resolving power, $d_i = d - 0.020$) are given by the range of measurement errors, $\Delta\lg D_i$ - 0.015, but the maximum error between different clusters is given by

$$\Delta\lg D_{ij} - \Delta\lg d_i + 0.5\,\Delta\lg I_j + \Delta\lg \bar{z}_j = 0.05.$$

Most of this is the photometric error $\Delta\lg I_j$. When it is eliminated, the estimated error drops to $\Delta\lg D_{ij}$ - 0.02.

Wilson finds seven coincidences between 22 measured D_i in the nearest cluster (Coma) and the discrete values given by Edelen's equation (2); that is,

$$\lg d_i + \lg \bar{z}_1 = K_1 + \frac{1}{2}\lg n(N+1) \pm 0.005 \qquad (6)$$

where K is a fitting constant.

$$K_1 = \lg 2 - 0.5 \lg \xi_1 - \lg(Sc/H) = 3.5121 \qquad (7)$$

and the values of n involved are 1,4,6,8....17. It is remarkable that 26 measured values of $\lg d_i + \lg \bar{z}_j$ in the other 7 clusters also fit equation (6) with the same constant, K_1. He finds 34 measured values fitted by equation (6) with a different constant, $K_2 = 3.6131$, and a third set of 26 fitted by $K_3 = 3.6869$. These are interpreted as three different values of ξ. Altogether, over 75% of the ~~xxxxxxx~~ meausred values D_{ij} are fitted by equation ( ) using 3 constants, K_1, K_2 and K_3, as summarized below.

Cluster	$\lg \bar{z}_j$	Values of n used with K_1	K_2	K_3	No. of D_{ij} unfitted
1. Coma		1,4,6,8..17			
2.					
3.					
4.					
5.					
6.					
7.					
8.					
Mean ellipticity of galaxies fitted in Coma Cluster		1.16 ±	2.21 ±	3.12 ±	

Evidence of a physical difference between the K_1, K_2 and K_3 sequences is shown by the different mean ellipticities in the Coma cluster. This might account for different values of ξ in terms of different stages of evolution, and suggests looking for color or absolute magnitude differences between the three sequences. There was some discussinn as to whether discrete sizes of E0 = E4 galaxies imply discrete masses and luminsoties. Edelen emphasized that the theory leading to equation (2) involves only the surface layer and does not yet lead to anyconclusions about the mass or dynamics of the interior. Moreover, the empirical formulae for surface brightness all involve two parameters ($I(r_e)$ and r_e in de Vaucouleurs' equation (4), I_o and a in Hubbl's $I = I_o/(r+a)^2$ or Baum's $I = I_o/2r(r+a)$), so that a given edge brightness can be associated with a wide variety of interior luminosity distributions.

A second consequence of the brightness formulae concerns the possible emissivity outside the boundary of a galaxy defined by Edelen. In fact, if the formulae are correct be yond the boundary, they correspond to emissivity per unit volumn proportional to r^{-3} in 3 dimensions, and this may be modified by projection so that equation (2) is not correct for 2-dimensional (projected) images.

The procedure for selecting values of K so that equation (6) fits the observations appears somewhat arbitrary, and there was extensive discussion of the statistical significance of the fit and of the values of K so derived. Wilson emphasized the fit in terms of success in predicting D_{ij} in seven clusters Ij = 2....8) from equation (6) using the value K_1 determined in the Coma cluster (j = 1) and allowing the tolerance, $\delta(\pm 0.005$ in equation (6)) to vary from ± 0.005 to ± 0.02. In the full range of observed lg D_{ij} from 3.77 to 4.9294 the 17 intervals of width 2δ add to 34δ. Hence, for a uniform distribution of random values of D_{ij}, the expected ("probable") number of coincidences with equation (6) among N_o random values is

$$N(\text{prob.}) = 34\delta\ N_o/1.15 \qquad (8)$$

(For large δ the intervals overlap at large $\underline{n}$, involving a ~~dmwx~~ downward correction to equation (8) that has been included in Table 2. The number of "increases," N(Obs) = coincidences with observed D_{ij} within $\pm \delta$, is considerably larger than (Nprob.), as shown in Table 2. Since fluctuations about the expected N(prob) might account for this, a random-number Monte Carlo calculation was performed 25 times. Carlstedt described the computer program whereby residuals $x_i = 1/2$ lg $n(\bar{n}+1)$ were computed for $N_o = 76$ random numbers and the number of residuals δ counted. The largest in the 25 trials are listed under N(max) in Table 2.

Table 2. Observed or Random Coincidences with Equation (6)
(N(max) normalized to N_o = 108 in 25 fields)

Tolerance δ	N(obs) for K_1	K_2	K_3	N(prob)	N(max)	N(obs)/N(prob)
±0.0050	26	34	26	16.8		1.55
±0.0075	39			24.0		1.63
±0.010	52			31.0		1.68
±0.015	70			46.0		1.53
±0.020	82			58.0		1.41

Wilson notes that the ratio of N(obs) to N(prob) reaches a maximum at $\delta = \pm 0.01$, which tends to confirm both the error estimates and validity of Eq. (6). However, Scott pointed out that the uniform probability distribution is artificial in the calculation of N(prob) and N(max). In addition, different values of K might give a better fit to Eq. (6), and an even better fit might be obtained with some other formula, such as K' & lg n(n+1), appropriate for projected spheroids of discrete radii. Wilson said trials had shown that the alternative formulae n(n+1), $1/n(n+1)$ and $1/\sqrt{n(n+1)}$ do not fit as well as $\sqrt{n(n+1)}$. It was agreed that <u>the analysis should be extended</u> in the following ways: (1) N(obs) = number of coincidences with Eq. (6) among all 130 observed values of D_{ij} with K varying in small steps over a wide range. A plot of N(obs) vs K should show a general rise toward small K since large n gives a greater density of predicted discrete values of lg D, but there will be maxima in N(obs) at several values of K. If there are maxima more distinct than those at K_1, K_2, and K_3, they should be considered also. (2) Similar N(obs) vs K plots should be made for alternative discretization formulas, each with a wide range in K and in δ. (3) Calculations of N(prob) and N(max) should be made for non-uniform distributions of $\overline{D_i}$

(a) Gaussian distribution about $\overline{\lg D_{ij}}$ = mean of all 130 observations with the same variance.

(b) A skew uni-model distribution matching the observations in 3 moments.

(c) A multi-model distribution matching the distribution of observed $\lg D_{ij}$ smoothed over intervals about 1/11 of the full range in lg D.

Efron proposed a direct test of discreteness in the distribution n(D) based on the differences, $y_i = D_{i+1} - D_i$ in the ordered sequence from smallest (D_1) to largest (D_N). These differences will be exponentially distributed with mean and variance = $1/Nf_i$ where f_i is the underlying distribution (constant, if n(D) is uniform). If $N > 100$, the correlation between y_i and y_j is small, and the chi-square test for uniform distribution is

$$\sum_i (\lg y_i - \lg N - \overline{\lg y_i})^2 = \chi^2_{n-1} \quad (1.62) \qquad (9)$$

Tests for other smooth distributions can be similarly formulated. Scott said that the "Neyman smooth test" first published about 1938 is based on similar considerations and should be applied. She proposed that the smooth distribution n(D) derived for E0 - E3 galaxies in the H-M-S catalog by Neyman and Scott be tested against Wilson's observed D_{ij}.

Harris proposed a statistical test allowing for a "background contamination" on which ~~clumping~~ "clumping" of observed values of lg D is superposed, this clumping involving a spread $\pm\,\delta$ in χ = lg D around predicted values $a_n = \frac{1}{2} \lg n(n+1)$. He divided the intervals $a_i - a_{i-1}$ in half by the points b_i and considered two hypotheses:

(1) A smooth distribution $f(x)$

(2) A distribution $\mu_i\, f(\chi)$ in the interval b_i to b_{i+1} and $\lambda_i f(x)$ in the small interval $a_i - \delta$ to $a_i + \delta$, where $\mu_i < 1$ and $\lambda_i \gg 1$.

In order to conserve probability density so that hypotheses (1) and (2) an comparable

$$\int_{b_i}^{b_{i+1}} f(x)\,dx = \mu_i \int_{b_i}^{b_{i+1}} f(x)\,dx + \lambda_i \int_{a_i-\delta}^{a_i+\delta} f(x)\,dx \qquad (10)$$

The <u>amount</u> of clumpiness or discretization is defined as

$$\emptyset_i = \lambda_i \int_{a_i-\delta}^{a_i+\delta} f(x)\,dx/\mu_i \int_{b_i}^{b_{i+1}} f(x)\,dx \qquad (11)$$

and, if $\beta_i \equiv (b_{i+1} - b_i)/2\delta$ (12)

$$\lambda_i = \beta_i\, \emptyset \qquad (13)$$

$$\mu_i = \beta_i(1 - \emptyset_i)/(\beta_i - 1) \qquad (14)$$

where μ_i is considered to be 0 in the small interval $a_i - \delta$ to $a_i + \delta$.

A more specific hypothesis, somewhat simpler to test, is (2) with $\emptyset_i = \emptyset$ = constant, and $\beta_i = \beta$ = constant, in which case $\delta = (b_{i+1} - b_i)/2\beta$

$$\simeq (a_{i+1} - a_i)/2\beta = (1/4\beta)[\lg(i+2) - \lg i]$$

and δ is not exactly constant.

The Neymann-Pearson liklihood test can be used to compare hypothesis (1) with hypothesis (2s) on the basis of N observations, $x_1 = \lg D_1$, x_2, ... x_N, by evaluating the ratio

$$(2s,\ 1) = f_2(\chi_i)/ \quad f(\chi_i) = \frac{f_2(\chi_i)}{f(\chi_i)} \qquad (15)$$

where $f_2(\chi) = \mu f(\chi)$ and (clumping $= \lambda f(\chi)$ around each a_i) and $\gg 1$ implies hypothesis (2s); $\ll 1$ implies hypothesis (1).

//

Each coincidence of an observed $\chi_i = \lg D_i$ with $a_j \pm 1$ (a "sucess" or "fit" in Table 2) introduces factors in the numerator and demonimator of equation (15)

$$f_2(\chi_i)/f(\chi_i) = \lambda = \beta\phi \tag{16}$$

There are N(obs) such successes, and N_o - N(obs) "failures" where

$$f_2(\chi_i)/f(\chi_i) = \beta(1 - \phi)/(\beta - 1) \tag{17}$$

Hence, $\lg \quad = N(\mathrm{obs}) \lg \beta\phi + [N_o - N(\mathrm{obs})] \lg [\beta(1-\phi)/(\beta-1)]$ (18)

where β and ϕ must be consistent with equations (10), (11) and (12). Harris is evaluating these integrals so that Eq. (18) becomes a sum of terms involving the "successes" and ~~f~~ "failures" in each interval b_i to b_{i+1}.

G. de Vaucouleurs described the Reference Catalog of Bright Galaxies soon to be published by the University of Texas Press. It includes, among other things:

lg D for 2300 galaxies

lg R = lg D - lg (minor axis)

lg D(0) = lg D - 0.4 lg R, the square-on diameter mean surface brightness and colors corrected for redshift, obscuration, inclination redshifts of the nucleus for 959 galaxies

morphological types, radio fluxes, etc.

All published data on diameters have been included, and converted to the same scale -- that is, referring to the same isophote (the absolute surface brightness of which is not yet determined.) Correction is made for the "Holmberg effect" by which minor diameters are systematically measured too small relative to ~~jax~~ major diameters. Intercomparisons allowed ~~fairlx~~' fairly reliable estimates of measurement errors and weights for the tabulated

Table 3. Discrete Redshifts of Clusters

Cluster Name	No. of z measures	$\bar{z}$	σ	lg $\bar{z}$	lg $\bar{z}$ - lg CN(N+1)	Value of n with C_1	C_2

diameters and other parameters. One statistical result linked with the Holmberg effect is that diameters measured on small-scale plates are always too small. This is best illustrated by a plot of d_s vs d_L, where d_s is the image diameter measured on a small-scale plate and d_L is the value (for the same galaxy) measured on a large scale plate. The two values are proportional for $d_s > 1$ wm., but d_s is far too small for d_L in smaller galaxies. Wilson is confident that measured image diameters are proportional to angular diameter well below d = 1 mm. in his measures but agreed that check plates should be taken into various scales to confirm this. Hubble noted a different effect in photographic photometry of spectra is carried to intervals on the plate much smaller than 1 mm.

Wilson described two other sets of discrete quantities that are not yet predicted or explained theoretically: the observed values of K in Eq. (6) fit the formula $K = 1/3 \lg \upsilon(\upsilon + 1)$, with $\upsilon = 2$ for K_1, $\upsilon = 3$ for K_2, $\upsilon = 4$ for K_3, and so on. In addition to this pattern, the values of K needed to fit measured diameters in 8 different clusters suggest discrete values of cluster redshifts, $\bar{z}_j$, or of $u = \bar{z}/(1 + \bar{z})$ such that $u = C\,N(N+1)$. As shown in Table 3, all ~~kmmaxm~~ known cluster redshifts (30 in number) fit this expression, with N = 3... 19, and two values of C.

Wilson ~~f~~ interprets this as an indication of regularity in cluster spacing. One implication of this is that the regions occupied by clusters tend to be of the same volume, or a discrete set of volumes, which implies further that the angular diameters of nearby clusters and the angles between their centers have discrete sizes. The closest packing of equal sized spheres involves a lining up, one behind the other, that can also be checked observationally. Questions arise as to whether every possible cluster location in such an array is actually occupied.

G. de Vaucouleurs summarized briefly the evidence for a supergalaxy, or organized grouping of nearby clusters of galaxies, indicated in part by deviations from the Hubble law, $cz/(1+z) = H\ d$ (where $d \equiv$ distance) such that $\underline{H}$ is not a constant, but a function of direction and distance. Distances based on absolute magnitudes estimated from morphological types indicate that our galaxy is located in a flattened supergalaxy with poles nearly in the plane of the Milky Way of galectic coordinates $l^{I} = 15^{o}$ $b^{I} = +5^{o}$. The plane of the supergalaxy, roughly perpendicular to the plane of the Milky Way, defines a supergalactic latitude, $\underline{B}$, and deVaucouleurs has analyzed redshifts in the zone $|B| < 30^{o}$ in a manner similar to Oort's analysis of galactic rotation in terms of stellar radial velocities. This indicates a 500 km/sec circular velocity of our local group in the plane of the supergalaxy, and a distortion of the Hubble Law in the direction of the supergalaxy center (near the Virgo cluster) at distances near 10^{7} passes, which is interpreted as the distance to the center.

Scott discussed subclustering in terms of counts of galaxies in rings around ~~?~~ selected points on Palomar Atlas prints in 7 different clusters. After proper allowance is made for the varying density of galaxies, she and Abell conclude that there is minor subclustering within two of the clusters; probably not sufficient to account for the large velocity dispersions that seem inconsistent with stability.

SUMMARY

There is a theoretical basis for expecting certain types of galaxies to occur only in discrete linear sizes. Observations tend to confirm this in individual clusters, although the effects of projection have not yet been taken into account. Possible errors in photometry still reduce confidence in comparison between diameters in different clusters, but this is countered by several unexpected regularities, including the several sequences in discrete sizes, and the discrete values of cluster redshifts.

It can be argued that these reaularities are a result of chance among a limited number of observations. Although the statistical analysis makes this appear unlikely, further observations of cluster redshifts and diameters of specific isophotes in elliptical galaxies are needed in order to settle the matter.

Implications are far reaching: on the evaluation of galaxies and their morphology, the variety of stable forms may be limited; the luminosity function may have a sharp cut-off at the low end with the smallest size set by $n = 1$; clusters of galaxies may be found to occupy volumes in a specific pattern; and the mathematics of cosmology may require major revision to allow for the discrete nature of mass distribution ∅ in the universe.

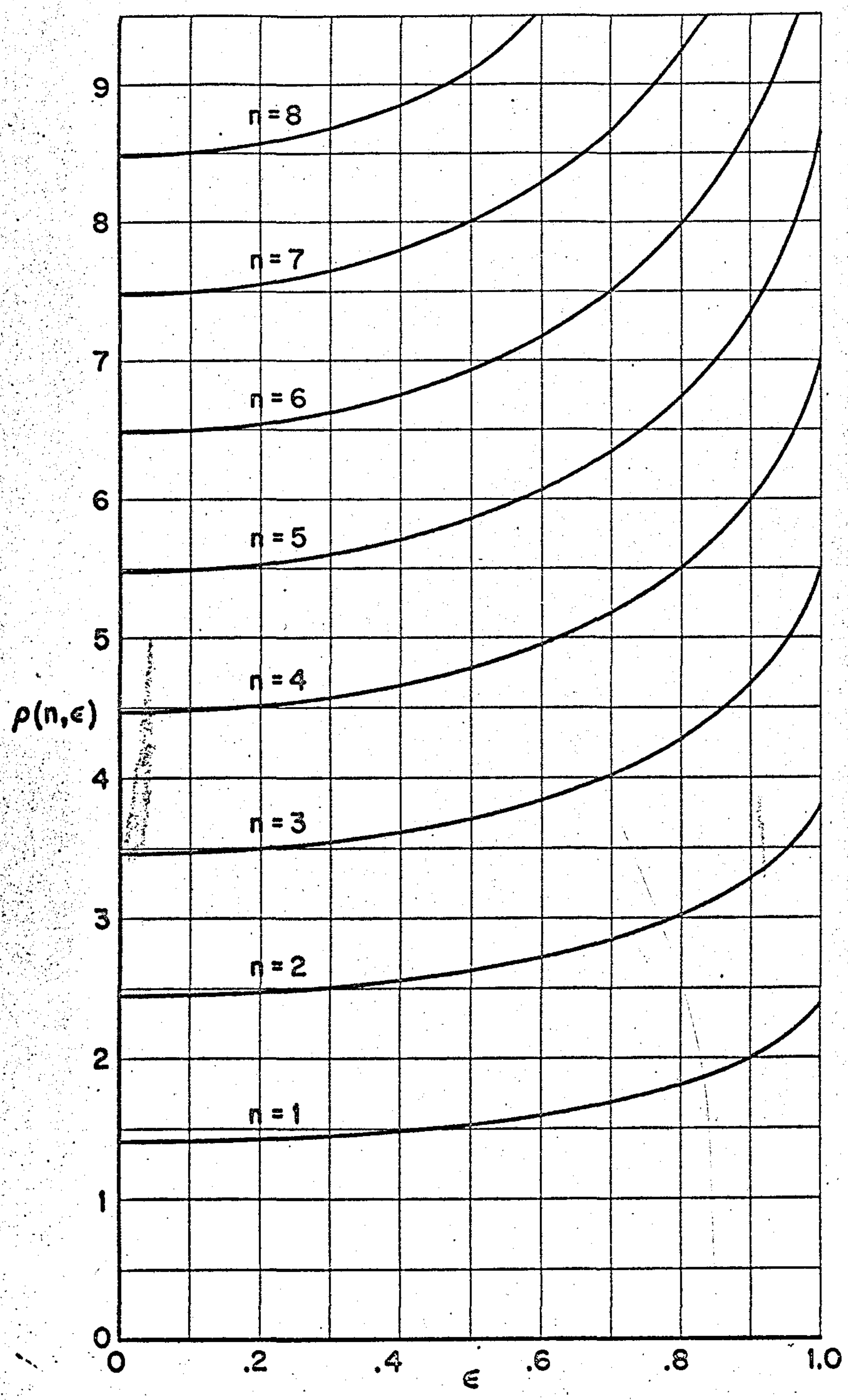
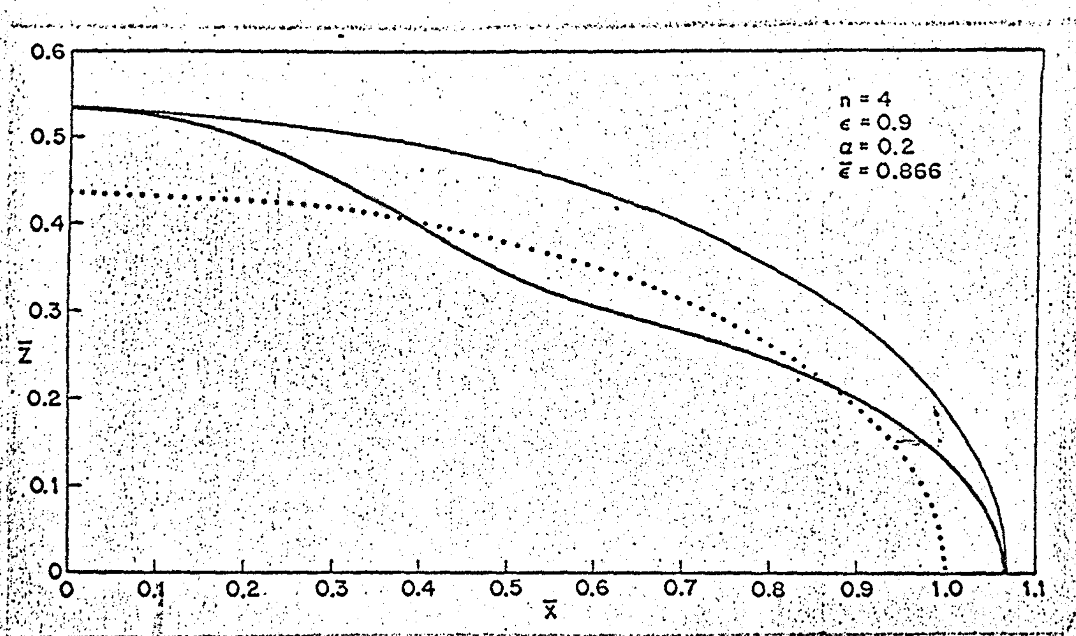
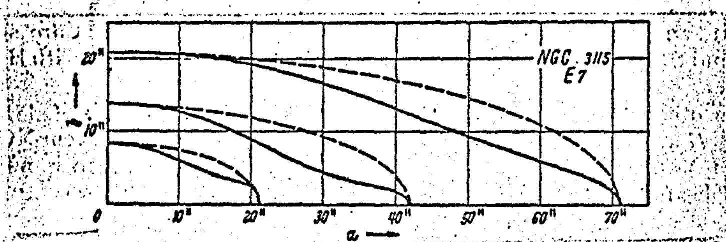


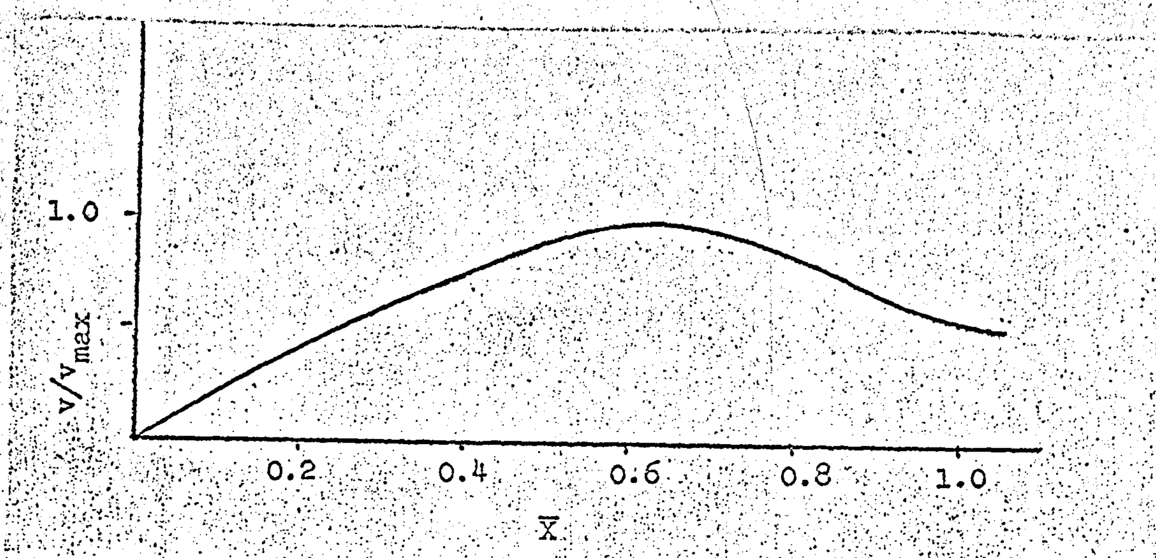
Fig. 1 — The discretization function $\rho(n, \epsilon)$



A. Typical Equilibrium Surface for n=4, ε=0.9



B. Some isophotes of NGC 3115, after Oort.



C. Typical Circular Velocity in the Boundary Layer.

Fig. 2

Table 1. Fitting of Observations to Eq. (6)

		Values of n used with			No. of D_{ij}	
Cluster	$\lg \bar{v}$	K_1	K_2	K_3	Total	not fitted
1. Coma	3.8084	5,5,8,8,8,8	3,7,7,12	1,4,6,7,8,10, 16	22	6
2. 2322+1425	4.1201	4,6,8,8,12	10,10	7, 9,13	20	10
3. UMa I	4.1838	6,7,11,15	13,16	6,6,7,7,10, 11,13	13	2
4. Cor Bor	4.3355	8,8,8,8,10,10 10,12,14,14,16	6,7,8,8,8,9 10,11,11,14,15	3,5,6,8,10 13,13	29	8
5. Shane Cloud	4.4523	9,11,12,13,15	7,8,8,8,9,9 12,15	8,9,9,10,11 14	19	5
6. Boötes	4.5951	9,10,14,17	8,10	6,7,9,15,17	11	2
7. UMa II	4.6113	9,13,16	9,9,12,12	6,9,14	11	2
8. Hydra	4.7843	10,12,15	12	7,9,10	5	0
Mean ellipticity for galaxies fitted in the Coma Cluster		1.16 $\pm$ 1.	2.21 $\pm$ 1.	3.12 $\pm$ 1.		

Table 2. Observed *vs*. Random Coincidences with Eq. (6)

Tolerance, δ	$N(obs)/N_o$ for K_1	$N(obs)/N_o$ for K_2	$N(obs)/N_o$ for K_3	$\frac{N(Prob)}{N_o}$	$N(obs)/N(Prob)$ for K_1	$N(obs)/N(Prob)$ for K_3	$N(Max)/N_o$ for K_1	$N(Max)/N_o$ for K_3	Standard Score for K_1
$\pm$ 0.0050	0.3145	0.2405	0.2405	0.15	2.10	1.60	0.21	0.24	5.25
$\pm$ 0.0075	0.4348	0.3885	0.3608	0.22	1.98	1.64	0.28	0.30	6.68
$\pm$ 0.010	0.5273	0.5088	0.4810	0.30	1.76	1.60	0.36	0.37	5.43
$\pm$ 0.015	0.6475	0.6475	0.6475	0.42	1.54	1.54	0.50	0.55	4.42
$\pm$ 0.020	0.7308	0.7955	0.7585	0.52	1.41	1.46	0.59	0.67	4.85

Note: The fit for K_1 is best, for K_3 is poorest. The ratio N(Obs)/N(Prob) is a measure of the importance. The standard score is $[N(Obs) - N(Prob)]/\sigma_N$. Both N(Max) and σ_N were based on $N_o = 76$ in 25 trials.

Table 3

MEAN CLUSTER REDSHIFTS OBSERVED AND CALCULATED

Cluster	Name	Number of Redshifts	$\overline{V}$	$\log \overline{u}_o$	M	N	P	$P - \log \overline{u}_o$	References
	Virgo	73	1136	2.4234	3	2	2.4225	-0.0009	
0316+4121	Perseus	7	5435	1.7494	7	2	1.7534	+0.0040	
0123-0137	"NGC 541"	43	5439	1.7494	7	2	1.7534	+0.0040	(8)
1257+2812	COMA	50	6432	1.6780	4	4	1.6779	-0.0001	(9)
1627+3937	Abell 2199	19	9028	1.5344	9	2	1.5474	0.0130	(10)
1603+1755	HERCULES	15	10775	1.4597	10	2	1.4603	0.0006	(11)
2308+0720	PEGASUS II	3	12821	1.3874	11	2	1.3811	-0.0063	
2322+1425		2	13187	1.3757	11	2	1.3811	0.0054	
1145+5559	U.M. I	4	15269	1.3149	12	2	1.3086	-0.0063	
0106-1536	Haufen A	2	15872	1.3012	12	2	1.3086	0.0074	
1024+1039	LEO	1	19489	1.2147	7	4	1.2306	0.0159	
1239+1853		2	21533	1.1741	14	2	1.1795	0.0054	
1520+2754	CORBOR	8	21651	1.1719	14	2	1.1795	0.0076	
0705+3506	GEMINI	2	23366	1.1360	15	2	1.1215	-0.0145	
0348+0613		1	25644	1.1038	15	2	1.1215	0.0177	
1513+0433	Shane Cloud	1	28333	1.0640	16	2	1.0671	0.0031	
1431+3146	Boötes	2	39367	0.9356	10	4	0.9375	0.0019	
1055+5702	U.M. II	2	40860	0.9213	19	2	0.9219	0.0006	
1153+2341	Abell 1413	2	42784	0.9037	7	6	0.9083	0.0046	
1534+3749		3	45951	0.8767	20	2	0.8785	0.0018	
0025+2223		2	47836	0.8616	11	4	0.8583	-0.0033	
1228+1050		2	49514	0.8487	11	4	0.8583	+0.0096	
0138+1840		1	51908	0.8312	21	2	0.8371	0.0059	
1309-0105		1	52362	0.8280	21	2	0.8371	+0.0091	
1304+3110	Coma B	1	54917	0.8104	21	2	0.8371	-0.0129	
0925+2044		1	57498	0.7937	22	2	0.7975	0.0038	
1253+4422		1	59382	0.7819	12	4	0.7858	0.0039	
0855+0321	HYDRA	3	60860	0.7730	12	4	0.7858	+0.0128	

$$P = 4.2799 - \log M(M+1) - \log N(N+1)$$

$$\log \overline{u}_o = \log [(C+\overline{V})/\overline{V}]$$