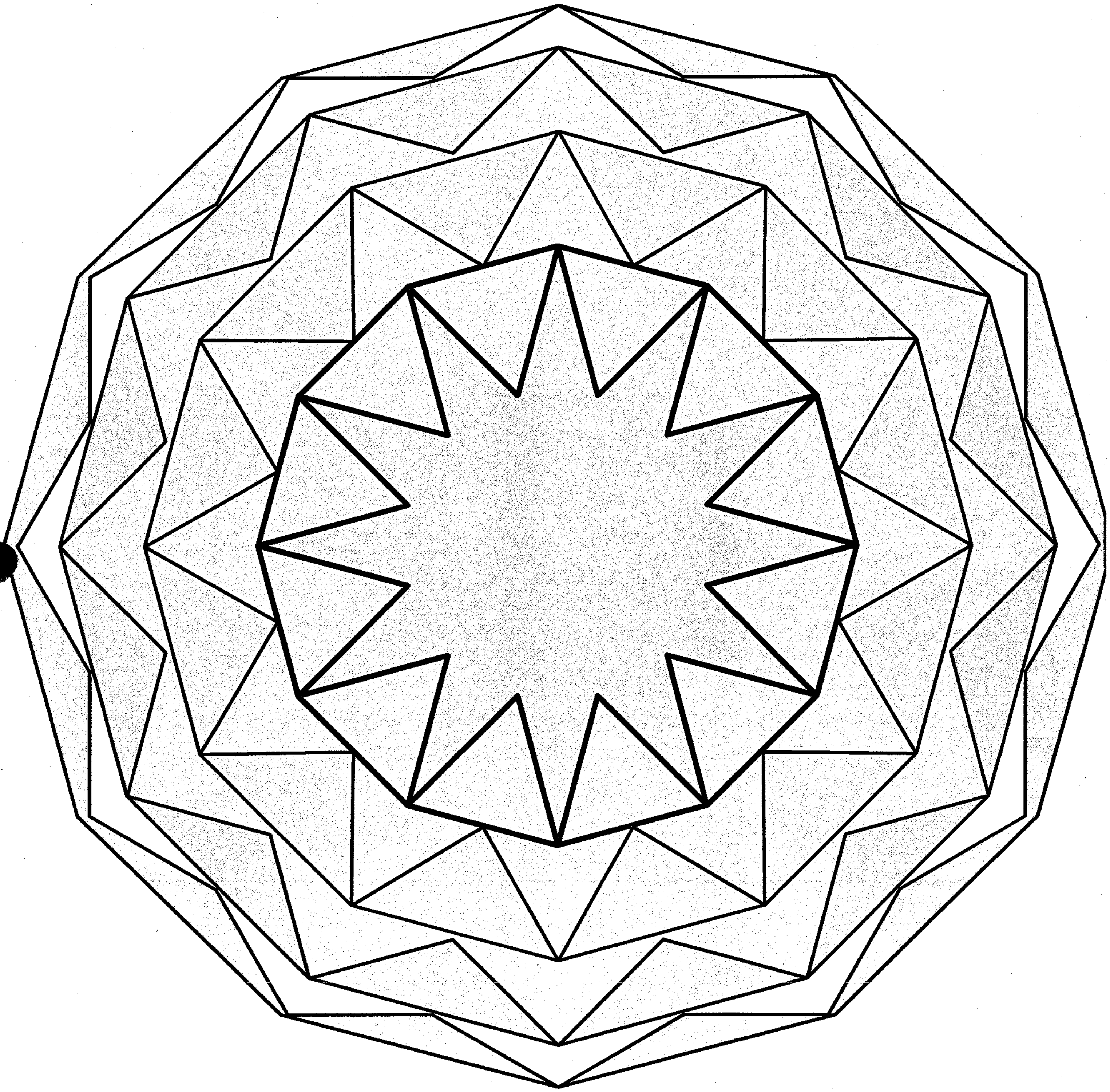
POLYSTARS

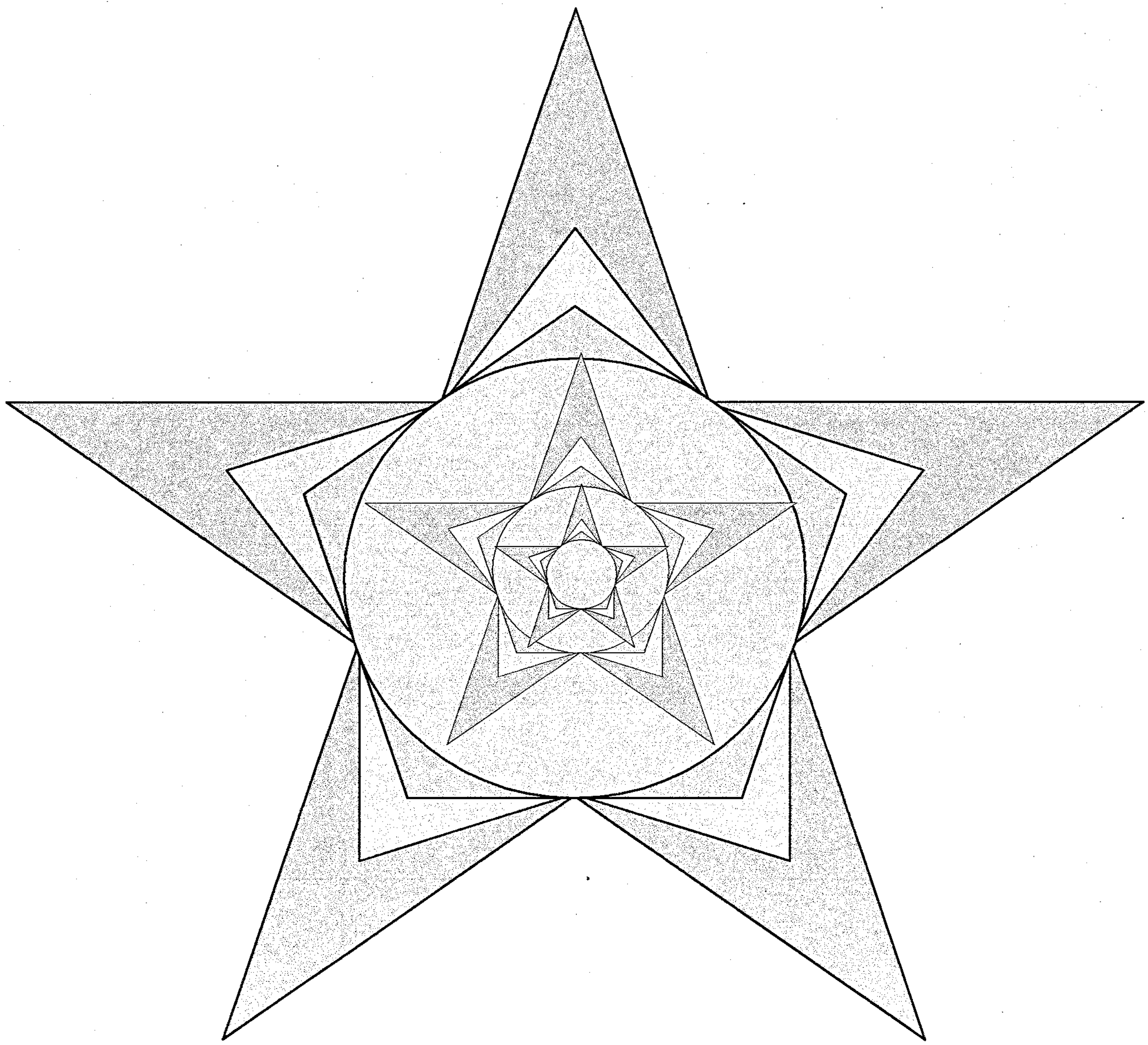
Software

Probably Corel Draw v3 or v5

Use v9

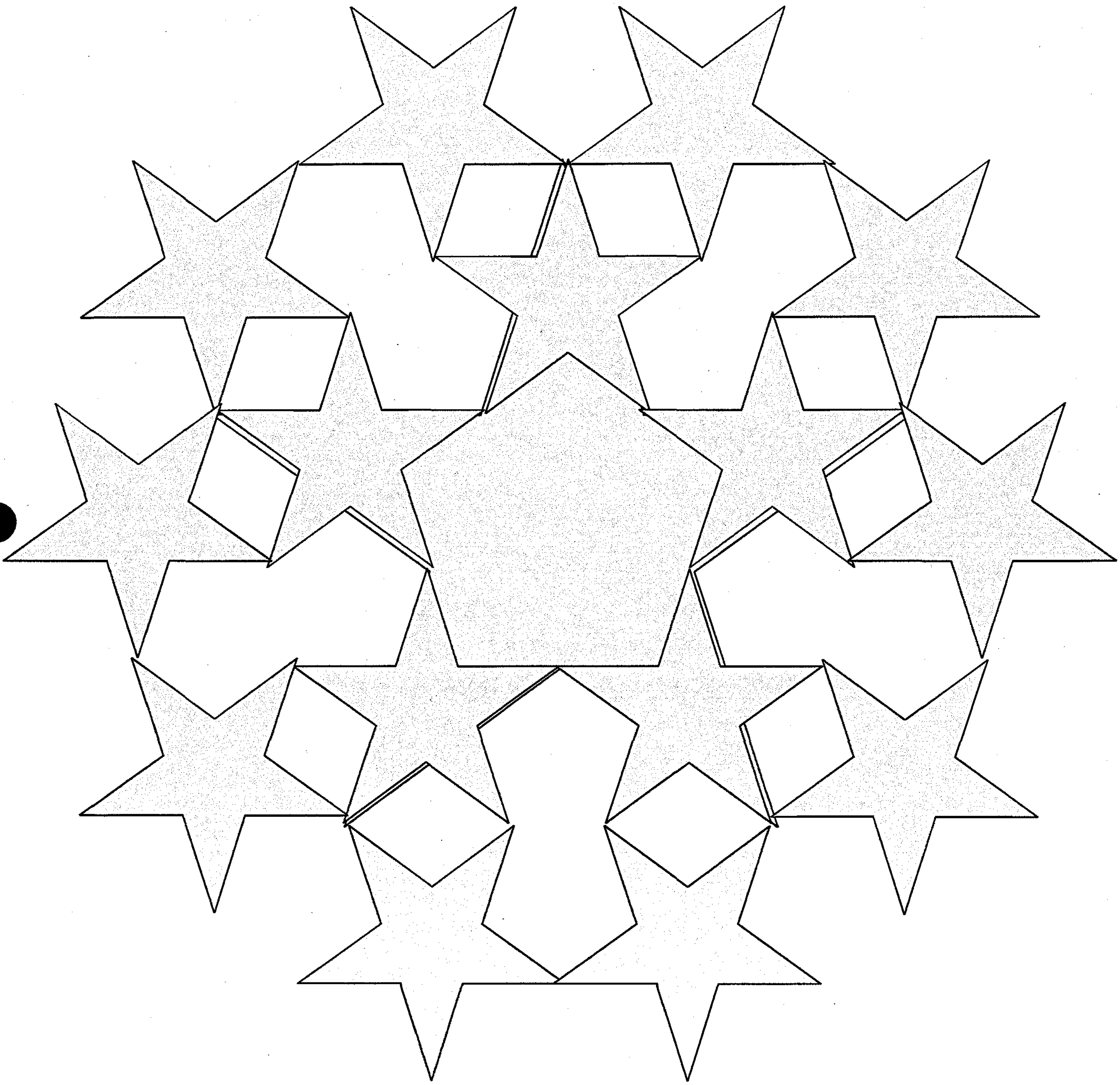


...IEWE 02/01/18



SKETCH

~ Fivec

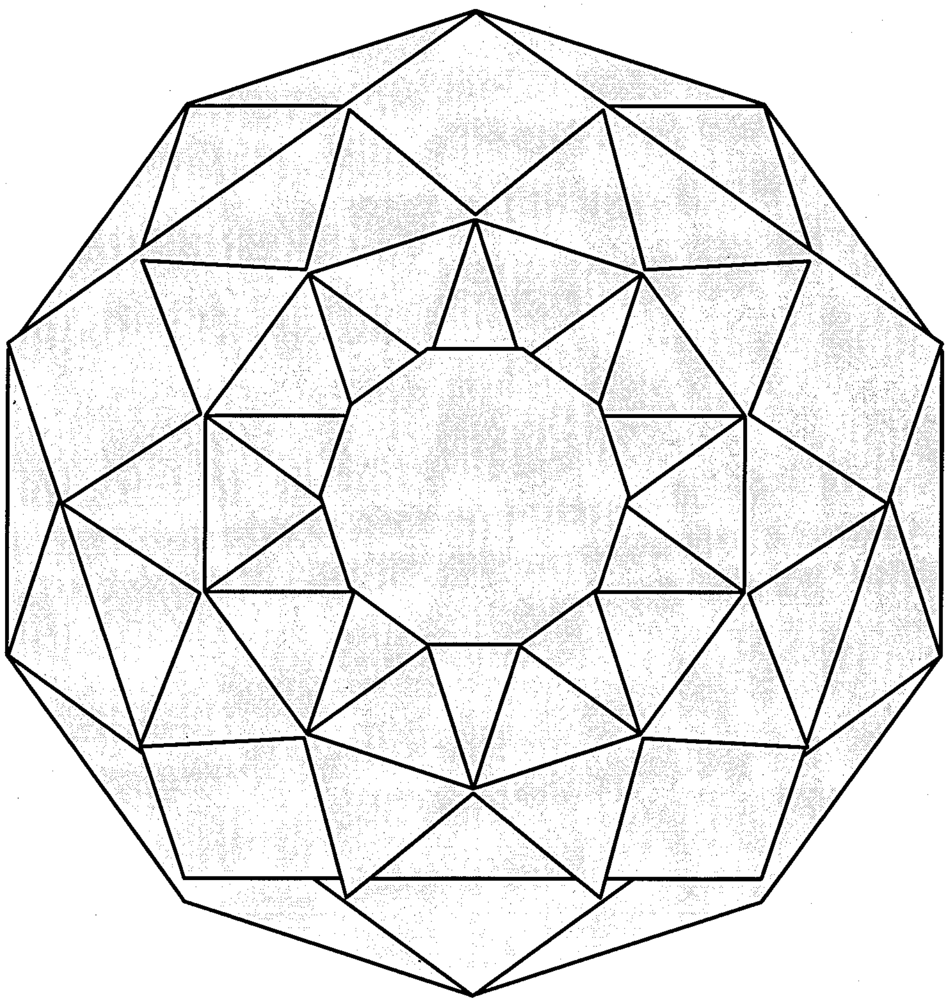


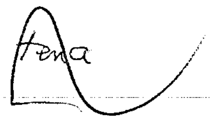
INSIGNIA FOR A 15 STAR GENERAL

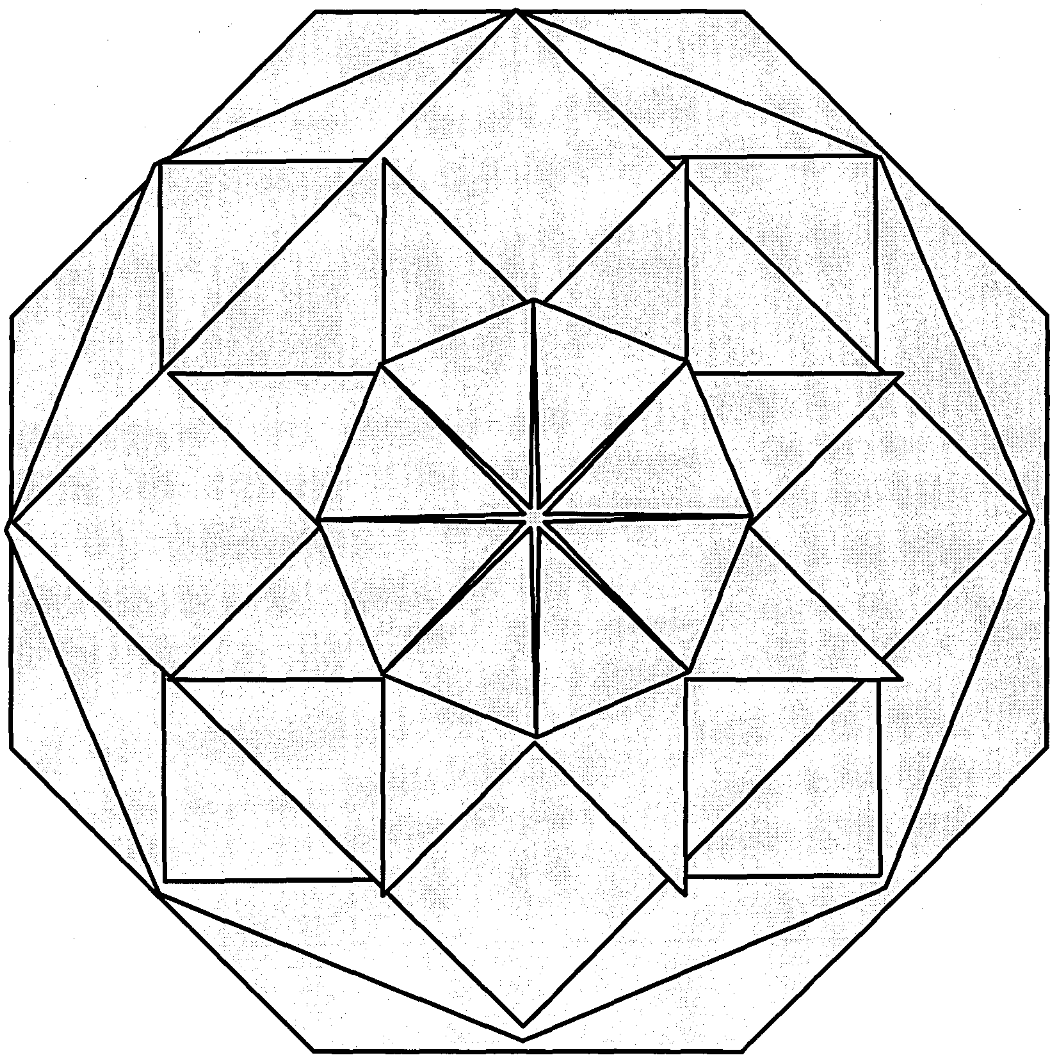
FIVE

) HP-PCL XL;1;1;Comment Copyright Hewlett-Packard Company 1989-1998

0F╡ó4@se:)↓↔-Bvi;*→▲.é⌡ßP§☻♦#å µR▬♥♦$Æ ╟Z☺♠ ù ╤┐↕☻!G┬ñ5&
0I╜⌐6'
1)↑∟,>ob8(↑∟+=na8¶☻♥#˜≡█M¶☻♥"|ϵ┘L►☺♣▼Ä ╟W►☺♣▼î —V&







eighta

Polystars as metaphors for mutuality: containing and being contained

There is a fulcrum – P_0 – that separates outsid from inside

By rotation all can radiate, or balance along the same axes,
or alternate inward and outward

[There is no way for all to be inward if ∃ an outside]

There is a reciprocity between a shape parameter and a size parameter

%'s are shape parameters [n is also a shape parameter
but is related to $2n$]

Nomenclature:

The fulcrum, the boundary between inside and outside, the "skin".

Designated by [illegible] $P_0 = q_0$ The basic polygon $= 100 \cdot \cos\left(\frac{180}{n}\right) = q_0$

X_i is for exterior, W_i is for interior

q_i = i sides skipped ; ~~p_i = i vertices skipped~~
exterior $\dot{q}_i$ rotated exterior for the same i, $p_i = q_i$ % numerically the same

The symmetries or folds:

- CF = center fold ⟷ 0% in ~~CF(n) = 50% for all~~ n
- VF = opposite vertex forld [n odd] $VF(n) = \frac{100 \cdot [illegible]}{1 + 2\cos\left(\frac{180}{n}\right)}$
- SF = opposite side fold [n even] = ~~33.33% for all even~~ n

Other stars:

X_1, the first outside stars is "100%", = a polygon with $2n$ side

W_1, the first inside star $= 100 \cdot \left(2\cos\left(\frac{180}{n}\right) - 1\right)$

$X_1 \longleftrightarrow W_1$

X_2, the second outside star is P rotated by $\frac{180}{n}$

W_2, the second inside star is $p_1 = \frac{100 \cdot \cos\left(2 \cdot \frac{180}{n}\right)}{\cos\left(\frac{\pi}{n}\right)}$ $(\doteq q_1)$

$X_2 \longleftrightarrow W_2$, $(p_1 \longleftrightarrow P_{rot})$

Additional balances:

$q_1 \longleftrightarrow P_2$

$q_2 \longleftrightarrow P_3$

....

ext int

Polystars & Flowers

Polystars map the morphology of many species of flowers

STAR POINTS

EVERY POLYGON HAS AVAILABLE 2 FO's, FOLDOUT SYMMETRICAL POINTS

ONE FROM THE CENTER CFØ $\mathring{R}$

FOR n - even ONE FROM THE OPPOSITE SIDE SFØ

FOR n - odd ONE FROM THE OPPOSITE VERTEX VFØ

The "SKIP" POINTS

q's the extended sides

$q = 0$, the polygon itself

$q = 1$, one side skipped, etc. .. n

p's the interior skipped vertices

$p = 0$, the polygon itself

$p = 1$, 1 vertex skipped, $p = 2$, 2 skipped ... n

The number of q stars possible is $\frac{n-4}{2}$ for n - even   first is $n = 6$

$\frac{n-3}{2}$ for n - odd   first is $n = 5$

The number of p stars possible is $\frac{n-4}{2}$ for n - even

$\frac{n-3}{2}$ for n - odd

Questions: What resonances occur for a given n?

What resonances occur between values of n?

---

e.g. $3q_1 + 5q_0 \doteq 4q_2 - 1q_3$ for $n = 9$ $9.0011 = 9.0078$

For $n = 9$, $R_3 \cdot R_2 = R_3 + R_2 = 4.4115$

$(2.8794) \times (1.5321) = (2.8794) + (1.5321)$

---

r = Radius of inscribed circle, $\bar{R}$ = radius of circumscribed circle

STAR POINT FORMULAE :- RADII BASED on $\bar{R}=1$, radius of circumscribed circle

$\mathring{R}$ = CFO = $2r$

$\gamma = \frac{180}{n}$

$\check{R}$ = VFO for n-odd = $\left(2+\frac{1}{\cos\gamma}\right)r = \bar{R}(1+2\cos\gamma)$ $\bar{R} = \frac{r}{\cos\gamma}$

E: $\mathfrak{R}$ = SFO for n-even = $3r = 3\bar{R}\cos\gamma$

$R_q = \frac{\cos(q\gamma)}{\cos((1+q)\gamma)}\, r$ q = 0, polygon itself (circumscribed circle)

FO~p1

$R_{p_1} = \frac{1}{\cos^2\gamma}$ (r=1)

$\bar{R}_q = \frac{\cos(q\gamma)\cos\gamma}{\cos[(q+1)\gamma]}\,\bar{R}$

FO-p2 ≡ $R_{q=1}$

$R_{p_1} = \frac{1}{\cos\gamma}$ for $\bar{R}=1$

ITERATIONS OF POLYGONS AND STARS

While geometry basically involves continuous parameters such as length, angle, area, etc., some important geometric properties are functions of discrete parameters. In particular, many of the important properties of polygons and polyhedra and the stars that may be constructed on them are functions of discrete variables, such the number of sides, edges, vertices, etc. This essay inspects some functions of discrete parameters associated with polygons and their two dimensional stars. In the following only regular polygons and stars with number of sides > 4 are considered.

By extending the sides of a polygon to points of intersection, polygonal stars may be constructed, and by connecting the points of intersection larger polygons of the same number of sides as the original may be formed. These two steps can be iterated to generate a set or family polygon-stars. Alternately, by connecting the vertices of a polygon, inner polygonal stars may be constructed whose sides create smaller polygons similar to the original polygon. These steps may also be iterated to create a family of polygon-stars. A polygon-star family will be determined by **n**, the number of sides of the polygon, and by **q**, the number of vertices or sides skipped in the star constructions.

Since the extended sides of a triangle or square do not intersect, no iterated families of polygon-stars may be constructed on them. The first polygon permitting a polygon-star family is the pentagon. Both pentagons and hexagons support a single family of iterated polygon-stars. Heptagons and octagons support two families, nonagons and decagons three families. In general the number, N, of distinct polygon-star families that may be constructed expressed in terms of the number, n, of sides of the original polygon is given by:

$$N = (n-4)/2 \text{ for n even and } N = (n-3)/2 \text{ for n odd}$$

It can be shown that if **r** is the radius of inscribed circle of a polygon or star and **R** is the radius of the circumscribed circle of the polygon or star, then

$$\frac{\mathbf{r}}{\mathbf{R}} = \frac{\cos(\mathbf{q}+1)\varphi)}{\cos(\mathbf{q}\varphi)}$$

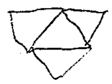
where $\varphi = 180°/n$ and **q** is the family order number, q = 0 for polygons, q = 1 for stars constructed from one side or vertex skipped, q = 2 for two sides or vertices skipped, etc. Polygon-stars are thus a two parameter family, functions of **n** and **q**.

An interesting question arises. For any given value of n, when, if ever, will a polygon that is a member of one family coincide with a polygon that is a member of a different family? Stated mathematically, for two different families with numbers $\mathbf{q_1}$ and $\mathbf{q_2}$, and with **u** and **v** both integers, when will

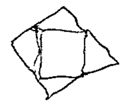
$$\left\{\frac{\cos((q_1+1)\varphi)}{\cos(q_1\varphi)}\right\}^u = \left\{\frac{\cos((q_2+1)\varphi)}{\cos(q_2\varphi)}\right\}^v$$

Or putting this metaphorically, considering the initial polygon as a fundamental frequency, do any of either the 'harmonics' or 'sub- harmonics' of one sequence coincide with those of another sequence, that is, when do resonances occur?

For The triangle and square, the stars and polygons are the same



polygon to
star
rotate 60°



Polygon to
star
rotate 90°

$n := 3 .. 16$ $q := 0 .. 6$

$$S_{n,q} := \frac{100 \cdot \cos\left[(q+1)\cdot\frac{\pi}{n}\right]}{\cos\left(q\cdot\frac{\pi}{n}\right)}$$

The values in these tables are 100 times the ratio of the radius of the inscribed circle to the radius of the circumscribed circle of the polystar.
n = number of sides or vertices of the basic polygon.
q = the number of sides or vertices skipped in the construction of the polystar, with q = 0 for the basic polygon.

n

q =

S =

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	50	~~100~~	~~200~~	~~50~~	~~-100~~	~~200~~
4	70.7107	~~$8.6593\cdot10^{-15}$~~	~~$1.1548\cdot10^{18}$~~	~~141.4214~~	~~70.7107~~	~~$2.5978\cdot10^{14}$~~
5	80.9017	38.1966	~~100~~	~~261.8034~~	~~123.6068~~	~~80.9017~~
6	86.6025	57.735	~~$1.2246\cdot10^{-14}$~~	~~$-8.1659\cdot10^{17}$~~	~~173.2051~~	~~115.4701~~
7	90.0969	69.2021	35.6896	~~100~~	~~280.1938~~	~~144.5042~~
8	92.388	76.5367	54.1196	~~$1.6\cdot10^{-14}$~~	~~$-6.2499\cdot10^{17}$~~	~~184.7759~~
9	93.9693	81.5207	65.2704	34.7296	~~-100~~	~~287.9385~~
10	95.1057	85.0651	72.6543	52.5731	~~$1.9815\cdot10^{-14}$~~	~~$-5.0468\cdot10^{17}$~~
11	95.9493	87.6769	77.8434	63.4356	34.2585	~~100~~
12	96.5926	89.6575	81.6497	70.7107	51.7638	~~$2.3658\cdot10^{-14}$~~
13	97.0942	91.1956	84.5339	75.8927	62.4233	33.9918
14	97.4928	92.4139	86.7767	79.7473	69.5895	51.2858
15	97.8148	93.3955	88.5579	82.7091	74.7238	61.8034
16	98.0785	94.1979	89.9976	85.043	78.5695	68.8812

$n := 3..16$

CF is the polystar whose vertices come to the center of the q = 0 polygon when folded inward
SF is the polystar whose vertices come to the opposite side of the q=0 polygon when folded in n-even
VF is the polystar whose vertices come to the opposite vertex of the q=0 polygon when folded in n-odd
EX is the polystar exterior to q=0 that is identical to the polygon q=0 for 2n
IN is the polystar that with respect to q=0 "balances" EX

$$CF(n) := \frac{100}{2\cdot\cos\left(\frac{\pi}{n}\right)} \qquad SF(n) := \frac{100}{3\cdot\cos\left(\frac{\pi}{n}\right)} \qquad VF(n) := \frac{100}{1+2\cdot\cos\left(\frac{\pi}{n}\right)} \qquad EX(n) := 100\cdot\cos\left(\frac{\pi}{2\cdot n}\right) \qquad IN(n) := 100\cdot\left(2\cdot\cos\left(\frac{\pi}{n}\right)-1\right)$$

CF(n) =	SF(n) =	VF(n) =	n =	EX(n) =	IN(n) =
100	~~66.6667~~	50	3	86.6025	$2.2204\cdot 10^{-14}$
70.7107	47.1405	~~41.4214~~	4	92.388	41.4214
61.8034	~~41.2023~~	38.1966	5	95.1057	61.8034
57.735	38.49	~~36.6025~~	6	96.5926	73.2051
55.4958	~~36.9972~~	35.6896	7	97.4928	80.1938
54.1196	36.0797	~~35.1153~~	8	98.0785	84.7759
53.2089	~~35.4726~~	34.7296	9	98.4808	87.9385
52.5731	35.0487	~~34.4577~~	10	98.7688	90.2113
52.1109	~~34.7406~~	34.2585	11	98.9821	91.8986
51.7638	34.5092	~~34.1081~~	12	99.1445	93.1852
51.4964	~~34.3309~~	33.9918	13	99.2709	94.1884
51.2858	34.1906	~~33.9~~	14	99.3712	94.9856
51.117	~~34.078~~	33.8261	15	99.4522	95.6295
50.9796	33.9864	~~33.7659~~	16	99.5185	96.1571

POLYSTARS

The shape function of a polystar is taken to be the ratio of the radius of its inscribed circle (r) to the radius of its circumscribed circle (R). [The program setting is $\frac{100 \cdot r}{R}$]

- q_0 The Polygon of n sides: , $a = \frac{180}{n}$

$$\frac{r_0}{R_0} = \cos a$$

- q_1 The Polystar of one vertex (or side) skipped.

$$\frac{r_0}{r_1} = \cos a, \quad \frac{r_0}{R_1} = \cos 2a, \quad \text{hence } \frac{r_1}{R_1} = \frac{\cos 2a}{\cos a}$$

- q_2 The polystar with two vertices skipped

$$\frac{r_0}{r_2} = \cos 2a, \quad \frac{r_0}{R_2} = \cos 3a \qquad \frac{r_2}{R_2} = \frac{\cos 3a}{\cos 2a}$$

- q_3 $\quad \frac{r_0}{r_3} = \cos 3a, \quad \frac{r_0}{R_3} = \cos 4a \qquad \frac{r_3}{R_3} = \frac{\cos 4a}{\cos 3a}$

- Generalizing $\quad q_b = \frac{\cos[(b+1)a]}{\cos[ba]} = \frac{r_b}{R_b}$

- CF center fold

$$R_c = 2r_0, \quad \frac{r_0}{r_c} = \cos(a) \quad \therefore \frac{r_c}{R_c} = \frac{1}{2\cos(a)}$$

- SF Even values of n only

$$R_s = 3r_0 \qquad \frac{r_0}{r_s} = \cos(a) \qquad \frac{r_s}{R_s} = \frac{1}{3\cos(a)}$$

- VF odd values of n only

$$R_v = 2r_0 + r_v = r_v(2\cos a + 1)$$

$$\frac{r_0}{r_v} = \cos(a) \qquad \frac{r_v}{R_v} = \frac{1}{2\cos(a)+1}$$

N

- The 2n polygon [set 100 in program]

% for 2n =

$$\frac{r_0}{R_0} = \cos\left(\frac{a}{2}\right) = \sqrt{\frac{1+\cos a}{2}}$$

- The symmetric star to the 2n polygon

$$\frac{r_0}{\bar{R}_0} = \cos a$$

$$\bar{R}_0 - r_0 = x$$

$$\bar{r} = \bar{R} - 2x = \bar{R} - 2(\bar{R} - r_0) = \bar{R} - 2(\bar{R} - \bar{R}\cos(a))$$

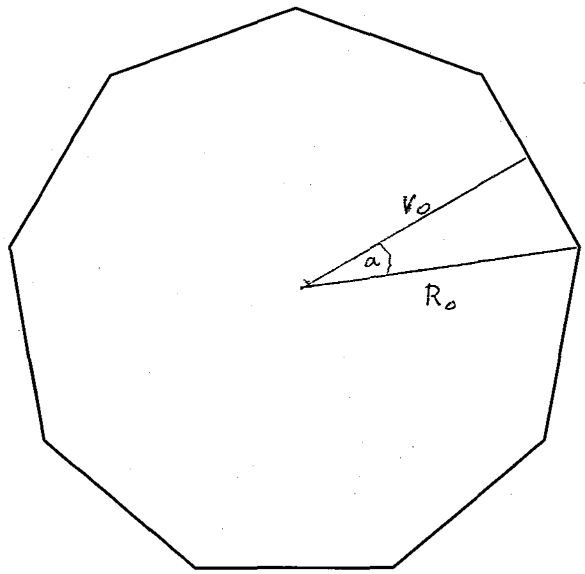
$$= \bar{R}(2\cos a - 1)$$

$$\frac{\bar{r}}{\bar{R}} = 2\cos(a) - 1$$

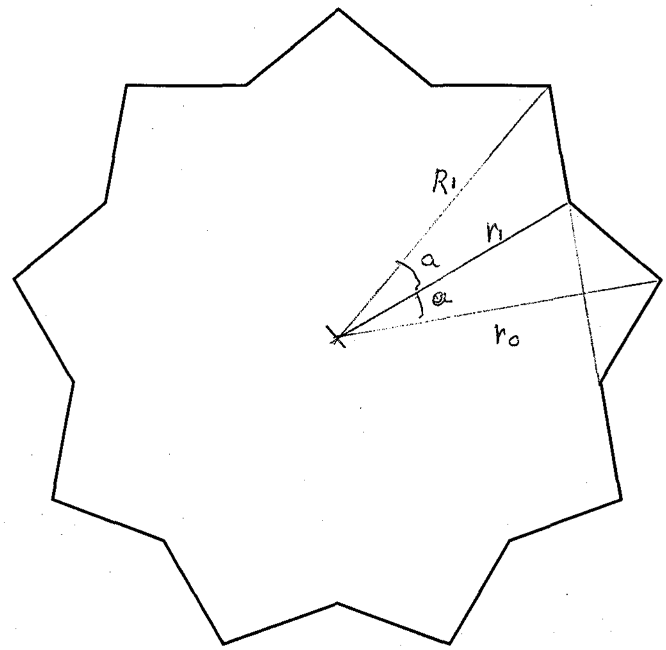
N = no of sides of polygon, S = number of stars by side extension

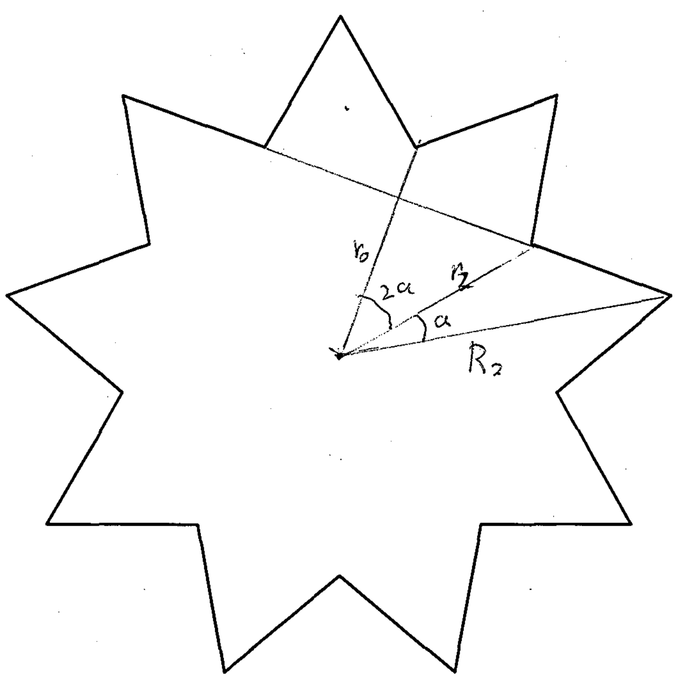
N	S
3	0
4	0
5	1
6	1
7	2
8	2
9	3
10	

9_0

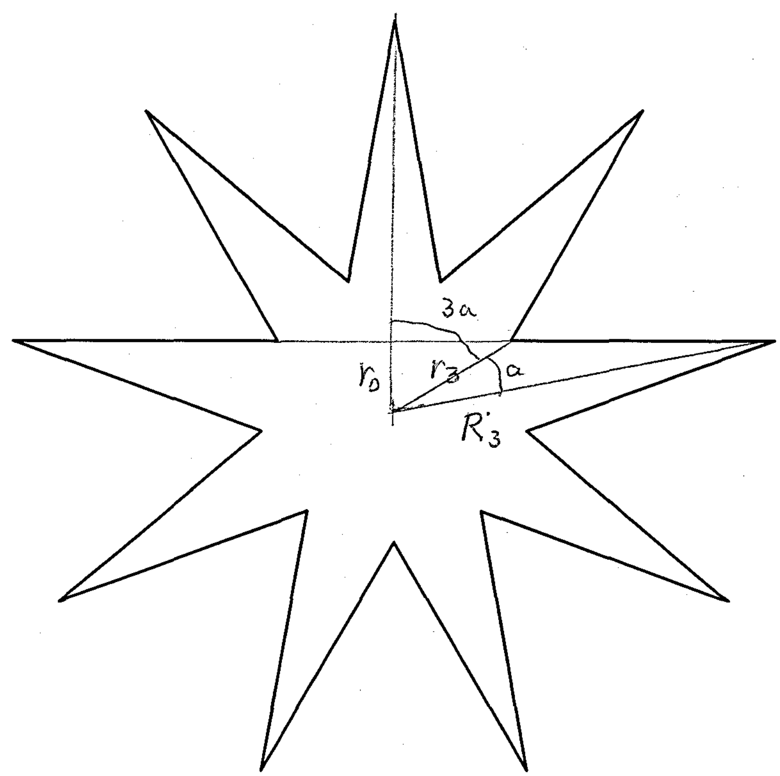


9_1





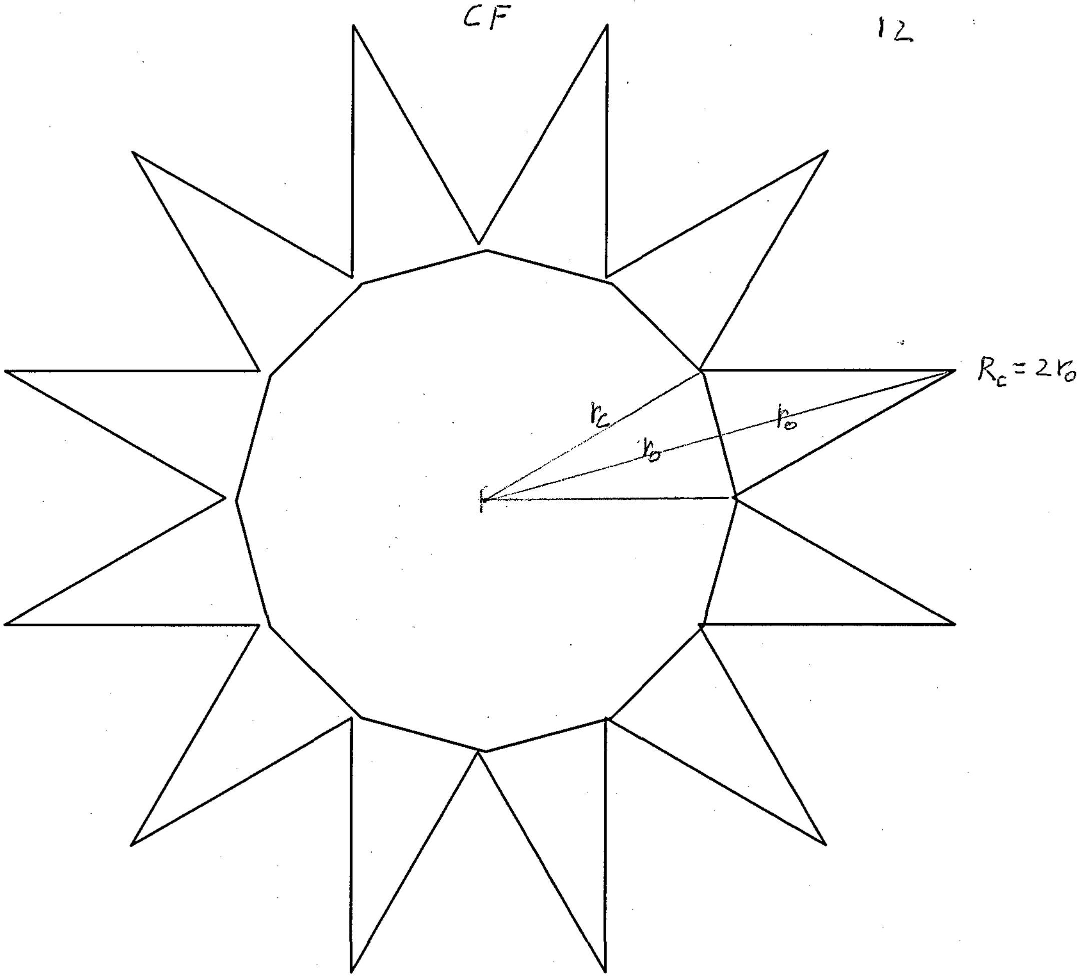
9_2

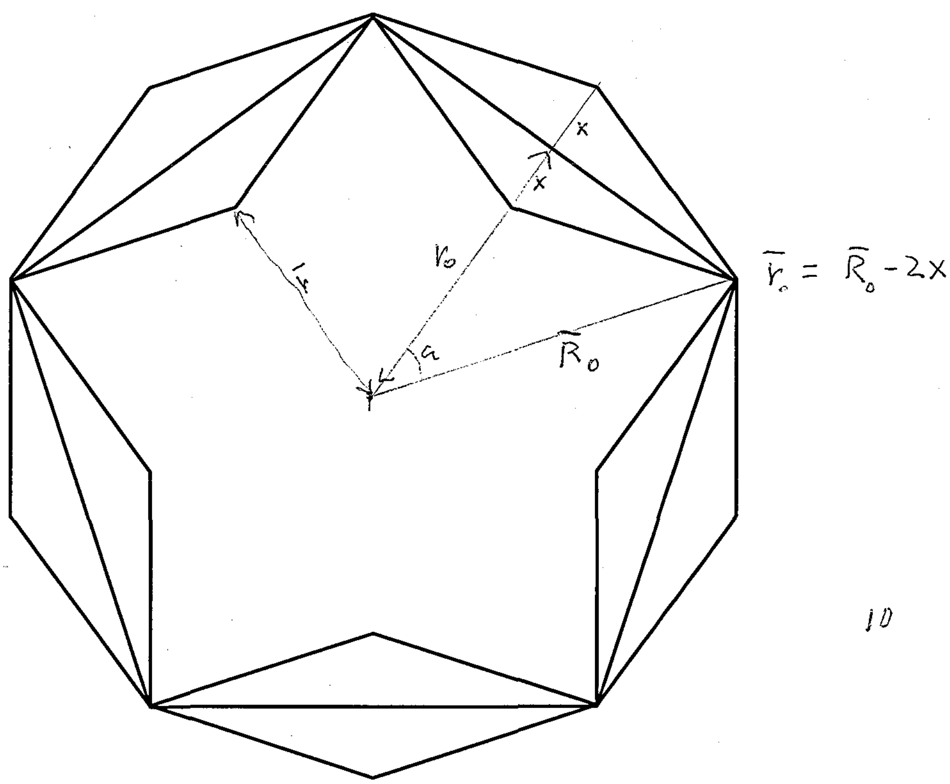


9_3

9 → 3 stars

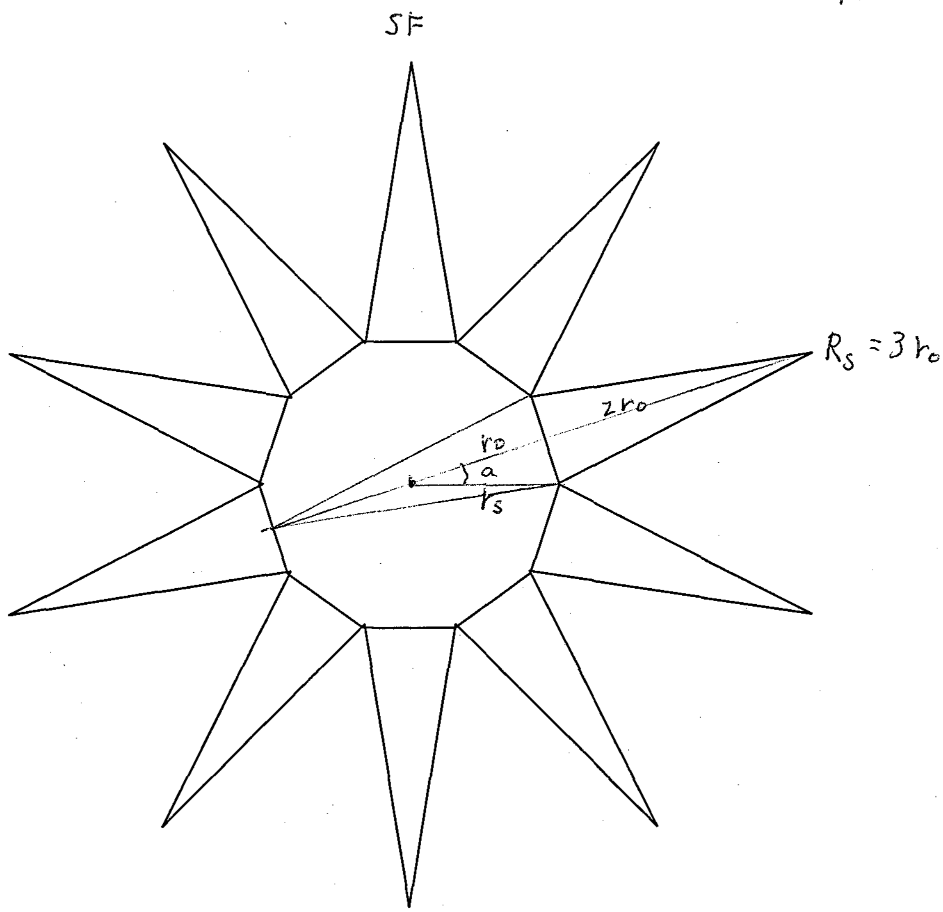
CF 12

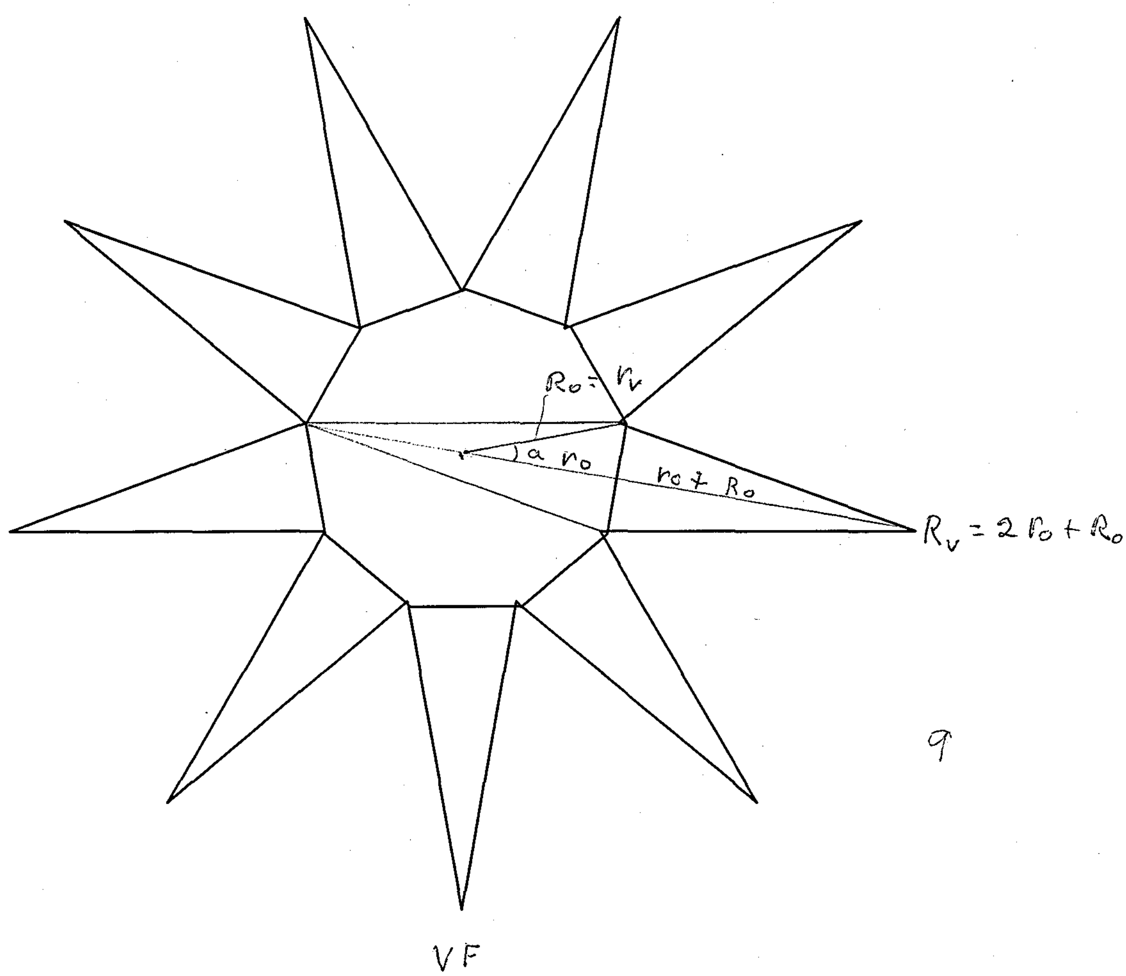




10

~ 1N


SF
$R_s = 3r_o$
$2r_o$
r_o
a
r_s


$R_o = r_v$
a r_o
$r_o + R_o$
$R_v = 2r_o + R_o$
VF

%ϕ = / 100
Star Trimmer radii

n =	3	4	5	6	7	8	9	10	11
X_5				38.5 SF					
X_4		~41.4	q_1 rot 38.2 VF	q_1 rot 67.7 CF					
X_3		47.1 SF	61.8 CF	73.2					
X_2	q_0 rot 50 VF	q_0 rot 70.7 CF	q_0 rot 80.9	q_0 rot 86.6					
X_1	100 CF	100	100	100	100				
→ $P = q_0$	50	70.7	80.9	86.6	90.1				
W_1	0 CF	47.1 SF	~61.8	~73.2					
W_2		~41.4	38.2 = q_1	57.7 = q_1					
W_3		0	0	38.5 SF					
W_4				0					

$\overset{1N}{\sim}$ (to 100) $W_1 = 100\left[2\cos\left(\frac{\pi}{n}\right) - 1\right]$

$q_0 = 100\cos\left(\frac{\pi}{n}\right)$

$q_z = \frac{100\cos\left[(z+1)\frac{\pi}{n}\right]}{\cos\left(z\frac{\pi}{n}\right)}$

$CF = \frac{100}{2\cos\left(\frac{\pi}{n}\right)}$

$VF = \frac{100}{1 + 2\cos\left(\frac{\pi}{n}\right)}$ n-odd

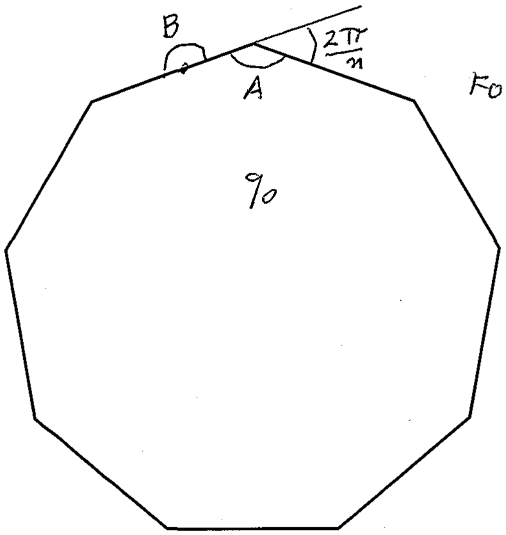
$SF = \frac{100}{3\cos\left(\frac{\pi}{n}\right)}$ n-even

$\overset{1N}{\sim}$ = symmetry to 100

EX = "100" $\cos\left(\frac{\pi}{2n}\right)$

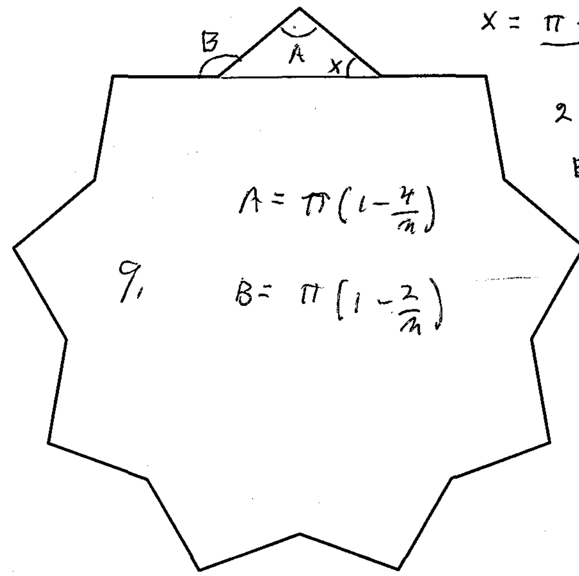
ANGLES

POLYSTAR ANGLES



For q_0 $\quad A = \pi\left(1 - \frac{2}{n}\right)$

$B = \pi$



q_1

$A = \pi\left(1 - \frac{4}{n}\right)$

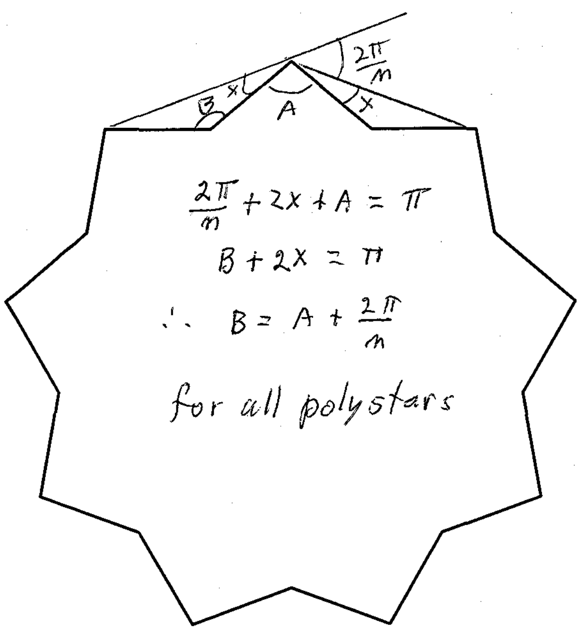
$B = \pi\left(1 - \frac{2}{n}\right)$

$X + B = \pi$

$X = \frac{\pi - A}{2}$

$2B - A = \pi$

$B = A + \frac{2\pi}{n}$



$\frac{2\pi}{n} + 2X + A = \pi$

$B + 2X = \pi$

$\therefore \quad B = A + \frac{2\pi}{n}$

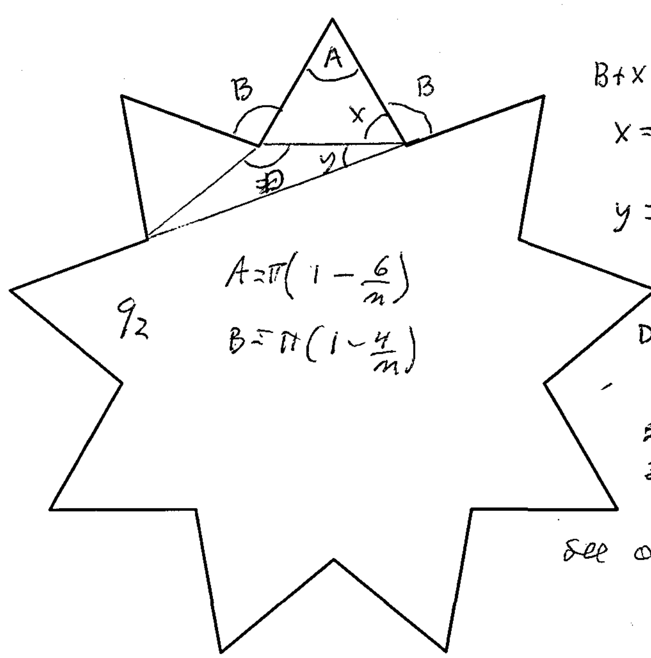
for all polystars

In General

for q_z

$A = \pi\left(1 - \frac{2(z+1)}{n}\right)$

$B = \pi\left(1 - \frac{2z}{n}\right)$



q_2

$A = \pi\left(1 - \frac{6}{n}\right)$

$B = \pi\left(1 - \frac{4}{n}\right)$

$B + X + y = \pi$

$X = \frac{\pi - A}{2}$

$y = \frac{\pi - D}{2}$

$D = 2\pi - (B + 2X)$

~~$2B = \pi + A$~~

~~$3B = \pi + 2A$~~

see over

g_2 $B + x + y = \pi$

$2x = \pi - A$

$2y = \pi - D$

$D = 2\pi - B - 2x = 2\pi - B - \pi + A = \pi - 2y$

$\therefore\ B - A = 2y$

$2\pi = 2y + 2x + 2B$

$2\pi = B - A + \pi - A + 2B$

$\pi = 3B - 2A$ and $B = A + \frac{2\pi}{n}$ $\Big\}$ $B = \pi\left(1 - \frac{4}{n}\right)$, $A = \pi\left(1 - \frac{6}{n}\right)$

A and B as functions of n etc

EX [i.e 2n] $A_x = \pi - \frac{2\pi}{2n} = \pi\left(1 - \frac{1}{n}\right)$

$B_x = \pi\left(1 - \frac{1}{n}\right) + \frac{2\pi}{n} = \pi\left(1 + \frac{1}{n}\right)$

IN $B_w = A_x$ $B = \pi\left(1 - \frac{1}{n}\right)$

$A = B - \frac{2\pi}{n} = \pi\left(1 - \frac{3}{n}\right)$

CF $A = \frac{2\pi}{n}$, $B = \frac{4\pi}{n}$

VF n odd, SF n even $A = \frac{\pi}{n}$, $B = \frac{3\pi}{n}$

FOR ALL POLYSTARS $B - A = \frac{2\pi}{n}$

$A + B = 2\pi$ for EX

$= 2\pi\left(1 - \frac{2}{n}\right)$ for IN

$= \frac{6\pi}{n}$ for CF

$= \frac{4\pi}{n}$ for VF, SF

$= 2\pi\left(1 - \frac{2z+1}{n}\right)$ for g_z

For a circle $A = B$

$\frac{A}{B} = \frac{n-2}{n}$ for g_0, $= \frac{n - 2(z+1)}{n - 2z}$ for g_z

$\frac{A}{B} = \frac{n-4}{n-2}$ for g_1, $= \frac{n-6}{n-4}$ for g_2

$\frac{A}{B} = \frac{n-1}{n+1}$ for EX

$\frac{A}{B} = \frac{n-1}{n-3}$ for IN

$\frac{A}{B} = \frac{1}{2}$ for CF

$\frac{A}{B} = \frac{1}{3}$ for SF, VF

n := 3.. 16 ANGLES FOR CF = C AND VF(n odd), SF (n even) = F 2002-03-28

$AC(n) := \frac{360}{n}$ $BC(n) := \frac{720}{n}$ $A(n) := 0$ $B(n) := \frac{360}{n}$ $AF(n) := \frac{180}{n}$ $BF(n) := \frac{540}{n}$

AC(n) =	BC(n) =	n =	A(n) =	B(n) =	AF(n) =	BF(n) =
120	240	3	0	120	60	180
90	180	4	0	90	45	135
72	144	5	0	72	36	108
60	120	6	0	60	30	90
51.429	102.857	7	0	51.429	25.714	77.143
45	90	8	0	45	22.5	67.5
40	80	9	0	40	20	60
36	72	10	0	36	18	54
32.727	65.455	11	0	32.727	16.364	49.091
30	60	12	0	30	15	45
27.692	55.385	13	0	27.692	13.846	41.538
25.714	51.429	14	0	25.714	12.857	38.571
24	48	15	0	24	12	36
22.5	45	16	0	22.5	11.25	33.75

CONDITIONS FOR SYMMETRY
AC(n) = B(n)

Note for n = 3 VF ≡ q_0
n = 5 VF = q_1
n = 7 VF = q_2

Note: for n = 4. CF = q_0
n = 6 CF = q_1

for CF $\frac{A}{B} \equiv \frac{1}{2}$

for VF, SF $\frac{A}{B} \equiv \frac{1}{3}$

VF, SF do not do symmetries

$n := 3 .. 16$

ANGLES FOR EX (2n) AND IN

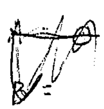
2002 – 03 – 28

Criterion for symmetry: AX = BI

$AX(n) := 180 \cdot \left(1 - \frac{1}{n}\right)$ $BX(n) := 180 \cdot \left(1 + \frac{1}{n}\right)$ $AI(n) := 180 \cdot \left(1 - \frac{3}{n}\right)$ $BI(n) := 180 \cdot \left(1 - \frac{1}{n}\right)$

AX(n) =	BX(n) =	n =	AI(n) =	BI(n) =
120	240	3	0	120
135	225	4	45	135
144	216	5	72	144
150	210	6	90	150
154.286	205.714	7	102.857	154.286
157.5	202.5	8	112.5	157.5
160	200	9	120	160
162	198	10	126	162
163.636	196.364	11	130.909	163.636
165	195	12	135	165
166.154	193.846	13	138.462	166.154
167.143	192.857	14	141.429	167.143
168	192	15	144	168
168.75	191.25	16	146.25	168.75

For $n = 3$ EX = CF



$n := 3..16$ $z := 0..5$ ANGLES FOR q's 2002-03-28

$$AQ_{n,z} := 180 \cdot \left[1 - 2 \cdot \frac{(z+1)}{n}\right] \qquad BQ_{n,z} := 180 \cdot \left(1 - 2 \cdot \frac{z}{n}\right)$$

z

AQ = (n)

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	60	-60	-180	-300	-420	-540
4	90	0	-90	-180	-270	-360
5	108	36	-36	-108	-180	-252
6	120	60	0	-60	-120	-180
7	128.571	77.143	25.714	-25.714	-77.143	-128.571
8	135	90	45	0	-45	-90
9	140	100	60	20	-20	-60
10	144	108	72	36	0	-36
11	147.273	114.545	81.818	49.091	16.364	-16.364
12	150	120	90	60	30	0
13	152.308	124.615	96.923	69.231	41.538	13.846
14	154.286	128.571	102.857	77.143	51.429	25.714
15	156	132	108	84	60	36
16	157.5	135	112.5	90	67.5	45

z

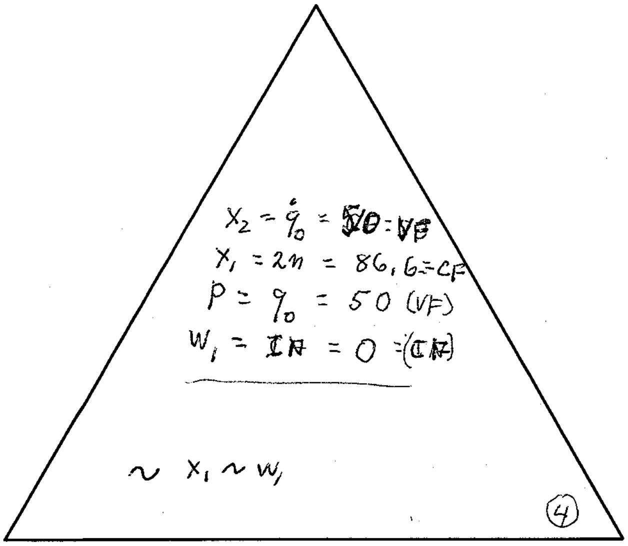
BQ = (n)

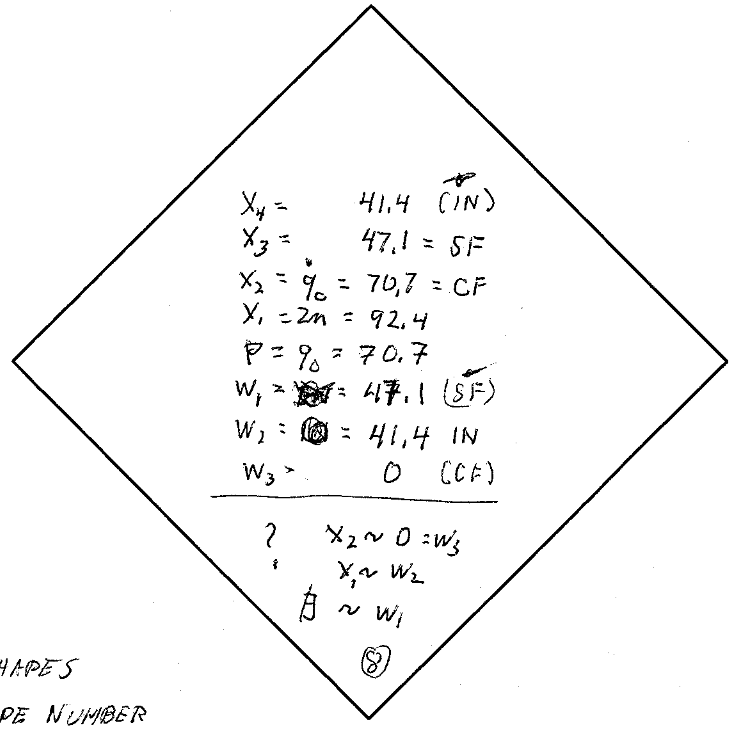
	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	180	60	-60	-180	-300	-420
4	180	90	0	-90	-180	-270
5	180	108	36	-36	-108	-180
6	180	120	60	0	-60	-120
7	180	128.571	77.143	25.714	-25.714	-77.143
8	180	135	90	45	0	-45
9	180	140	100	60	20	-20
10	180	144	108	72	36	0
11	180	147.273	114.545	81.818	49.091	16.364
12	180	150	120	90	60	30
13	180	152.308	124.615	96.923	69.231	41.538
14	180	154.286	128.571	102.857	77.143	51.429
15	180	156	132	108	84	60
16	180	157.5	135	112.5	90	67.5

CONDITION FOR SYMMETRY

$A = B$

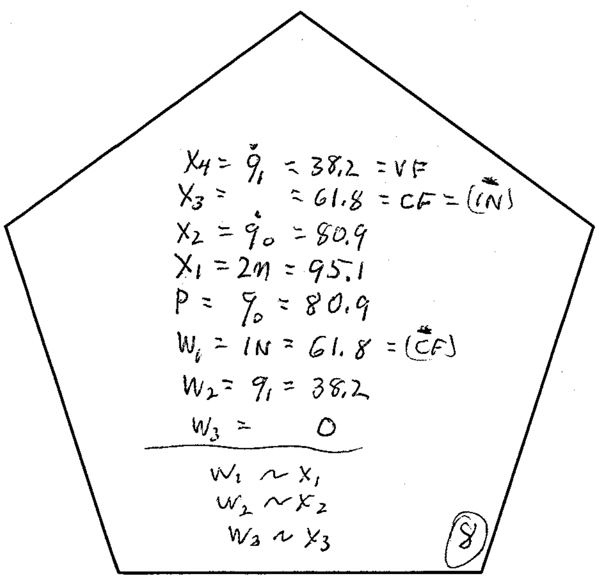
For all n: $AQ_z = BQ_{z+1}$

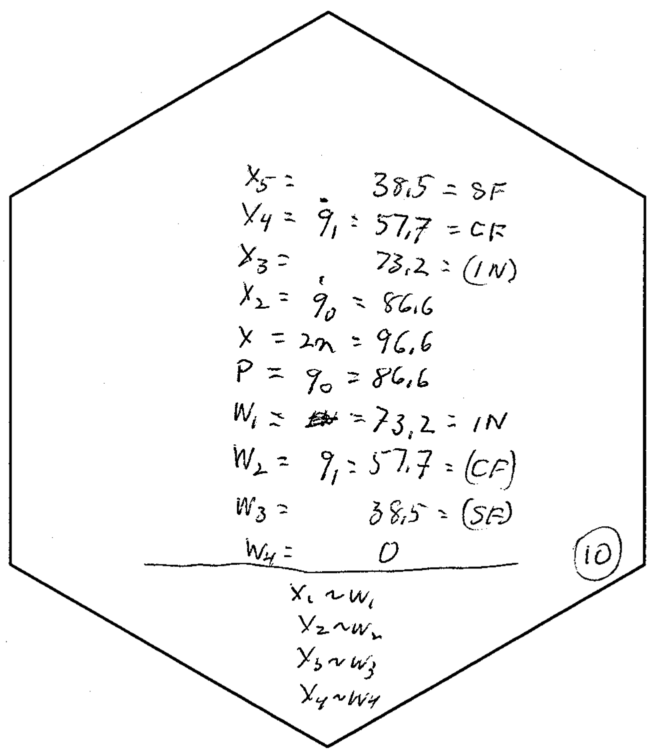


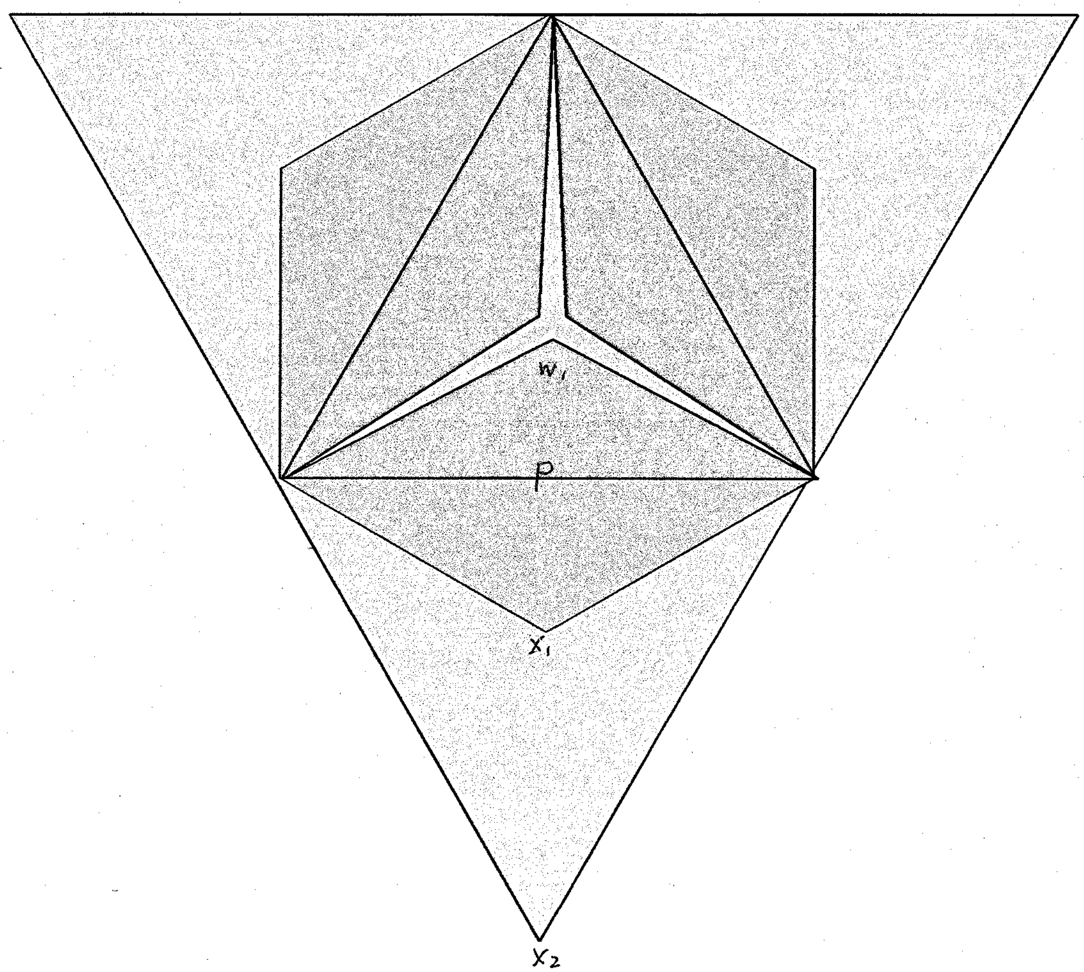


SAME SHAPES
BY SHAPE NUMBER

SYMMETRIES about P ~







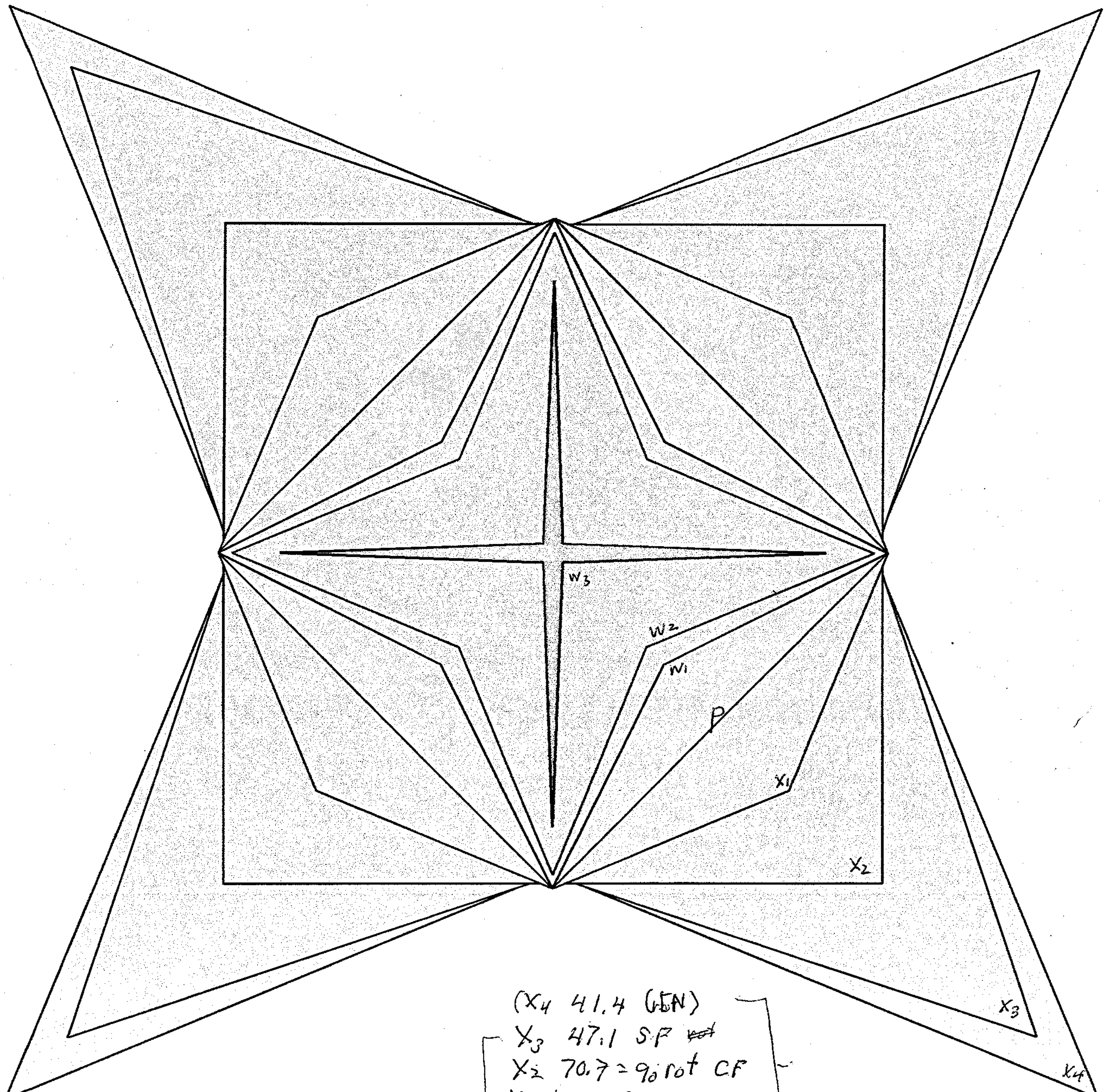
Same shape sym
P X2 O – X1

X2 50 90 rot VF ✓
X1 ~~100~~ 86.6 CF 2n = EX ✓
P 50 = 80
W1 O (CF)

(4)

rot 60

THREEF



(X_4 41.4 (GEN))
X_3 47.1 SF ~~rot~~
X_2 70.7 = 9₀ rot CF
X_1 ~~100~~ 2n = 92.4.
P 70.7 = 9₀
(W_1 47.1). (SF)
W_2 41.4. 100 sym IN
W_3 O (CF)

sym

Same shape

P	X_2
W_2	X_4
W_1	X_3

sym

X_2	O
X_1	W_2

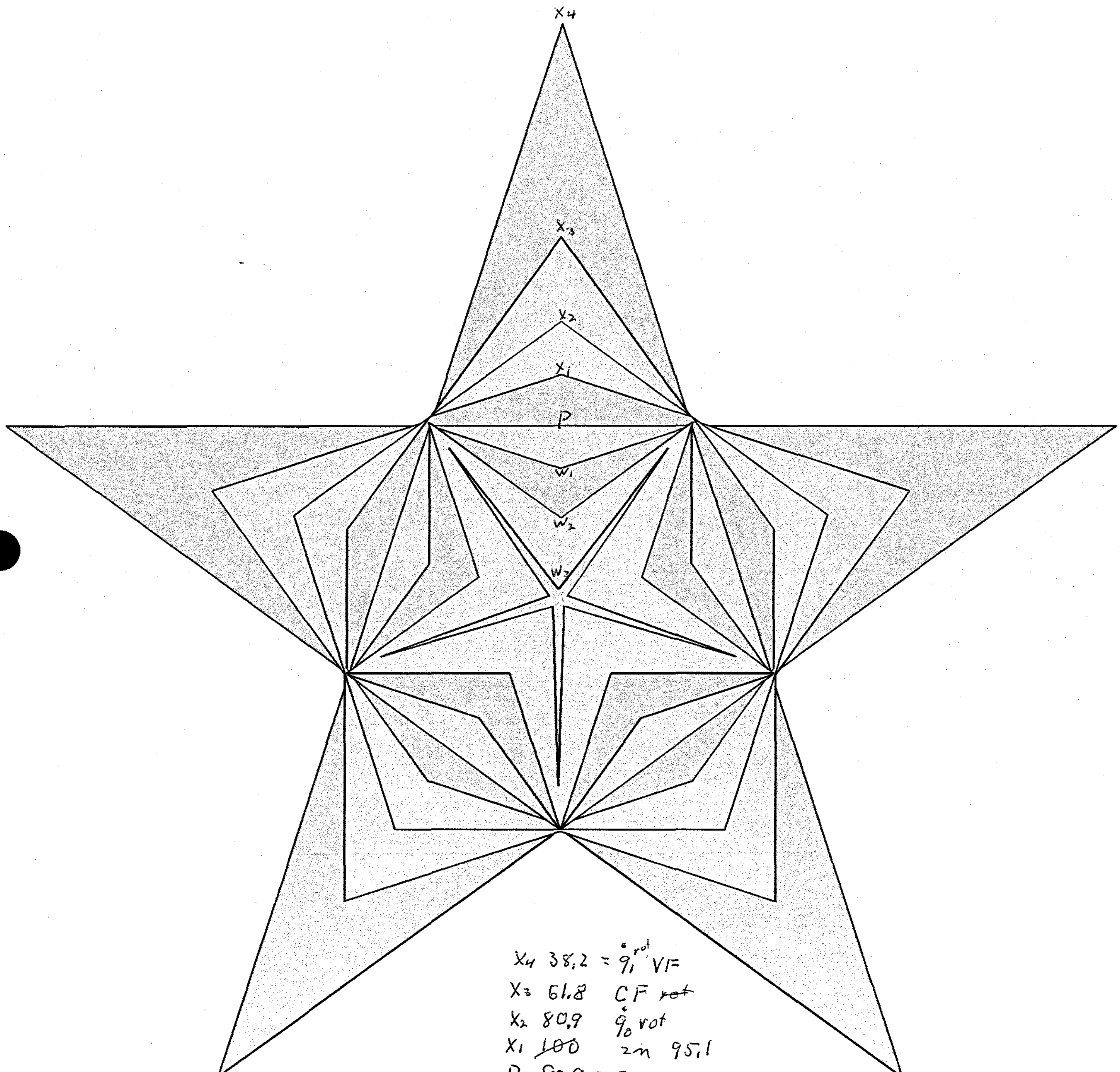
8

∃? ~ W_1?

rot 45

Maltese Cross?

FOURF2



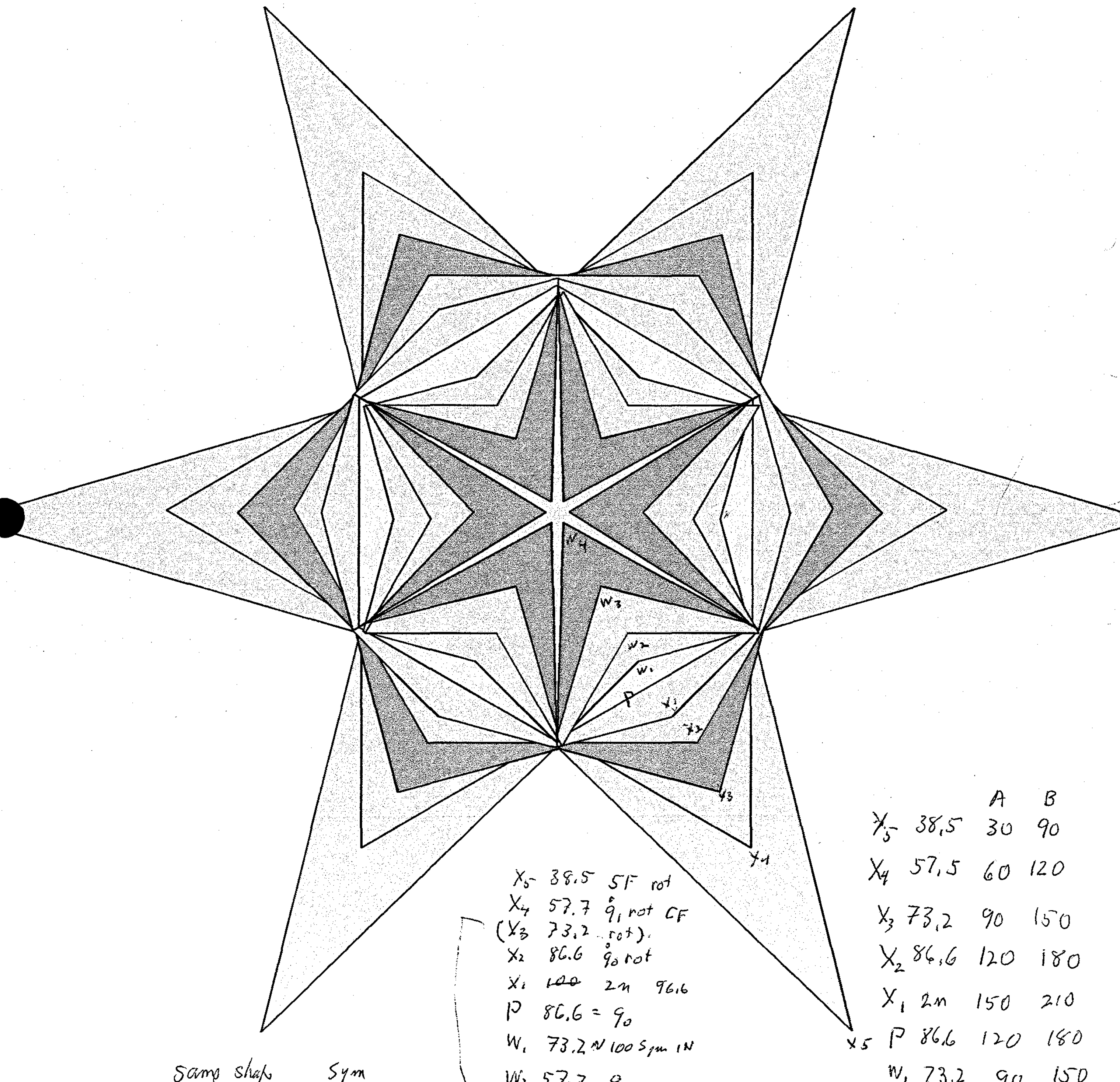
X_4 38.2 = $\dot{9}_1$ rot VF
X_3 61.8 CF ~~rot~~
X_2 80.9 $\dot{9}_0$ rot
X_1 ~~100~~ 2n 95.1
P 80.9 = 9_0
W_1 61.8 n100sym IN
W_2 38.2 = 9_1
W_3 0

	Same shape		sym	
80.9	P	~~X_2~~	2n	W_1
38.2 W_2	VF/ 9_1	X_4	X_2	W_2
61.8	W_1	X_3 CF	X_3	0

(8)

rot 36°

FIVE F



		A	B
X_5	38.5	30	90
X_4	57.5	60	120
X_3	73.2	90	150
X_2	86.6	120	180
X_1	2n	150	210
P	86.6	120	180
W_1	73.2	90	150
q_1 W_2	57.7	60	120
W_3	38.5	30	90
W_4	0	0	60

X_5 38.5 5F rot
X_4 57.7 q_1 rot CF
(X_3 73.2 rot).
X_2 86.6 q_0 rot
X_1 ~~100~~ 2n 96.6
P 86.6 = q_0
W_1 73.2 N 100 Sym IN
W_2 57.7 q_1
(W_3 38.5)
W_4 0

Same shape		Sym	
P	X_2	W_4 0	X_4
W_1	X_3	W_3	X_3
W_2	X_4	W_2	X_2
W_3	X_5	W_1	X_1

(10)

rot 30°

SIX F

$X_6 = \dot{q}_2 = 35.7 = VF$

$X_5 = CF = 55.5$

$X_4 = \dot{q}_1 = 69.2$

$X_3 = (1N) = 80.2$

$X_2 = \dot{q}_0 = 90.1$

$X_1 = 2n = 97.5$

$P = q_0 = 90.1$

$W_1 = 1N = 80.2$

$W_2 = q_1 = 69.2$

$W_3 = (CF) = 55.5$

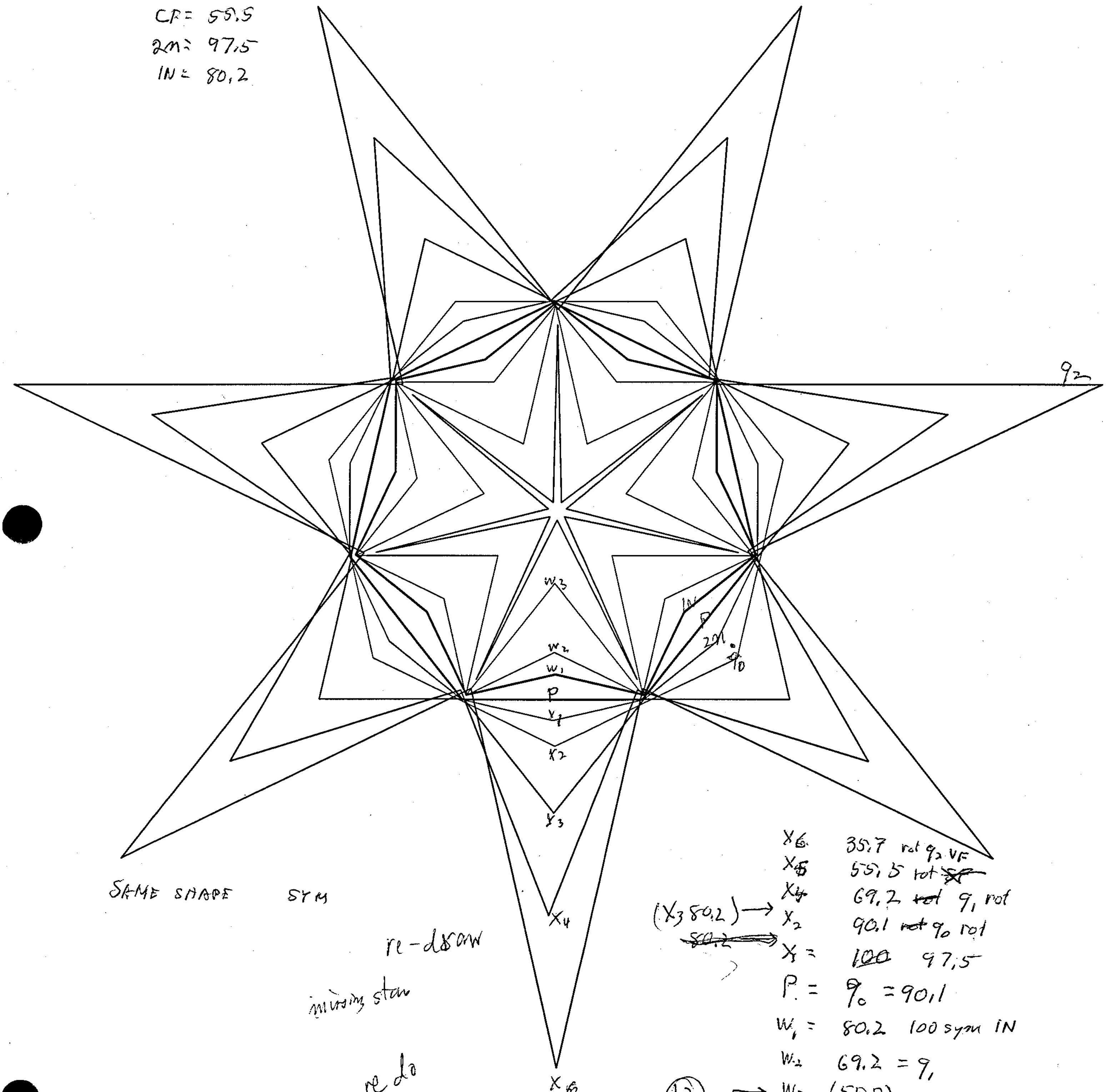
$W_4 = q_2 = 35.7 = (VF)$

$W_5 = \quad = 0$

(12)

P = q_0 = 90.1
q_1 = 69.2
q_2 = 35.7

VF = 35.7 = q_2
CF = 55.5
ZM = 97.5
IN = 80.2



SAME SHAPE SYM

re-draw

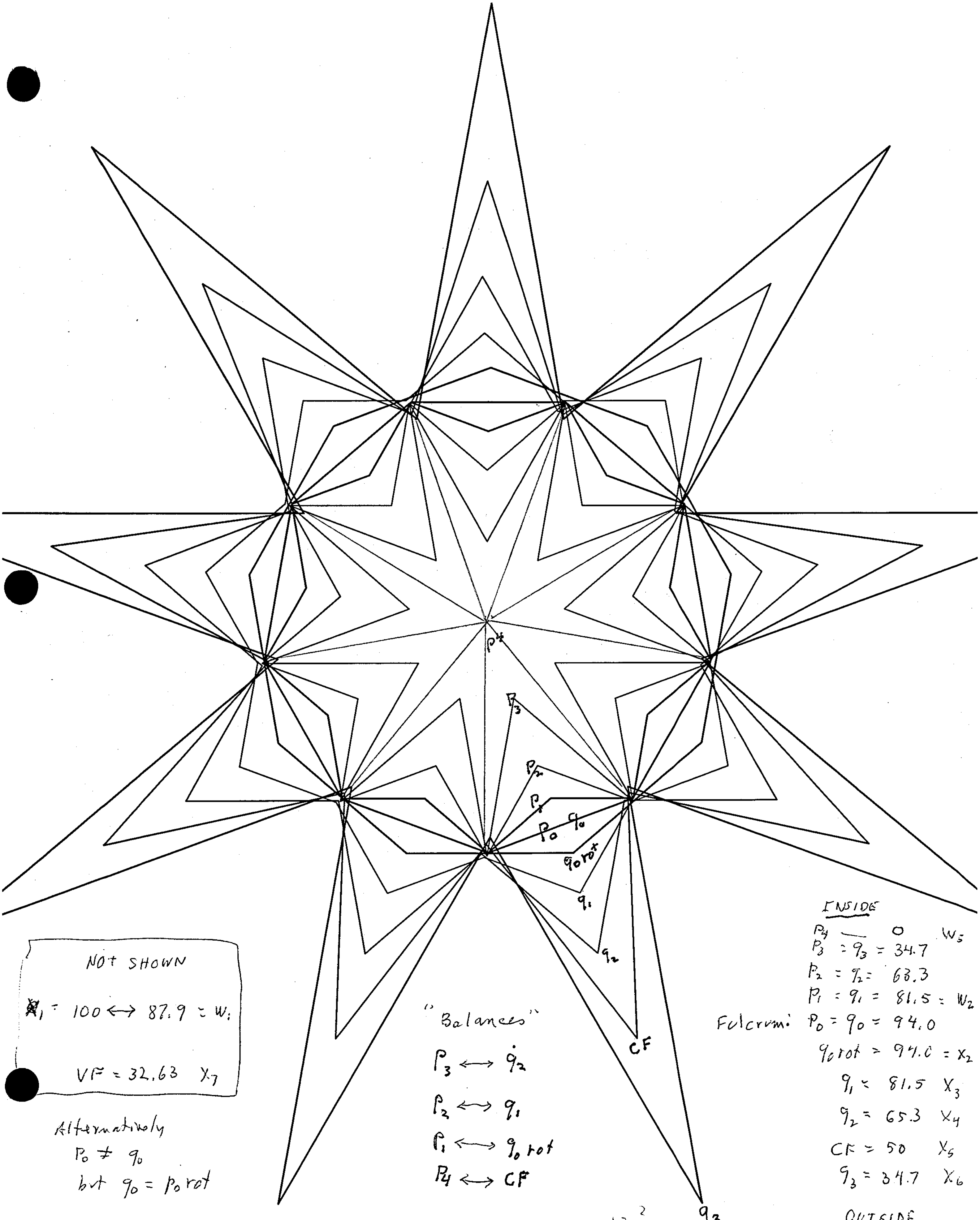
missing stars

re do

X6 35.7 rot q_2 VF
X5 55.5 rot ~~CF~~
X4 69.2 ~~rot~~ q_1 rot
(X3 80.2) → X2 90.1 ~~rot~~ q_0 rot
~~80.2~~ → X1 = ~~100~~ 97.5
P = q_0 = 90.1
W_1 = 80.2 100 sym IN
W_2 69.2 = q_1
(12) → W_3 (55.5)
W_4 35.7 = q_2
W_5 0
rot 25.714286°

SEVENF

NINEF



NOT SHOWN

~~X~~$_1$ = 100 ⟷ 87.9 = W_1

VF = 32.63 X_7

Alternatively
$p_0 \neq q_0$
but $q_0 = p_0$ rot

"Balances"

$p_3 \longleftrightarrow q_2$

$p_2 \longleftrightarrow q_1$

$p_1 \longleftrightarrow q_0$ rot

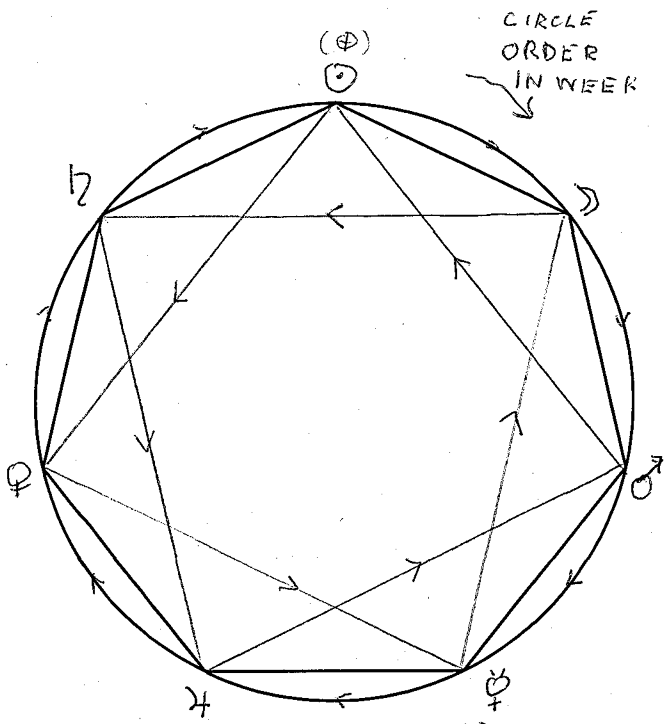
$p_4 \longleftrightarrow$ CF

INSIDE

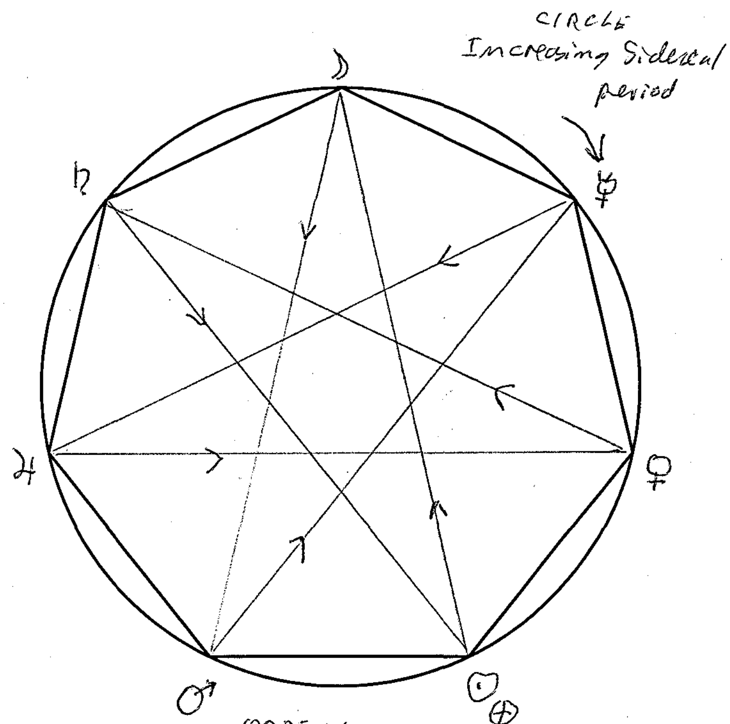
p_4 — 0 W_5
$p_3 = q_3 = 34.7$
$p_2 = q_2 = 63.3$
$p_1 = q_1 = 81.5 = W_2$
Fulcrum: $p_0 = q_0 = 94.0$
q_0 rot = 94.0 = X_2
q_1 = 81.5 X_3
q_2 = 65.3 X_4
CF = 50 X_5
q_3 = 34.7 X_6

OUTSIDE

See 1999 #7

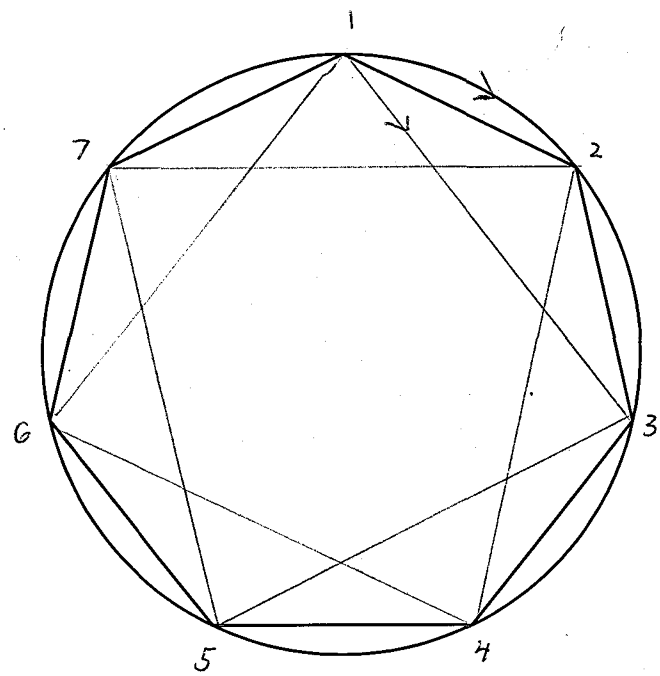


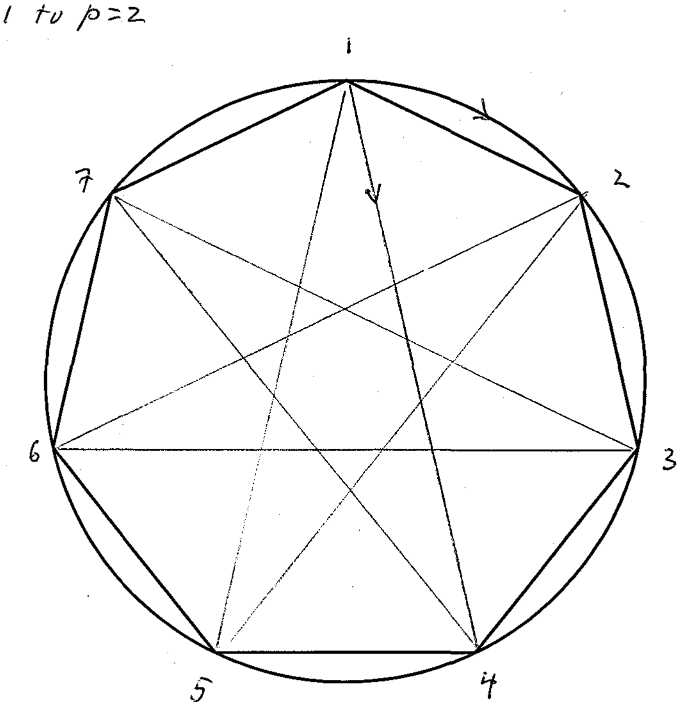
STAR: SIDEREAL PERIOD OR DISTANCE (FROM SUN)



STAR: ORDE IN WEEK

TO INVERT, SWITCH
FROM, $p=1$ to $p=2$





→ 1, 3, 5, 7, 2, 4, 6

→ 1, 4, 7, 3, 6, 2, 5

$\frac{1}{7} = 0.1428571$

If $1 \equiv 8$ mod(7) : $1 \to 4 \to 2 \to 1 \to 5 \to 7 \to 1 \ldots$

Polygon Paths of Reciprocals

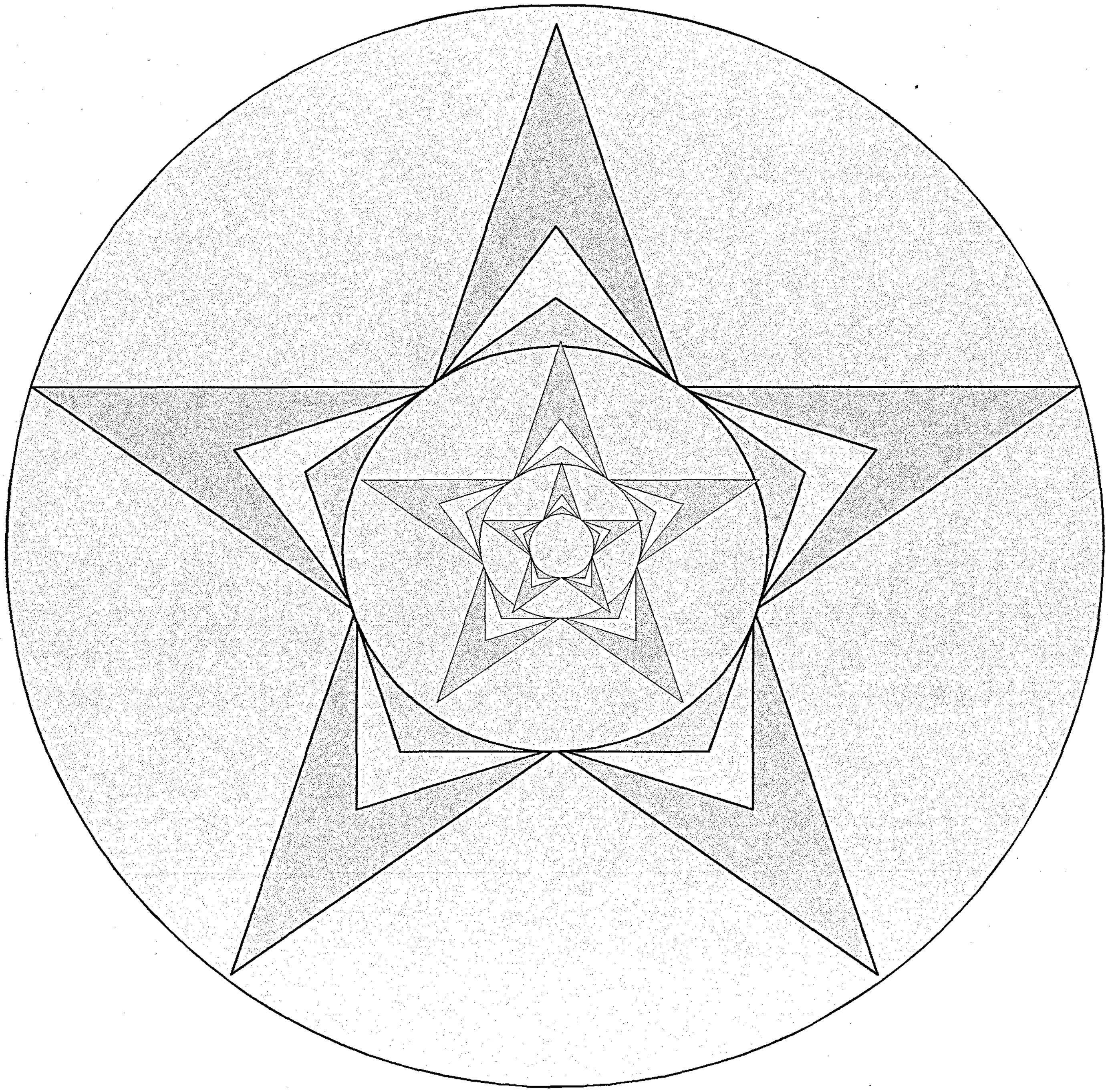
The Sidereal periods Synodic Period days

☽	27.321	
☿	87.97	115.88
♀	224.70	583.92
⊙	365.257	
♂	686.98	779.94
♃	4332.6	398.88
♄	10759.2	378.09

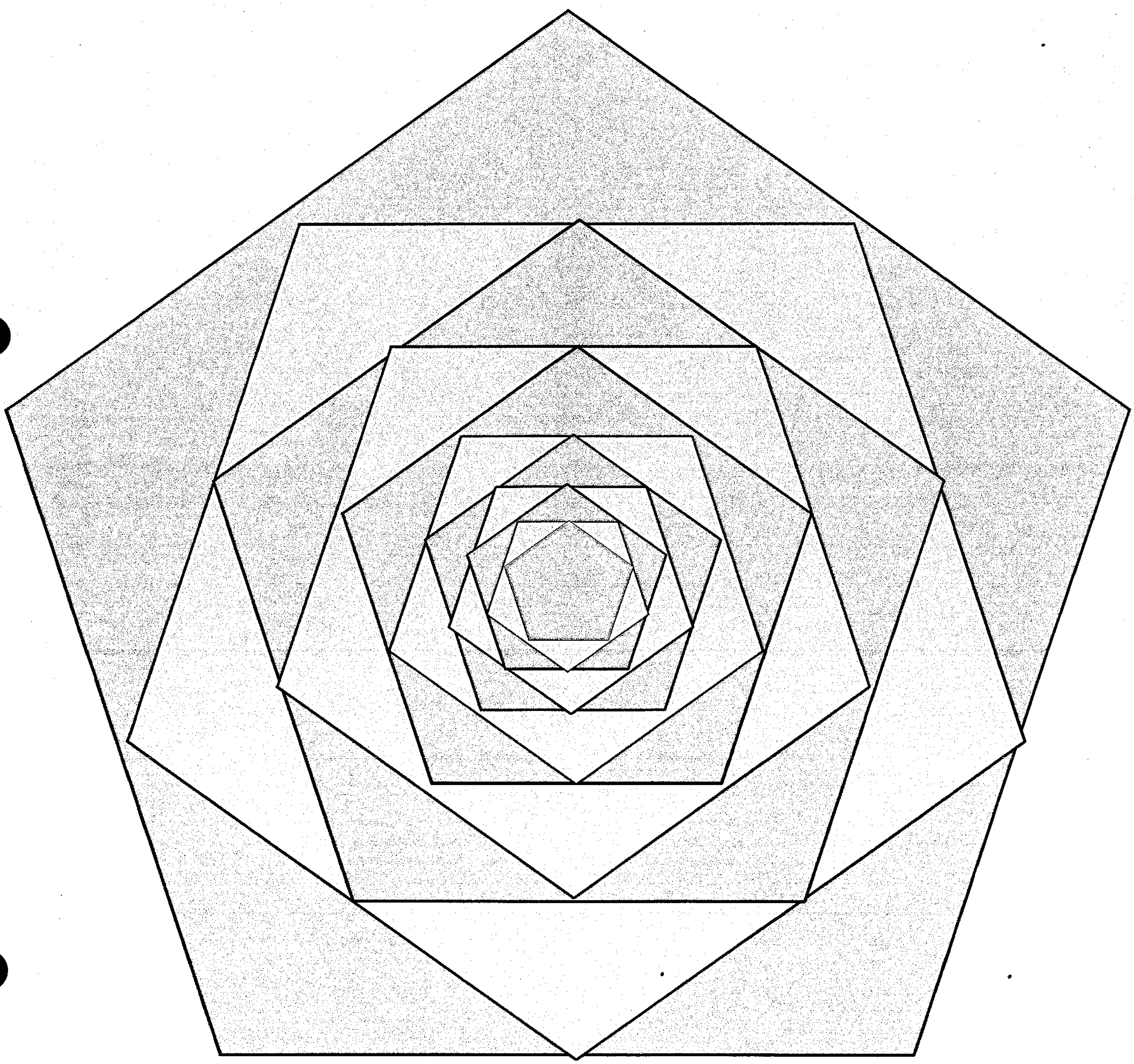
See 1994 #7

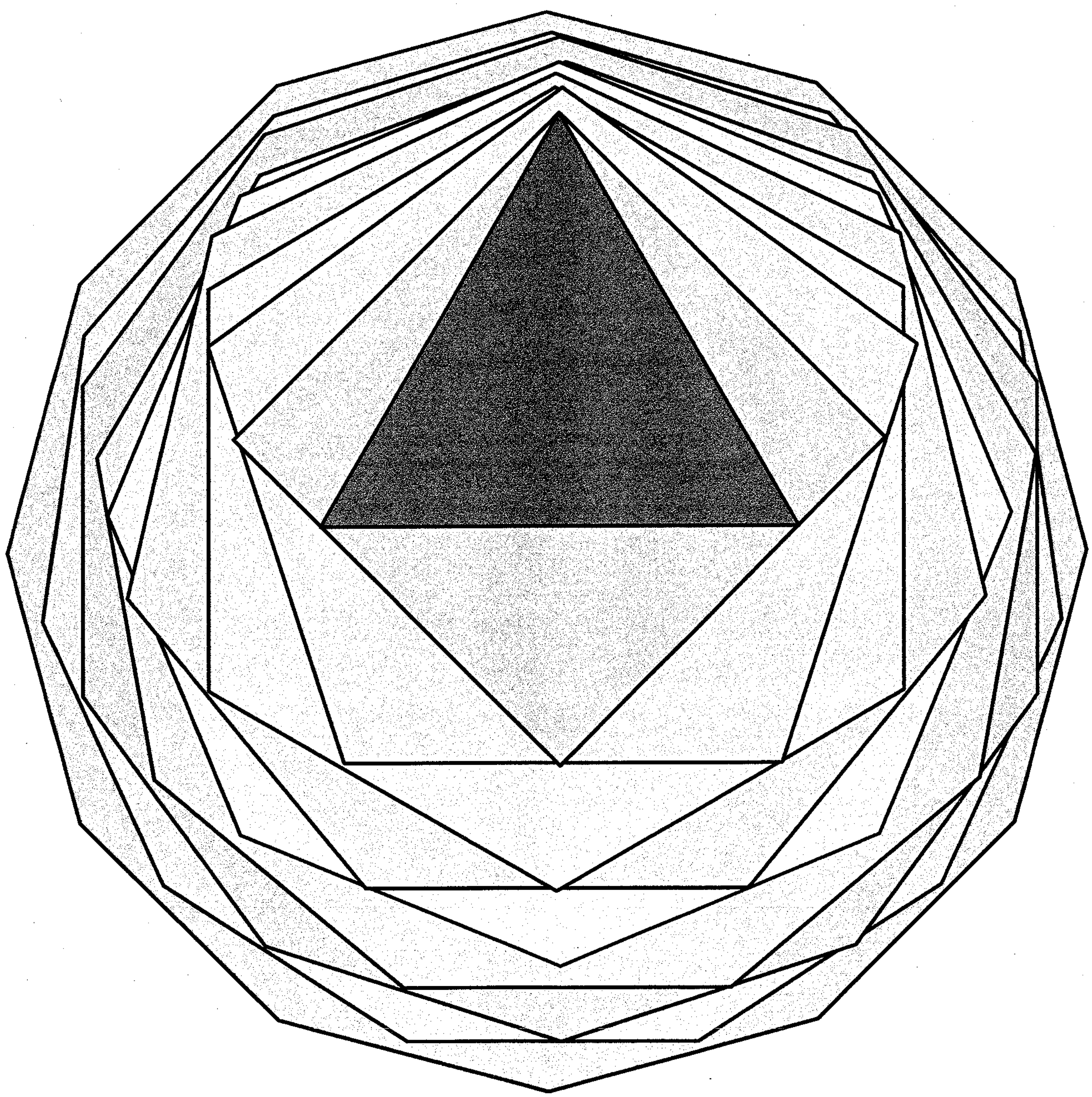
$n := 18 .. 36$

f1(g,n) =	f2(g,n) =	f3(g,n) =	f4(g,n) =	f5(g,n) =	f6(g,n) =
1.2299	1.742642	2.860562	5.485715	12.434969	33.855928
1.244121	1.79725	3.032563	6.029784	14.304026	41.173181
1.258506	1.85357	3.214905	6.627814	16.454015	50.071907
1.273058	1.911655	3.408212	7.285155	18.927161	60.893907
1.287778	1.971559	3.613142	8.007691	21.772038	74.054857
1.302668	2.033341	3.830394	8.801888	25.044518	90.060271
1.31773	2.097059	4.060709	9.674852	28.808873	109.524923
1.332966	2.162774	4.304872	10.634397	33.139035	133.196455
1.348379	2.230548	4.563716	11.689108	38.120049	161.984094
1.36397	2.300446	4.838125	12.848425	43.849742	196.993582
1.379741	2.372535	5.129033	14.122722	50.440646	239.569642
1.395694	2.446882	5.437432	15.523403	58.022206	291.347631
1.411832	2.523559	5.764376	17.063002	66.743324	354.316354
1.428156	2.602639	6.110978	18.755297	76.775283	430.894454
1.44467	2.684197	6.47842	20.615433	88.315111	524.023316
1.461374	2.76831	6.867956	22.660056	101.589451	637.280042
1.478271	2.85506	7.280914	24.907463	116.859011	775.01485
1.495364	2.944528	7.718703	27.377766	134.423686	942.518169
1.512654	3.0368	8.182815	30.093072	154.628448	$1.146224 \cdot 10^{3}$



REGRESSION





Alternat Shape Measures

1) $\frac{r}{R}$, the ratio of the radius of the inscribed circle r
to the radius of the circumscribed circle R
= a pure number

2) The interior and exterior angles
Since an angle is already a pure number
Several functions of angles qualify as shape indices

Creation of Polystars

1) rotate e.g $\dot{q}_0$
2) double e.g 2_n
3) skip all q_i $z > 0$
4) fold CF, (SF, VF)

are there fold outs?

Mitsubishi?

Condition for symmetry:

$X \quad w \qquad q_0 = \cos\frac{\pi}{m}$

$\frac{R}{\bar{R}} \quad \frac{r}{R}$

$$\bar{R} - R\cos a = R\cos a - r$$

$$\frac{\bar{R}}{R} - q_0 = q_0 - \frac{r}{R}, \qquad \frac{1}{\frac{R}{\bar{R}}} = 2q_0 - \frac{r}{R}$$

1) $\boxed{\frac{1}{X} = 2q_0 - w}$ $\qquad X = \frac{1}{2q_0 - w}, \quad W = 2q_0 - \frac{1}{X}$

---

for the first symmetry

$$\bar{R} - \bar{R}\cos a = \bar{R}\cos a - r$$

2) $1 = 2q_0 - w \qquad w = 1N$

This is same as 1)
if we take 2n as 100

Error at $n = 6 \quad X_3 \sim w_3 \quad ?$

INSTARSET2,~~WPD~~ MCD

STAR-POLYGON SETTINGS TABLES

2002-02-18

$n := 23$ $q := 10$

$$Q := \frac{100 \cdot \cos\left[(q+1)\cdot\frac{\pi}{n}\right]}{\cos\left(q\cdot\frac{\pi}{n}\right)} \qquad P1 := 100\cdot\left(2\cdot\cos\left(\frac{\pi}{n}\right) - 1\right) \qquad VF := \frac{\left(100\cdot\cos\left(\frac{\pi}{n}\right)\right)}{1 + 2\cdot\cos\left(\frac{\pi}{n}\right)}$$

$Q = 33.5416$ $P1 = 98.1372$ (≈ W_1) $VF = 33.2292$

P1 is the "balance" of 100

CF = 50 for all n ~~EF = 33.3333 for all even n~~ VF is only for odd n

$$CF = \frac{100}{2\cos\left(\frac{\pi}{n}\right)} \qquad SF = \frac{100}{3\cos\left(\frac{\pi}{n}\right)} \text{ even } n$$

$EX = 100\cos\left(\frac{\pi}{2n}\right)$ The first exterior polystar ≅ polygon of 2n sides

$IN = 100\,(2\cos(\alpha) - 1)$

Q MOD
~~INSTARSET,WPD~~

STAR-POLYGON SETTING TABLES

2002-03-04

$n := 3 .. 16$ $q := 0 .. 6$

$$S_{n,q} := \frac{100 \cdot \cos\left[(q+1)\cdot\frac{\pi}{n}\right]}{\cos\left(q \cdot \frac{\pi}{n}\right)}$$

The values in these tables are 100 times the ratio of the radius of the inscribed circle to the radius of the circumscribed circle of the polystar.
n = number of sides or vertices of the basic polygon.
q = the number of sides or vertices skipped in the construction of the polystar, with q = 0 for the basic polygon.

n

q =

S =

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	50	~~-100~~	~~200~~	~~50~~	~~-100~~	~~200~~
4	70.7107	~~$8.6593\cdot10^{-15}$~~	~~$-1.1548\cdot10^{18}$~~	~~141.4214~~	~~70.7107~~	~~$-2.5978\cdot10^{-14}$~~
5	80.9017	38.1966	~~-100~~	~~261.8034~~	~~123.6068~~	~~80.9017~~
6	86.6025	57.735	~~$1.2246\cdot10^{-14}$~~	~~$-8.1659\cdot10^{17}$~~	~~173.2051~~	~~115.4701~~
7	90.0969	69.2021	35.6896	~~-100~~	~~280.1938~~	~~144.5042~~
8	92.388	76.5367	54.1196	~~$1.6\cdot10^{-14}$~~	~~$-6.2499\cdot10^{17}$~~	~~184.7759~~
9	93.9693	81.5207	65.2704	34.7296	~~-100~~	~~287.9385~~
10	95.1057	85.0651	72.6543	52.5731	~~$1.9815\cdot10^{-14}$~~	~~$-5.0468\cdot10^{17}$~~
11	95.9493	87.6769	77.8434	63.4356	34.2585	~~-100~~
12	96.5926	89.6575	81.6497	70.7107	51.7638	~~$2.3658\cdot10^{-14}$~~
13	97.0942	91.1956	84.5339	75.8927	62.4233	33.9918
14	97.4928	92.4139	86.7767	79.7473	69.5895	51.2858
15	97.8148	93.3955	88.5579	82.7091	74.7238	61.8034
16	98.0785	94.1979	89.9976	85.043	78.5695	68.8812

STARSET2.MCD STAR-POLYGON SETTING TABLES 2002-03-04

$n := 3 .. 16$

CF is the polystar whose vertices come to the center of the q = 0 polygon when folded inward
SF is the polystar whose vertices come to the opposite side of the q=0 polygon when folded in n-even
VF is the polystar whose vertices come to the opposite vertex of the q=0 polygon when folded in n-odd
EX is the polystar exterior to q=0 that is identical to the polygon q=0 for 2n
IN is the polystar that with respect to q=0 "balances" EX

$$CF(n) := \frac{100}{2\cdot\cos\left(\frac{\pi}{n}\right)} \qquad SF(n) := \frac{100}{3\cdot\cos\left(\frac{\pi}{n}\right)} \qquad VF(n) := \frac{100}{1 + 2\cdot\cos\left(\frac{\pi}{n}\right)} \qquad EX(n) := 100\cdot\cos\left(\frac{\pi}{2\cdot n}\right) \qquad IN(n) := 100\cdot\left(2\cdot\cos\left(\frac{\pi}{n}\right) - 1\right)$$

CF(n) =		SF(n) =	VF(n) =		n =	EX(n) =		IN(n) =	
100		~~66.6667~~	50	q_0	3	86.6025	q_0 6	$2.2204\cdot10^{-14}$	
70.7107	q_0	47.1405	~~41.4214~~		4	92.388	q_0 8	41.4214	
61.8034	IN	~~41.2023~~	38.1966	q_1	5	95.1057	q_0 10	61.8034	CF
57.735	q_1	38.49	~~36.6025~~		6	96.5926	q_0 12	73.2051	
55.4958		~~36.9972~~	35.6896	q_2	7	97.4928	q_0 14	80.1938	
54.1196	q_2	36.0797	~~35.1158~~		8	98.0785	q_0 16	84.7759	
53.2089		~~35.4726~~	34.7296	q_3	9	98.4808		87.9385	
52.5731	q_3	35.0487	~~34.4577~~		10	98.7688		90.2113	
52.1109		~~34.7406~~	34.2585	q_4	11	98.9821		91.8986	
51.7638	q_4	34.5092	~~34.1081~~		12	99.1445		93.1852	
51.4964		~~34.3309~~	33.9918	q_5	13	99.2709		94.1884	
51.2858	q_5	34.1906	~~33.9~~		14	99.3712		94.9856	
51.117		~~34.078~~	33.8261		15	99.4522		95.6295	
50.9796		33.9864	~~33.7659~~		16	99.5185		96.1571	

$2q_0 - IN = 1$