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Abstract
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Structural Parallels In Cosmic Aggregates

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Values of masses and radii derived from observations of eclipsing binary stars, large galaxies, and near-by clusters of galaxies, indicate the existence of a common universal bound for the maximum value of M/R for all such cosmic aggregates. This bound, when expressed in dimensionless units relative to the hydrogen atom, has a value of the order of 10^{39}. The limiting ratio between the gravitational radius and the metric radius for these aggregates suggests in each case the empirical relation

$$\frac{G\,M}{c^2 R} \sim \alpha^2$$

where G is the gravitational constant, c is the velocity of light, and α is the Sommerfeld fine structure constant.

The emergence of Eddington's dimensionless structural numbers from three types of independent astronomical observations may be interpreted either as supportive of Dirac's Principle or as further evidence for the existence of some basic structural law common to the microcosmos and macrocosmos.

1.06. Structural Parallels in Cosmic Aggregates, ALBERT G. WILSON,
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STRUCTURAL PARALLELS IN NON-DEGENERATE COSMIC BODIES

Albert G. Wilson
Douglas Advanced Research Laboratories

Schwarzschild's exact solution of the Einstein field equations leads to the prediction of a bound to the ratio of the gravitational radius to the linear radius for all gravitating systems, namely,

$$\frac{2GM}{c^2R} < 1$$

It is observed that this ratio for each of four species of non-degenerate cosmic body - main sequence stars, galaxies, clusters of galaxies, second-order-clusters - is bounded, and that the bound is closely the same for each species: $2GM/c^2R \leq 10^{-4.3}$. ($M/R = 10^{23.5}$ gm/cm or 10^{39} with respect to the mass and radius of the hydrogen atom.) The ratio of the observed bound for non-degenerate bodies to the Schwarzschild bound is of the order of the ratio of atomic to nuclear dimensions.

Assuming the Schwarzschild bound governs totally degenerate matter, the upper limit to observed masses of stars may be explained as the result of the Schwarzschild limit forbidding a mass of greater than about 10^{34} grams for a dense neutron fluid under initial conditions similar to those postulated in evolutionary cosmological models.

The $10^{-4.3}$ bound appears to play the limiting role for non-degenerate matter. The latter bound limits stellar matter under maximum non-degenerate density conditions to masses of about 10^{34} grams ~~in accordance~~ consistent with observed main sequence stellar masses.

A basic question is raised by the existence of the $10^{-4.3}$ bound for aggregates other than stars. Some generalized form of degeneracy for larger aggregates may be implied.

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Assuming the Schwarzschild bound to obtain under totally degenerate conditions, the upper limit to observed masses of stars may be explained as the result of the Schwarzschild limit forbidding a mass of greater than about 10^{34} grams for closely packed neutrons under initial conditions similar to those postulated in evolutionary cosmological models. Alternately, the observed upper bound to masses of stars can be explained by the necessity of any contracting protostar to lose mass as it evolves down along the non-degenerate bound to maximum non-degenerate density conditions.

A. J. Aug 1966

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liminary results show that the inclusion of these effects brings the cooling time of the white dwarfs into better agreement with the ages of the clusters containing them.

Photometric Methods for Measuring the Metal Content of K Giants. GEORGE WALLERSTEIN, *Astronomy Department, University of Washington,* AND H. L. HELFER, *Department of Physics and Astronomy, University of Rochester.*—Correlations are presented between spectroscopically determined ratios of iron-to-hydrogen for K giants and various combinations of color indices. Good correlations are found with the ultraviolet excess of the U, B, V system, two-color differences of the six-color system, one narrow-band system, and one system that combines $U, B, V,$ and six-color indices. Nevertheless, each system has its disadvantages, and it is suggested that a four-color system using the S-20 surface might provide the highest accuracy combined with the greatest efficiency of observing time.

The Effect of a Solar Wind Blast Wave on the Chromosphere. DONAT G. WENTZEL AND ALAN B. SOLINGER, *The University of Michigan.*—The enhanced solar wind following some solar flares is probably caused by coronal heating. The energy involved in the enhanced wind suggests that flare-induced coronal heating occurs over an area extending far beyond the flare area. The resulting disturbance of the underlying nonflaring chromosphere is computed on the assumption that the corona is heated suddenly. In this case, a shock moves downward from the heated region. The increase in the undisturbed chromospheric density ahead of the shock (ρ) tends to weaken the shock. In addition, magnetic fields may act as a guide for the shock. An increase in field strength along the shock's path constricts the area of the shock front (A) and tends to strengthen the shock. Both effects can be evaluated analytically, using Whitham's theory for shock propagation in inhomogeneous media (*J. Fluid Mech.* 4, 337, 1958), if both the shock and the chromosphere are considered to be isothermal. The change in Mach number (M) with ρ and A is given by

$$\left(1+\frac{1}{M^2}+\frac{1}{M}+\frac{2}{M-1}\right)dM+\frac{d\rho}{\rho}+\frac{dA}{A}=0.$$

For plane stratification or vertical magnetic fields ($dA=0$), the shock penetrates gas increasing in density by one to two orders of magnitude before attenuation becomes significant. Its effect may relate to the flare nimbus and to the disappearance of striation patterns. The strength of a shock entering the field of a sunspot ($dA/d\rho<0$) is attenuated much less. Flaring over sunspot umbrae may be initiated by such a shock.

Temperature Models for the Outer Solar Atmosphere. MARVIN L. WHITE AND KOO SUN KIM, *Air Force Cambridge Research Laboratories and Lowell Technological Institute Research Foundation.*—A re-examination of "observed" electron densities in the solar corona and corona extended plus Explorer X data shows that an inverse square law for the density variation is a good approximation. Such a law implies a constantly expanding model for the solar corona. An analytical expression is obtained for the temperature profile using this model. Without adjusting any constants, one obtains a temperature of 3500°K at 0.00025 solar radii above the limb, a maximum temperature of 1.65×10^6 °K at 0.7 solar radii and 2.16×10^4 °K at the earth's orbital distance. This temperature distribution is compared with one based on modifications of the hydrostatic model of Pottasch; a close agreement is found. The consequences of this agreement are discussed.

Structural Parallels in Nondegenerate Cosmic Bodies. ALBERT G. WILSON, *Douglas Advanced Research Laboratories.*—Schwarzschild's exact solution of the Einstein field equations leads to the prediction of a bound to the ratio of the gravitational radius to the linear radius for all gravitating systems, namely,

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Computations Concerning Use of the Princeton Nomographs for Eccentric Orbit Cases. ROBERT E. WILSON, *Georgetown College Observatory.*—Shifts in the depth and shape relations for eclipsing binaries in eccentric orbits are computed. These shifts depend on the eccentricity, longitude of periastron, radii of the components, orbital inclination, ratio of the luminosities, and whether primary eclipse is a transit or an occultation. The present computations supply a moderately large collection of sets of these seven input variables, along with the corresponding displacements of the depth and shape relations. Purposes are (1) to enable those attempting to solve light curves of eccentric systems to estimate the errors that would be made by ignoring the eccentricity and (2) to aid evaluations of solutions taken from previous investigators who have disregarded small eccentricities.

Results of the U. S. Naval Observatory Program of Visual Double-Star Observations. CHARLES E. WORLEY, *U. S. Naval Observatory.*—Intermittent series of visual double-star observations have been made at the Naval Observatory for more than a century. The present program was begun in 1961. More than 9000 measures have been made with the 12- and 26-in. refractors in Washington, and the 40- and 61-in. reflectors in Flagstaff, in this program. Automatic readout equipment developed in collaboration with A. H. Mikesell was used in the measures. Emphasis has been placed on pairs of astrophysical interest, including close, rapidly moving binaries, nearby stars, and intrinsically faint stars. With the 61-in. astrometric reflector, pairs of separation of 0.″06–0.″07 have been observed. Separations of 0.″11–0.″12 are possible with the 26-in. refractor. Intercomparison of the author's measures with those made by van den Bos show good agreement with little evidence of systematic differences.

The automatic equipment includes two gear-operated shaft encoders, a memory unit, a manual entry unit, and a solenoid-operated printer. The equipment has proved reliable in service, has eliminated reading errors, and has doubled the number of observations possible under good seeing conditions.

A New Catalogue of Hα Emission Objects in the Southern Hemisphere. JAMES D. WRAY, *Dearborn Observatory.*—Objective prism plates obtained by K. G. Henize at the Lamont Hussey Observatory have been searched for Hα emission objects, S stars, and carbon stars. A previous search by Henize of medium-exposure plates covering the entire southern hemisphere to $+10.5\ m_r$ at a dispersion of 450 Å/mm at Hα has yielded 1100 stars tentatively classified as early-type Hα emission stars. The recent search by the author of long-exposure plates covering the southern Milky Way from 240–10° galactic longitude to $+13.0\ m_r$ has revealed an additional 800 probable early-type emission stars, and a few new objects of type P, C, and S.

Coordinates (α, δ) accurate to $\sim\pm15''$ p.e. have been computed for all objects found on the long exposure plates by means of a 12 constant-plate reduction theory modified to account for tangential distortion introduced by the objective prism. All standard stars were of spectral type A0, providing a well-defined absorption line at Hα for measurement. Plate measures were obtained on a fully digitized Mann 621 comparator with punched card output. Use of a digital x,y plotter for identification of comparison stars, evaluation of plate reduction residuals, and measure data confirmation greatly facilitated the data reduction.

A preliminary analysis of the surface distribution of southern hemisphere early-type emission stars reveals the strong concentration of the galactic plane and the irregular distribution in longitude similar in nature to that found in the northern hemisphere by Merrill. The relatively high surface concentration in the Carina–Crux region previously examined by Bok is fully confirmed by this survey.

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STRUCTURAL PARALLELS IN COSMIC AGGREGATES

Albert G. Wilson

The parameter GM/R (where G is the gravitational constant, M the mass, and R the linear radius) may be determined for stars, galaxies, and clusters of galaxies independently of calibrations or assumptions concerning the distances to these bodies. Although the observables used for each of the three species of cosmic bodies are different, the derivation of GM/R depends in each case only on basic dynamical relations like Kepler's third law.

In the case of stars, M and R are determined directly from the orbital parameters of eclipsing binaries, through observations of the light curves and spectra. In the case of galaxies, the observables are the inclinations of lines in the equatorial spectra of the galaxy or the redshifts of individual bodies rotating with the galaxy supplemented with the appropriate angular dimensions. The observables in a cluster of galaxies are the individual member galaxy redshifts whose mean dispersions are proportional to GM/R by the virial theorem. The accuracy with which the observables in each case may be measured, the basic nature of the dynamical theories used in the reductions, and the non-dependence on distance, mentioned before, give GM/R observations a special usefulness in astronomy.

In Table 1 are listed the values of M/R, including the individual values of M and R when known, of stars, galaxies, clusters, and second order clusters. The entries in the R, M, and M/R columns are all logarithms to the base ten of the c.g.s. values. The dimensionless M/R column gives the $\log_{10}$ values in a system of units in which the unit mass is the mass of the hydrogen atom and the unit length is the radius of the first Bohr orbit. The column headed N gives the $\log_{10}$ of the numbers of atoms in a main sequence star, the number of stars in a galaxy, etc. These values, like all values in the table are order of magnitude values.

The values for stars and galaxies may be found in the published literature [e.g., Allen, Astrophysical Quantities]. The values of $\frac{M}{R}$ for clusters were derived using the virial theorem on the basis of published and unpublished redshifts. [ ] In the case of the coma cluster and four other clusters, the radii were determined from published [ ] and unpublished counts on 48" Schmidt plates. The value of M/R for second order clusters was synthesized from the observed dimensions of 2° order cluster cells, the numbers of constituent clusters, and the mass of individual clusters. The data are from Abell and de Vaucouleurs [ ].

The samples of the aggregates of each species whose values appear in the table were selected on the basis of possessing the largest M/R values in the attempt to determine whether an upper bound for M/R exists. No additional selectivity factors other than those which determined the choice of which binaries, which galaxies, and which clusters were observed in the first place affects the sample. Since observers tend to select objects on the basis of ease of observation, the biggest, brightest, and consequently most massive, objects are probably in the sample. We may conclude that it is a good sample to inspect for an upper bound. It is probably a poor sample for most other statistical purposes.

The salient feature of the table is that the independent upper bounds for M/R in the star sample, galaxy sample, and cluster sample, are nearly the same - all about $10^{23.5}$ grams/centimeter. The same is true for the second order clusters, but since M/R has not yet been directly measured for these aggregates, the determination is not independent. It thus appears that for all of these cosmic bodies there may exist some sort of <u>universal</u> <u>bound</u> governing the ratio of mass to size.

It is interesting to note the comparison of the value of this bound when expressed in dimensionless form, viz. $10^{38.9}$ with the values of some other basic dimensionless quantities of physics and cosmology.

As for example, $\log_{10} [\frac{e^2}{GM_{10} m_e}] = 39.356$, the ratio of coulomb to gravitational forces

$\log_{10} [\frac{C}{H r_e}] = 40.52$, the ratio of the "Hubble radius" to the electron radius.

↑ r_e

These numbers have a way of recurring, though their significance is not understood. Dirac [ ] surmised that all of the dimensionless numbers of the order of 10^{40} differed only by factors which are basic constants of the order of unity. Such as π, $\frac{\pi}{4}$, etc. Eddington felt these numbers which should include the above mass-radius ratio for non-degenerate cosmic bodies, reflect some basic - possibly number theoretic - property governing all macrocosmic and microcosmic structures; while Dirac held that these numbers reflect the age of the universe and their value changes with time.

Which of these viewpoints, if either, is correct remains to be seen, although the diverse origins of the same number lends support to the conjecture that the structure of cosmic bodies is related to the structure of atomic and sub-atomic bodies.

From the table we conclude

$$\frac{Mn}{Rn} \doteq 10^{39}$$

where N is a subscript referring to stars, galaxies, clusters, etc., The units are a_o for length and M_h for mass. In these same units, a_o, M_h, and a unit of time equal to the time τ_o for an electron to traverse 1 radian in the first Bohr orbit, $c = \alpha^{-1}$, where α is the fine structure constant and $G = 10^{-39.356}$, whence

$$(1) \quad \frac{GM_n}{C^2 R_n} \doteq \alpha^2 \cdot 10^{0.3} \doteq \alpha^2$$

The use of α^2 in the right member is to designate the order of magnitude. There may exist other factors of the order of unity. Equation (1) simply states that for all non-degenerate cosmic bodies the ratio of the gravitational energy $\frac{GM_n^2}{R_n}$ to the total energy C^2 is of the order of magnitude of the ratio of the range of nuclear forces to the range of coulomb forces. Or, said another way, the ratio of the gravitational radius to the linear radius is the same as the ratio of the radius of the electron to the Bohr radius, viz. $r_e/a_o = \alpha^2$.

Equation (1) may also be interpreted in terms of circular velocities or excape velocities. (A factor of two is unresolved.) GM/R is proportional to v_c^2. Hence for each cosmic body a circular velocity $v_c/C = \alpha$ is defined. The quantity, αc, is, of course, the velocity of the electron in the first Bohr orbit.

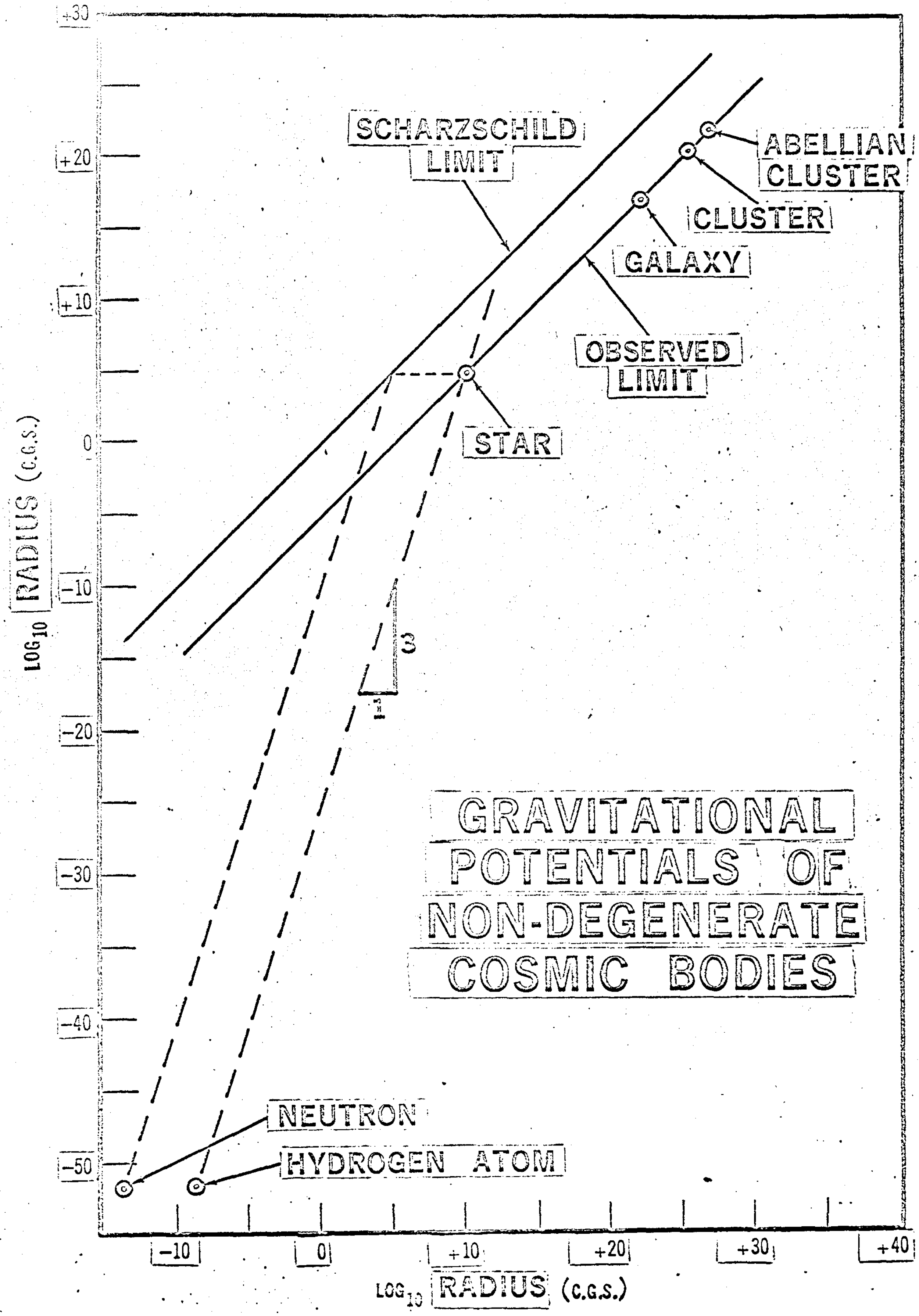
The underlying question as to why there should be a bound on the ratio of the gravitational radius to the linear radius has already been answered by relativity theory. Schwarzschild's exact solution of the Einstein field equations [ ] leads not only to the three classic tests of general relativity but on equating the inner and outer solutions on the boundary leads to the inequality.

$$(2) \qquad \frac{GM}{C^2 R} < \frac{1}{2}$$

It is thus not surprizing that we observe an upper bound to GM_n/c^2R_n in Table 1. The difference between equations (1) and (2), observed and theoretic, lies in the matter of degeneracy or stability. The observations were of non-degenerate systems. The theory applies to all spherical systems with static line element and perfect fluid approximation.

SUMMARY OF OBSERVATIONS

SYSTEM	R	M	M/R	N	(M/R) DIMENSIONLESS
Hydrogen Atom	-8.27640	-23.77642	-15.50002		
Stars					
V444 Cyg A	11.185	34.457	23.272	57.8	38.8
40 Ecl. Bin.	11.541	34.205	22.664		
Sun	10.843	33.299	22.456		
Galaxies					
M87	22.3	45.9	23.6	11.7	39.1
M31	22.2	44.8	22.6		
7 Galaxies	--	--	22.6		
Milky Way	22.26	44.30	22.04		
Clusters					
Coma	25.95	49.40	23.45	2.9	39.0
7 Clusters	--	--	22.59		
4 Clusters	25.54	48.08	22.54		
Second-Order Clusters					
Abellian Cell	26.0	49.2	23.2	1.0	38.7
Local Super-cluster	25.7	--	--		



GRAVITATIONAL POTENTIALS OF NON-DEGENERATE COSMIC BODIES

The possibility of the good life for any man depends on the possibility of realizing it for all men. And this is a function of society's ability to turn the energies of the universe to human advantage.

The problem of a comprehensive design science is to isolate specific instances of the behavior pattern of a general cosmic energy system, and to turn these to human use.

R. Buckminster Fuller

SIU

April 13, 1966.

Some Principles of Hierarchal Structure

A common feature to a large class of structures and organizations, both natural and arSificial, is hierarchal form. Whether the hierarchy is that of the cosmic environment: stars, galaxies, clusters of galaxies; or the social environment: neighborhoods, cities, metropolitan areas; there appear to be similar basic principles which define and limit each aggregate and lead to hierarchization. In order to approach the formulation of these principles it is instructive to inter compare the observables and descriptors of several types of hierarchies. Recent developments in astronomy hold some possible clues to the basic parameters which ~~giuxx~~ govern the sizes and energies of aggbegates.

draft: 3.27.66 Awilson

Lecture SIU - April 13, 1966

From time to time astronomers, who usually ~~have~~ are facing the stars with their backs turned on the human environment, turn around and point out something interesting that they have observed. They like to share their findings, but they recognize that they are generally regarded as being very much on the fringes of the practical world. However, one time when ~~the~~ astronomers turned around and muttered about their discoveries, they started a chain reaction in the world of practical affairs. The studies of the motions of planets by Tycho and Kepler led to the discovery of the laws of dynamics which led to the development of the science of mechanics, which in turn allowed the development ~~into~~ of scientific engineering, and finally, in the Eighteenth century launched the industrial revolution. This was followed by subsequent technological revolutions which have been sweeping us along ever since.

More recently, an astronomer turned around and muttered something about energy sources in stars and started a chain reaction in the minds of physicists which led to a different sort of chain reaction and started another revolution.

Today, with the advent of the so-called space age, I note with trepidation that astronomers have again turned around and are ~~talking~~ muttering more than ever before - ~~talking~~ in government committees, NASA staff meetings, Air Force planning groups - even discussing structures at SIU. The world has all the revolutions it needs right now. My advice is to get the astronomers back to their telescopes. ~~But whenever if you give them a chance to talk, they will.~~ Since there is a shortage of telescopes - this might be our most effective argument for obtaining additional funding.

SOME PRINCIPLES OF HIERARCHAL STRUCTURE

This evening, I would like to discuss one of the most universal phenomena concerning structure with which we are acquainted - the phonomena of the hierarchal ordering of aggregates. We encounter hierarchal structure ubiquitously in our internal and external environments. It is basic to our thought patterns and to our classification systems. We organize ourselves hierarchically in our social order, in government, in the military, in corporations. We observe the ~~wide-spread~~ existence of hierarchal structure in the biological world. We observe hierarchal structure in inanimate matter. But In spite of the ubiquity of this phenomena, at the present time we have no comprehensive explanations as to why nature, including ~~we~~ man, ~~ourselves~~, organizes in a hierarchal manner.

Perhaps there is no single principle or meta-principle underlying and causing hierarchal organization. There may be as many reasons for it as there are hierarchies. But whenever diverse agencies employ the same technique, there must be something of value in that technique. We may, accordingly, reasonably inquire what common value giving features ~~giving value is it possible to~~ may be abstracted from different hierarchies. Whether hierarchies have a common cause or merely share certain common features is a metaphysical question. Our present concern is not to explore that question, but merely attempt to identify similarities and differences in hierarchies whenever possible. Not initially to seek explanations, but rather, to observe any relationships and patterns in structure that may be evident in ~~the~~ whatever data ~~which~~ is well established and generally available.

This may prove to be a very important quest, especially in our times when the complexities of life are increasing, and available space is decreasing. Every possible economy, every possible bit of guidance, which can be ascertained may be basic to our survival tomorrow. It will pay us to explore whatever organizational principles exist in the universe, be they informational, physical, psychological, social, or whatever.

Before I go any further in a discussion of hierarchal structure, I had best define how I shall be using the term. A hierarchal structure is a structure which consists of a set of aggregates, the elements of each aggregate being themselves aggregates, whose constituent elements are in turn aggregates, etc. This sequence may or may not terminate on either the small scale end, or the large scale end.

Because the study of inanimate matter has proven far simpler than the study of bio-organisms or social organisms, the easiest place to begin is perhaps with material aggregates. It has been recognized for over two centuries that the cosmos might be constructed along hierarchal lines. The first surmise in this connection was purely speculative and was proposed by the Swedish philosopher, Swedenborg. In 1750, the Alsatian physicist, Lambert, hypothesized that the universe was constructed hierarchically. He was impressed with the fact that the newly invented telescopes had revealed satellite systems for the planets Jupiter and Saturn which resembled miniature solar systems. Lambert pursued the analogy between the orbiting satellites around Jupiter and Saturn and the orbiting planets around the sun. He speculated that perhaps the sun, itself, could be a satellite revolving about some distant center in the universe in a planetary-like orbit. (He did not, of course, know ~~realize~~ that the sun ~~indeed~~ moves in a planetary-like orbit about a galactic center thirty thousand light years distant in the direction of the constellation of Sagittarius. All of this was to be discovered later.) Lambert extended his speculations postulating an entire hierarchy where the center about which the sun moved in a planetary orbit itself moved about some even more remote center, etc., etc. Subsequent developments in astronomy have shown that the universe, even though not constructed along the lines imagined by Lambert, was indeed hierarchal.

In 1826 a German physician named Olbers became interested in the question of the extent of the universe of stars. Through a simple calculation he showed that if the universe were composed of stars like the sun, uniformly distributed, and if there were an infinite number of them ~~in extent~~, that the brightness of the sky should be as bright as the sun everywhere. But since the sky is dark at night, possibly the universe was not infinite. Olbers preferred to hold to the infinity of the universe and assume there was some other cause for the darkness. He postulated there to be some intervening cosmic dust which cut off the light from distant stars. The urge to preserve the infinitude of the universe led other astronomers to seek causes for what had come to be called Olbers' paradox. Early in this century a Swedish mathematician, C. V. Charlier, proposed a solution in which he showed that if the universe, instead of being composed of an infinite distribution of stars, were hierarchically structured, these stars being grouped into galaxies, and these in turn grouped into super aggregations, etc., that we could have any brightness of the night sky and yet have an infinite number of stars in the universe. Shortly after the work of Charlier, the general theory of relativity was introduced which provided alternative solutions to Olbers' paradox.

Relatavistic models of the universe were based on assumptions of uniform density and hierarchal structure has not so far been used as a base of relatavistic cosmological considerations. However, The recent establishment of the existence of second order clusters of galaxies by Abell requires that hierarchization be taken into account in all realistic models.[9] Professor Shapley, the Emeritus Director of the Harvard Observatory, has long been intrigued with the hierarchization of matter and has written two books in which he describes this interesting phenomenon. [1] The first slide summarizes Shapley's classification of the material systems found in the universe. Shapley has assigned an index designating the order or rank of an aggregate in the hierarchy. He gives the fundamental particles composing the atoms an index of -4, the atoms -3. Next come the molecular systems, including crystals and colloidal systems; then meteoritic associations, built up from molecular systems; satellite systems; stars; star clusters; galaxies; clusters of galaxies; the metagalaxy; and the universe: each level being an aggregate or set whose elements are in turn the aggregates of order one less. This classification shows us that in the scale interval of the universe with which we are familiar, the scale-wise structure is definitely hierarchal. We have no reason to assume that the largest aggregate that we now know is the largest which exists (saving the term universe for the last). Although arguments from analogy are often persuasive, arguments based solely on analogy cannot definitively establish whether the hierarchy continues to larger and larger aggregates, and there may be no way to establish whether or not the universe is hierarchal ad infinitum. Shapley's table illustrates the different known aggregates of matter in order of size and mass. It is not proper to assume that all aggregates listed be given equal weight in this hierarchy. Later we shall see that there are basic aggregates which we may call primary and the others must be regarded as satellitic aggregates.

1 off

CLASSIFICATION OF MATERIAL SYSTEMS

(AFTER SHAPLEY)

-5 Quarks

-4 Corpuscles (Fundamental Particles)

-3 Atoms

-2 Molecules

-1 Molecular Systems

+1 Meteoritic Associations

+2 Satellitic Systems

+3 Stars and Star Families

+4 Stellar Clusters

+5 Galaxies

+6 Galaxy Aggregations

+7 The Metagalaxy

+8 The Universe: Space-Time Complex

+9 ....

Four basic questions arise. First, why is matter organized in a hierarchal manner. Second, why do the particular aggregates having the masses and sizes which they have occur in nature and why not other aggregates with different masses and radii. Or, specifically, why does a star or a galaxy have the mass and radius it has? Third, since we do not encounter other bodies in unlimited assortments, we may ask do other bodies exist, but have escaped observation, or is the hierarchal structure truly discrete and completely represented by known aggregates. And the fourth question, how far does the hierarchal structure extend both down in scale and up in scale. Is it open ended or does it terminate.

In looking at the cosmic portion of total interval of observable levels in the hierarchy, we find two advantages. First, there exists a descriptor which may serve as the basis for comparisons which may be readily deduced for all aggregates in the hierarchy from simple accurate observations; and second, we have approximate spherical symmetry in all aggregates. The relations may be expected to be simpler and depend on fewer parameters in the case of the cosmic aggregates than in the case of terrestrial aggregates.

The basic descriptor available to us for comparisons of cosmic bodies is the simple ratio of the mass (M) to the radius (R) of the aggregate. Its evaluation depends in each case on Kepler's Third Law, but in each case on an independent technique. Kepler's Third Law is one of the most powerful tools available to the astronomer.

$$P^2 = \frac{4\pi^2 a^3}{G(M_1+M_2)} \qquad \text{or} \qquad \frac{G(M_1+M_2)}{v_0^2\, a} = 1$$

This law, first discovered by Kepler, and later modified by Newton, allows the astronomer to measure the masses of interacting heavenly bodies provided he knows their distances of separation and either their velocities or periods of rotation about one another. Usually, if we are to determine some explicit property of a celestial body such as the mass or the linear size, we have to know the distance to the body. But a useful thing about the ratio of mass divided by radius, it may be determined without having to know the distance to the object under study. This is a tremendous advantage because of the difficulties and uncertainties in determining distances to celestial bodies, especially the more remote ones whose distances can only be determined through iterated calibrations and compounding of several methods of distance determination. The observations required for determination of M/R are straight forward being mostly observations of ~~spectral~~ radial velocities, angular dimensions, and light variations. The observations of radial velocities which are determined from the doppler shift of spectral lines can be made with as high precision as any observations in astronomy.[4] The ratio of mass to the linear radius is determined in different ways using different techniques for each of the four aggregates which are available to observation. For stars the ratio of the mass to the radius may be determined in the case of a type of star known as an eclipsing variable or eclipsing binary. These are a pair of stars orbiting about one another in a plane which happens to pass through the earth. In this case, we see the stars eclipsing one another. Aside from the sun our knowledge of accurate masses and radii of stars are limited to those of eclipsing binary stars. I will not go into the details of the determination, other than to say it is an observation involving the period, light curve and the spectral orbit of the stars. ~~Now~~ For galaxies, $\frac{M}{R}$ may be derived in at least two ways and with less certainty, in a third way. One basic way of determining the ratio of the mass to the radius ~~are~~ is to observe the spectra of the rotating galaxy placing the slit along the equator and measuring the inclination of the spectral lines. This angle of inclination together with the angular radius and the linear value of the Doppler velocity, allow us to determine the ratio of $\frac{M}{R}$. Again, no knowledge of the distance is required. To determine the $\frac{M}{R}$ ratio for a cluster of galaxies we employ what is known in mechanics as the virial theorem which gives the value of $\frac{GM}{R}$ in terms of the dispersion of the velocities of the members of the cluster. Since velocities can be determined from the redshifts which are directly observable, it is possible without any assumptions whatsoever concerning the distance to the cluster, to evaluate the $\frac{M}{R}$ for the cluster directly. In the case of the second order clusters the same technique can be used but also the mass and radius can be extrapolated from counts of the number of clusters in the second order cluster, from masses of clusters, and observed angular dimensions converted to linear dimensions by redshifts. The extrapolation method, however, is not independent.

We thus have three independent types of astronomical observations for the three species of aggregates, stars, galaxies, and clusters which allow us to determine the ratio of $\frac{M}{R}$ for each species directly from observation. I wish to emphasize again that the methods, ~~though all based on Kepler's law,~~ are independent, are based on different observables, and involve essentially no theoretical assumptions beyond Kepler's Law.

When one compares the values of the mass to radius ratio for the different aggregates, a very interesting coincidence is observed. On the basis of the sample of all available eclipsing binaries (and the sun) we find that the maximum value assumed by the ratio $\frac{M}{R}$ in metric units of grams per centimeter is $10^{23.3}$. For the available sample of galaxies whose mass to radius ratio has been determined, we find that the maximum value which occurs is $10^{23.6}$ grams per centimeter. The mass to radius ratio determined by the virial theorem for all of the clusters of galaxies for which sufficient data is available again gives $\frac{M}{R}$ equal to $10^{23.5}$. The super clusters of galaxies can be studied by taking the values for clusters of galaxies and multiplying by the number of clusters in the super cluster and using the proper mass for a cluster and the observed radius for the super cluster. Again we come up with $10^{23.2}$ for the $\frac{M}{R}$ ratio.

We thus find the rather startling result that the maximum ratio of mass to radius for every species of non-degenerate cosmic aggregate that we know has the same value in grams per centimeter, namely, $10^{23.5}$.

The fact that this ratio is bounded is not completely unexpected. The German astronomer Schwarzschild in 1916 obtained an exact solution of the Einstein field equations of general relativity under certain assumptions, including spherical symmetry. The Schwarzschild solution led to the three famous predictions of the general theory of relativity. These predictions consisted of 1) the advance in the perihelion of the planet Mercury, that is the prediction that the major axis of the planet's orbit rotates in space in a manner different from that predicted by classical Newtonian theory. The second prediction was that a ray of light passing near a massive body, like the sun, would be deflected. This may be tested by making observations of the star field surrounding the sun during a total eclipse and comparing the same star field photographed in the night sky six months later. The third prediction was the so-called Einstein or gravitational redshift. The frequency with which an atom radiates is different when in a strong gravitational field than when in a weak field so that a spectral line coming from an atom on the sun would be shifted in frequency with respect to one originating in a laboratory on the earth. These three effects have been observed. But in addition to these three classical predictions of general relativity the Schwarzschild exact solution makes a fourth prediction. This is the prediction that the quantity

$$\frac{GM}{c^2R} < \frac{1}{2}$$

Here we have multiplied the ratio of M/R by two universal constants: G, the universal gravitational coupling constant, and c, the velocity of light. The resulting product is dimensionless. {energy ratio $\frac{GM^2}{R} / c^2 M$ or grav. rad / Newton rad}

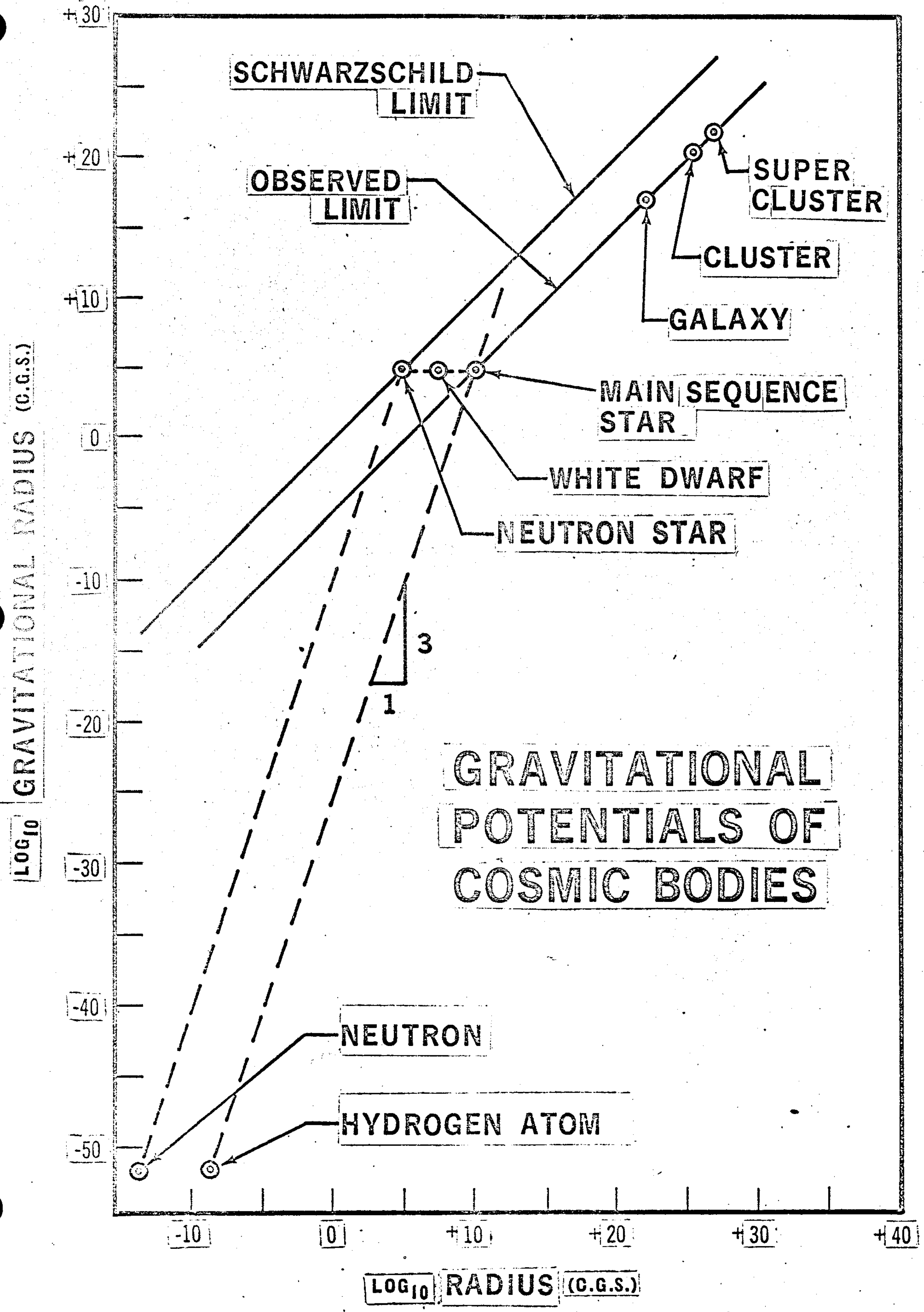
There are several ways of interpreting the Schwarzschild limit. Without going through the details of the derivation, we may see that the limit is an immediate consequence of classical Newtonian theory and the relativistic assumption that there exists a limiting velocity for material objects, namely the velocity of light. Classical Newtonian mechanics leads to a formula

$$\sqrt{\frac{2GM}{R}} = V_x$$

where V_x equals the velocity of escape. In the case of the earth, substituting the mass of the earth and the radius of the earth in this equation we find the velocity of escape is about 11 kilometers per second. For the sun the value is about 620 kilometers per second. For the Moon it is in the neighborhood of 2 kilometers per second. What the Schwarzschild limit implies is that no aggregate in the universe can assume values of $\frac{M}{R}$ which make the velocity of escape greater than the velocity of light. Another way of looking at the escape velocity is, if a particle is released at a very large distance from a body and allowed to fall freely it will accelerate until the speed with which the falling body strikes the surface of the planet is equal to the escape velocity. Consequently the Schwarzschild limit states that any body which is accelerated only by gravity has a limiting velocity of c.

Recapitulating, we have from the general theory of relativity that the ratio of $\frac{M}{R}$ is bounded and the dimensionless quantity $\frac{GM}{c^2R}$ has the bound of one-half. Observations show that $\frac{M}{R}$ is indeed bounded but that the quantity $\frac{GM}{c^2R}$ for the all observed nondegenerate systems has the bound, not of one-half, but of a quantity which has the value of about $10^{-4.3}$ (from $\frac{G}{c^2} \times 10^{+23.9}$). Why this discrepancy? It is here that the matter of degeneracy comes in. If we assume a model in which hydrogen atoms are spheres whose radii are of the order of 10^{-8} centimeters and if these spheres are packed solidly as one would pack cannon balls or marbles, a large aggregate of hydrogen molecules can be assembled. The question is, assuming an aggregating principle-like gravity which assembles atoms until a large mass has been built up, how big may the mass be? The slide shows us what will happen.
[2] Across the bottom of the slide is plotted the logarithm of the radius of the aggregate, cosmic or atomic, in centimeters; vertically is plotted the quantity $\frac{GM}{c^2}$ which is called the gravitational radius. [80×50 cycle log-log plot] Multiplying the mass by the fundamental constants $\frac{G}{c^2}$ converts the dimension mass into the dimension length hence the name mass radius or gravitational radius. We are thus able to compare masses and lengths and the ratio GM/c^2R becomes dimensionless. The mass of close packed hydrogen atoms under consideration would grow up along the dotted line passing through the hydrogen atom and having a slope of 3 to 1. This is a line of constant density. Growth could continue until encounter with the Schwarzschild limit. Growth is not possible beyond that. No physical body can be any larger than that determined by this limit. However, if it is the second or $10^{-4.3}$ observed limit which really governs any aggregate of closely packed

2


SCHWARZSCHILD LIMIT
OBSERVED LIMIT
SUPER CLUSTER
CLUSTER
GALAXY
MAIN SEQUENCE STAR
WHITE DWARF
NEUTRON STAR
3
1
GRAVITATIONAL POTENTIALS OF COSMIC BODIES
NEUTRON
HYDROGEN ATOM
+30
+20
+10
0
-10
-20
-30
-40
-50
-10
0
+10
+20
+30
+40
LOG10 GRAVITATIONAL RADIUS (C.G.S.)
LOG10 RADIUS (C.G.S.)

hydrogen atoms, then the mass could become no larger than determined by the intersection of the constant density hydrogen line with the observed limit. It is precisely at this intersection that we observe the aggregate we call stars. The mass determined by the intersection of the two lines has a value of 10^{33} to 10^{34} grams. This is the observed mass for ordinary stars. Hence, we have here a partial answer to one of our three basic questions, why stars have the masses which they are observed to have. Alternatively, observed stellar mass may derive from the intersection of a constant density line for close-packed nuclear particles with the Schwarzschild limit. If instead of taking hydrogen atoms we take neutrons or nuclei of hydrogen atoms and pack them closely we find the same cutoff mass, 10^{34} grams, from closely packed neutrons being cutoff at the Schwarzschild limit. This notion follows conditions hypothesized for the initial state of an expanding or "big bang" cosmology. Thus there exists a parallel between atomic size with the observed limit for non-degenerate cosmic aggregates ~~in the universe~~ and neutron size with the theoretical relatavistic Schwarzschild limit. In fact the $10^{-4.3}$ bound is very closely equal to the ratio of the size of the nucleus to the first Bohr radius of the atom. We are here apparently encountering a manifestation of atomic ~~dimensions and~~ ratios on a cosmic scale.

The use of close packed models, though quantitatively consistent for the features we are discussing, is not a correct model for a star. We have ignored the thermal energies which are of the same order of magnitude as the gravitational energies. But to move a stellar point down by 0.30 (log 2) on our log-log diagram is to move it through the diameter of the dot representing the star. So for our present discussion we may ignore the details of stellar structure and consider a star to be a close-packed aggregate of hydrogen atoms.

This is a rather exciting parallel. For several decades cosmologists have suspected that there exist~~s~~ ~~a~~ relationships between the properties of structures ~~which are observed~~ on the cosmic scale and the basic properties of the atom, which is the fundamental building block of all larger aggregates. Over thirty years ago, ~~In the twenties,~~ Eddington pointed out the identity between basic dimensionless numbers associated with the properties of the atom~~s~~, and basic dimensionless numbers associated with the cosmos. For example, $e^2/Gm_pm_e = 10^{39}$ and $cH/r_e = 10^{39}$ where $r_e = e^2/m_ec^2$ is the radius of the electron. These numerical identities have been regarded by many physicists as merely coincidences. Yet, when dealing with numbers of the order of magnitude of 10^{+39}, it is a little difficult to account for two such numbers coming from two spins of a wheel of chance - unless there are only a very few numbers on the wheel and that would be an even more remarkable situation. But now we have further evidence in slide 3 for the existence of basic relationships between atomic and cosmic structure. Another of the dimensionless numbers considered by Eddington is the Sommerfeld-Fine structure constant (α) which was first discovered in atomic spectra. (The reciprocal of this number has a value of about 137.) The ratio of the size of the first Bohr orbit in the hydrogen atom to the electron radius is equal to the square of this number = $10^{+4.27}$ and it is quite possible that this is the reciprocal ~~same number~~ of our $10^{-4.3}$. If so, we may write $GM/c^2R \leq \alpha^2$.

Now let us return to the concept of degeneracy. Whenever the spheres of hydrogen atoms become more closely packed than their unperturbed radii permit or whenever the electrons present do not have a suitable number of states to occupy, a condition of matter which we call degeneracy arises. It is like having to stack cannon balls together in a space which is so small that the cannon balls would have to intersect each other in order to be squeezed into the space. This, or course, creates very high densities of matter, much larger than any occurring in normal solid state. We actually do observe bodies in the universe which do have these high densities and manifest the property of degeneracy. These are stars which are known as white dwarf stars. They are located on the diagram between normal stars and neutron stars. They have values of $\frac{M}{R}$ which are greater than the values observed for the nondegenerate aggregates, the main sequence stars, galaxies, etc., but less than the Schwarzschild limit. We are led to ~~conclude~~ surmise that the smaller $10^{-4.3}$ limit applies to nondegenerate aggregates of matter, while the Schwarzschild limit applies to degenerate aggregates and is the ultimate limit. In other words, if one regards ~~the~~ neutrons as the ultimate cannon balls, they cannot be packed more closely than their radii allow, nor in ~~masses~~ aggregates greater than the Schwarzschild limit will allow. ~~We thus have the limiting properties of space are related to the properties of matter through the density and energy limits shown on the diagram.~~ Furthermore, whereas the Schwarzschild limit corresponds to the velocity of light, the $10^{-4.3}$ observed limit corresponds to a velocity which is equal to the velocity of light divided by the Eddington number, 137. This has a value of around 2,200 kilometers per second. Hence the maximum escape velocity from any nondegenerate star, galaxy, cluster or super cluster, is the same and equals about 2,200 kilometers per second. This is the fastest that one would expect to find any material body in the universe being accelerated by the ~~gravity~~ gravitational field of a non-degenerate body. It is interesting to note that the circular velocity corresponding to this value of αc is exactly the velocity with which an electron moves in the first Bohr orbit. Another parallel between the atomic and cosmic structure.

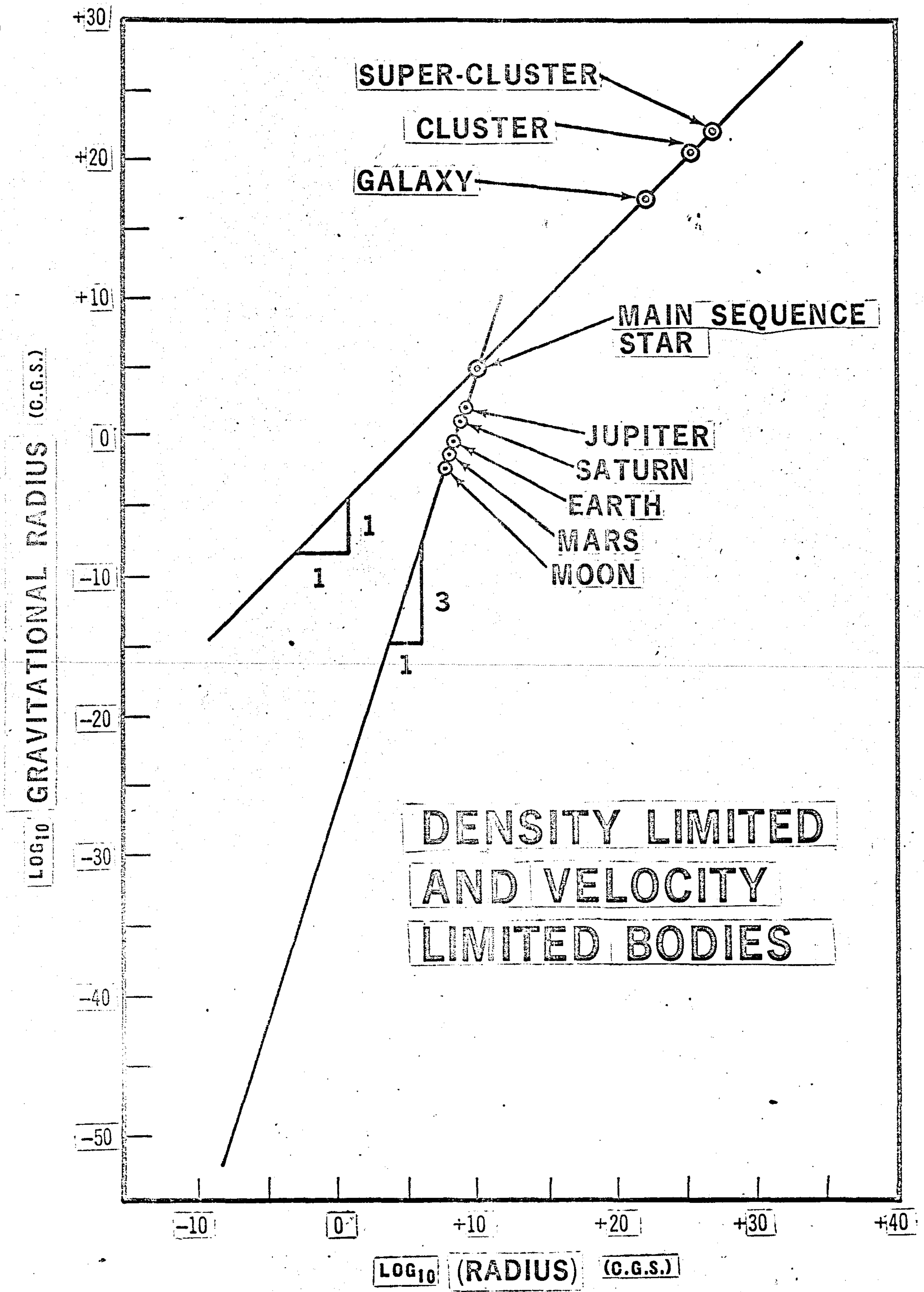
On the basis of the limits shown in the diagram can we get any clues toward ~~an~~ some answer to the question why hierarchization occurs. ~~Considering further, the answer is, yes.~~

3

The third slide has the same axes as the second. The 1:1 slope line is the observed limit for all nondegenerate bodies and the 3:1 slope line is the observed distribution for non-degenerate solid bodies with densities the order of hydrogen atom density. These solid bodies, the planets Jupiter, Saturn, etc., do not all have the same density, but on the scale of this diagram they are approximately the same. The densities range from about one to six. and ~~Hence, these bodies~~ are all on essentially the same constant density line. We see that there are two types of limiting aggregates in the universe: those falling along the constant density line - planets and stars - these are density limited and are interpretable on a model of atoms close packed in volume. The second type of body, those which lie along the observed $10^{-4.3}$ limit with the slope one to one, are velocity limited. We have seen that the escape velocity for all of these objects is identical and of the order of 2100 kilometers per second. In the velocity limited bodies there is freedom of motion among the elements of the aggregate. On the other hand there is essentially no motion in the lithospheres of planets, and only fluid motion in the atmospheres. We can accordingly think of these two classes of bodies as (1) static bodies - those which are density limited, and (2) dynamic aggregates - those which are velocity limited.

There is yet another relationship governing the velocity limited bodies. That is this. Since M divided by R is the same for all these bodies and the mass of, say, a galaxy is equal to the mass of a star times the number of stars in the galaxy, it follows that the radius of a galaxy is equal to the number of stars in the galaxy times the radius of the star. The same is true for clusters, etc. In other words, instead of being close packed in volume, the objects which lie along the velocity limit line are <u>linearly close packed.</u> The diameter of any aggregate is equal to the diameter of the element composing that aggregate, times

3


SUPER-CLUSTER
CLUSTER
GALAXY
MAIN SEQUENCE STAR
JUPITER
SATURN
EARTH
MARS
MOON
1
1
3
1
DENSITY LIMITED
AND VELOCITY
LIMITED BODIES
+30
+20
+10
0
-10
-20
-30
-40
-50
-10
0
+10
+20
+30
+40
LOG_{10} GRAVITATIONAL RADIUS (C.G.S.)
LOG_{10} (RADIUS) (C.G.S.)

the number of elements in the aggregate. We thus have two types of packing - the velocity limited bodies are linearly packed, or 1-packed even though they are three dimensional bodies and occupy three dimensional space, the diameter assumed is just the linear extension of the particles making up the body. The solid bodies may be made by volume-close-packing of elemental spheres, or 3-pack either hydrogen atoms for nondegenerate, or neutrons for degenerate objects.

Several aggregates are known to exist which lie below the $10^{-4.3}$ observed limit for velocity limited bodies. These bodies are less than linearly packed. That is, the diameter of the aggregate exceeds the linear extension of the constituent particles. It thus appears that in nature dynamic aggregates are never pressed into a volume any smaller than one whose diameter is defined by linear packing. This observation may be of extreme importance in the design of all dynamic systems required to be collision free, which is essentially true of cosmic aggregates.

In slide 3, we can represent gravity by a vector force field which causes all bodies in the lower right part of the diagram to contract, i.e., to move to the left; and possibly to grow in mass accretively, i.e., to move upward. Motion will continue until one or the other limits - the density limit or the velocity limit is reached.

If the density limit is reached, the object may continue to grow in mass under gravitation, but will also have to increase in size. Mass and size may increase until the velocity limit is reached. Here in the corner made by the intersection of the two limits, we encounter a stable position. This corner is occupied by the stars.

Further growth along the density limit is impossible. To build a larger aggregate growth must ~~continue~~ proceed along the velocity limit. The aggregating force of gravity here effects a growth in linear size proportional to the growth in mass. This means that each addition of a unit of mass demands an increase in volume proportional to the square of the number of particles already present. Any cosmic body accreting along the velocity limit will therefore have to expand.

Growth along the velocity limit in effect amounts to an adjustment of the body to a density distribution which is such that the density at distance r from the center is proportional to r^{-2}. A body which may be stable under maximum constant density, when reaching this limit must expand and adjust to a r^{-2} density distribution.

Growth may not proceed smoothly up the velocity limit. Expansions will take bodies to the right of the limit. Such bodies, considered now as elemental particles, may accrete along a constant density line until the velocity limit is again reached. This process may be repeated. We can qualitatively account for hierarchization by speculating that this is how the two limits interact with gravity and build up higher order aggregates. The argument is quantitatively consistent as far as stars are concerned. Beyond the stars there remain many uncertainties and there is no clue as to what positions, if any, are stable.

A non-accreting body on the velocity limit, will under gravitational forces tend to contract. But in order to contract, it must lose [illegible] mass is expelled, in effect the size of the [illegible] increases. An object placed under such conflicting [illegible] will develop schizophrenia. It will explode. [illegible] of various types are frequently observed. These [illegible] to attempts to adjust to the velocity limit. The core drops below the line, the exploded shell moves to the right. Further contraction may ensue and the process repeated until a point of stability is reached. [3 off]

All of this suggests a general theorem underlying hierarchal structure.

1. If there exists an aggregating principle (such as gravity).
2. If there exists a maximum limiting density (slope 3).
3. If there exists a velocity (or energy) bound (e.g. $\frac{\frac{GM^2}{R}}{C^2M}$), (with slope < 3).

then, Matter will be (a) hierarchically structured, or (b) adjusting itself so as to be distributed in accord with the density distribution demanded by the energy bound. ~~which in general requires expansion~~.

Before this theorem can be completed, something further about stable positions along the velocity limit must be determined.

Recapitulating, In studying cosmic aggregates we have identified two types of limits which govern cosmic structure: a density limit, and a velocity limit. Where these two limits intersect a very basic and universal event occurs, namely the stars. We have further seen that the existence of these limits, together with an aggregating principle such as the law of gravity, can lead to hierarchic structure. We have not, however, been able to show why aggregates other than the basic aggregate of the star occur in nature. It seems as though some supplementary relationship such as a ~~universal cellerization of~~ "cell-nucleus" property for all aggregates needs to be postulated before we can reconstruct completely the observed hierarchal distributions.

Let us now turn from cosmic aggregates of the one-pack and three-pack varieties, keeping in mind what we have ~~learned~~ found, and consider ~~other~~ another types of aggregates ~~and hierarchies~~ composed of a different type of element. Let us consider, ~~for example~~ a typical human social organizations, ~~such as~~ the city (which has recently based its activities about the six-pack). Is it possible to detect anything in the structure and the behavior of humans and of the city which is similar to the two limits detected in the cosmos? We are certainly aware of one limit - the limit of maximum density. Human beings cannot live together in too compact a state. There is a certain minimum number of square feet required for life to be possible, even in a concentration camp or prison. ~~Precisely what~~ The precise value of ~~this basic~~ the minimum area required to sustain human life ~~is~~ may be hard to ~~isolate~~ measure, ~~and~~ It ~~may~~ probably depends upon several factors and is different for different cultures and levels of technology.

But we can assume that parallel to the density limit which exists for inanimate particles, there does exist a maximum density limit applying to human beings.

Are we able to detect any limit which parallels the velocity limit in the cosmic structure? The answer here is yes. There is definitely such a limit governing urban structure and this is the limit of the maximum acceptable commuting time. Finally, analogous to the aggregating principle which is at work in the cosmos, namely gravitation, there exists an aggregating principle among human beings. This is their natural gregariousness, their inclination to come together, for physical security, economic security, or emotional security. Since we have for a human aggregation like the city the three essential ingredients of the two types of limits and the aggregating principle, we might expect that an inequality similar to the ones discovered for cosmic aggregates may also exist.

> Let $\hat{\sigma}$ be the maximum possible density, and $\hat{T}$ be the maximum acceptable commuting time.
>
> A characteristic limiting velocity analogous to c exists within a city, call this v_c. This depends on the state of the art.
>
> $v_c\hat{T}$ defines a maximum length $\hat{R}$. The radius of the city R_c, must be less than $\hat{R}$.
>
> N, the population of the city, $= \pi R_c^2 \bar{\sigma}_c$, where $\bar{\sigma}_c$ is the <u>mean</u> density. Since $R_c < \hat{R}$ and $\bar{\sigma}_c < \hat{\sigma}$, we have
>
> $$N = \pi R_c^2 \bar{\sigma}_c < \pi \hat{R}^2 \hat{\sigma} = \pi v_c^2 \hat{T}^2 \hat{\sigma}$$
>
> Hence $\frac{N}{v_c^2} < \pi \hat{T}^2 \hat{\sigma}$ a bound, since the right member is bounded.

We thus see from these equations that there is a marked similarity between a human aggregation which is necessarily dynamic, and dynamic cosmic aggregations. Except for the fact that the city is two dimensional and the cosmic bodies are three dimensional, the equations are parallel in every sense.

If $M_c = N\bar{H}$, a height to make the city three dimensional, and $\hat{\rho}$ = volume density

$$\frac{M_c}{v_c^2 \bar{H}} < B = \pi \hat{\rho} \hat{T}^2, \quad [\hat{\rho} T^2] = [\frac{M}{L^3} T^2]$$

compared with

$$\frac{M}{c^2 R} < B = \alpha^2/G, \quad [\alpha^2/G] = \left[\frac{MT^2}{L^3}\right]$$

Thus $\frac{1}{\hat{\rho} \hat{T}^2}$ is analogous to the gravitational coupling constant.

Finally, it is reasonable to conclude that because of the existence of the aggregating principle which operates in human affairs, and the existence of a density bound, and a velocity bound, that and equations similar to those governing cosmic aggregates ~~some event~~ ~~is~~ are likely to occur. I think we may carefully explore further other parallels ~~safely reason~~ by analogy, and say that ~~the event which occurs at the intersection of the density and velocity bounds are the cities.~~ the parallels which exist in these aggregates may give clues which will allow us to further understand all of them.

THE 48-INCH SCHMIDT FOR LUNAR OBSERVATIONS

Photographing the moon against star field backgrounds is recognized as a most useful technique for measuring the difference between universal and ephemeris times, determining various higher order motions of the earth and moon, and investigating certain geodesic problems. Because simultaneous photographic observations of the moon and stars might be effectively used with new reduction techniques developed by Z. Kopal, for determining lunar librations from star positions independent of reference to the lunar limb, he recently requested that we investigate the feasibility of utilizing the 48-inch Palomar Schmidt as a moon-star camera. The Schmidt is rarely in use when the moon is above the horizon and good supplementary use could be made of the instrument if it should prove adaptable for lunar work.

The problem of photographing the moon against its star field background is complicated by the motion of the moon, the large difference in brightness between moon and stars, and the brightness of the sky in the neighborhood of the moon. It is also necessary that the moon and stars be photographed so that the position of the telescope when the moon is exposed be the same as the average position for the stars in order to minimize refraction and off-axis effects.

Mailed to Kopal
June 3, 1966

A highly successful, but sophisticated, camera for making simultaneous photographs of the moon and a star field has been developed by Markowitz (Astron. J. Vol. 59, 1954, pp. 69-73; Telescopes, G. P. Kuiper and Barbara M. Middlehurst, Editors, 1962, U. of Chicago Press, p. 107). In this camera the differential motion of the moon is corrected by continuously changing the tilt of a plane parallel glass filter. An exposure of from about 10 to 25 seconds is made of the star field with the moon being tracked by the changing tilt filter. The filter drive is set so that the filter is parallel to the photographic plate at exactly mid-exposure. [At the instant the filter is parallel, the moon image is exposed,] being recorded in its correct position with respect to the stars. Moon is exposed all along; The instant referred to is the "epoch" of the exposure. 2

In considering the Schmidt for this problem, it was felt that since the f/2.5 speed allows the recording of 12th magnitude stars (under dark sky conditions) in about 5 seconds, that the moon motion problem could be surmounted by driving the telescope at lunar rate and making a very short exposure. The star images would be only slightly trailed being still quite useful as fiduciary marks. The Schmidt has no declination rate control but for the short exposure, this should introduce very little blurring of the moon's image.

The moon brightness problem was to be solved by introducing a neutral filter having a dense central circle sufficiently large to mask the lunar image. Mr. George Kocher prepared a set of experimental filters by exposing 5 x 7 ~~103a-0~~ KODALITH ORTHO plates in contact with a template having a suitable central hole. The scale of the Schmidt is only 69" arc/mm and marginal at best for recording lunar features. Only the sharpest of properly exposed images could be of use for libration measures.

The sky problem could be handled by photographing the star field through a suitable filter. However, since this extends the exposure time and the star images are trails, the usual gains from reducing sky brightness cannot be effected.

Yellow filter is about as good as anything for sky - yet does not cut down on most stars appreciably

Time was secured on the nights of May 31 and June 1 through the courtesy of the Director of the Mt. Wilson-Palomar Observatories to make a feasibility study on the full moon. The 5 x 7 plate holder with field flattening lens was employed. The chief limitation to the experiment was the Schmidt shutter which has an operating time of three seconds and a minimum effective exposure time of some 4 to 5 seconds. The mounting of an auxiliary shutter would constitute a major and expensive modification for which neither time nor money was available.

The results were highly unpromising. It was not possible to record the lunar surface and stars on the same plate. The intense brightness of the sky in the neighborhood of the moon, although the transparency was good, vitiated the attempt.

The conclusions of the experiment are that the Schmidt is not useful as a moon-camera without extensive and expensive modifications. While it may be possible to adapt the Schmidt for simultaneous moon-star field photography through diaphragms to change the f ratio, building a system of dual shutters, introducing suitable combinations of filters, and a declination rate drive, none of this seems worthwhile. Since telescopes built for one purpose are rarely adaptable to quite different purposes, I recommend development of instruments following the successful designs of Markowitz.

timings

Albert G. Wilson

Report Mailed to
H. W. Babcock
June 14, 1966
To appear in Mt. Wilson
Year Book for 1965 - 1966

A. G. Wilson has been using the B Spectrograph with the 100 inch to observe the redshifts of bright galaxies in nearby clusters. The purpose of the program is to study the spatial distribution of clusters and investigate suspected regularities in redshift distributions. The nearby clusters so far observed appear not to be randomly distributed. Mean redshifts of clusters beyond the local Virgo-Ursa Major complex and closer than $z = \delta\lambda/\lambda = 0.09$ appear to possess an unexplained regularity which is closely represented by the one parameter expression,

$$\log_{10} z = -\frac{5}{3} + \frac{n}{4} \log_{10} 2 \;,\; n = -1, 0, 1, 2, \ldots 9$$

For most of the clusters in this range the relative error, $\delta z/z$, of this formula is less than one percent.

More distant clusters appear to be non-uniformly distributed, their redshifts showing a non-statistical banded distribution. Comparison with Schmidt's redshifts of radio sources shows the existence of a similar banded distribution for the radio sources. These distributions may be indicative of the clustering of both clusters and radio sources on a larger scale than that of any presently recognized aggregate of matter.

DISTRIBUTIONS OF REDSHIFTS OF RADIO SOURCES AND RICH CLUSTERS

A. G. Wilson

ABSTRACT

The distance distributions of radio-sources and rich clusters of galaxies, as derived from their observed redshifts, suggests non-uniformities on a scale of the order of 10^8 parsecs. This may be due to the existence of larger aggregates of matter than any recognized at present. Differences in the distributions of radio sources and cluster redshifts may be consistently interpreted as arising from an Einstein component in the observed redshifts, with the implication that the gravitational potentials of the radio sources are decreasing with time through expansion and/or mass loss.

JUNE 14, 1966

DISTRIBUTIONS OF REDSHIFTS OF RADIO SOURCES AND RICH CLUSTERS

for Liege

A. G. Wilson

The distribution of optical redshifts of radio sources is non-uniform (Figure 1), showing marked clumping into bands separated by distinct gaps. With the errors of individual redshifts not much larger than the line thickness; these bands and gaps appear not to be statistical fluctuations. Although the present total available sample of redshifts is small, the probability of statistical fluctuations creating this type of non-uniformity is remote. A Poisson $-\chi^2$ test shows probabilities of less than $1:10^6$ that the observed distribution would occur in a random sample with either uniform density or density increasing with square of distance.

There is the possibility that observational selectivity factors have generated the banded distributions. These selectivity factors are primarily those contributing to ease of observation. There is a declination factor, most of the sample being in the northern sky. There is a brightness factor, the optically bright objects being chosen to effect shorter exposure times. While the brightness factor has biased the sample toward emphasis on the giant D type radio galaxies, it is difficult to see how this factor or a declination factor could generate apparent bandings in a distribution which is in reality uniform.

If the distribution is not attributable to either statistical fluctuations or to observational selectivity factors, the next most likely hypothesis is that the radio sources are physically clustered. An investigation of the angular distribution of the sources indicates that physical clustering partially accounts for the clumping, but many sources having nearly equal redshifts are widely separated in angle. Consequently, assumption of large scale non-uniform distribution beyond the scale of recognized clusters seems necessary to account for the observed distribution.

A second type of cosmic object with large redshifts which also exhibits a similar banded redshift density distribution is the rich cluster of galaxies. In Fig. 2 are shown the mean redshifts of all clusters of galaxies in which redshifts of one or more individual galaxies have been measured. Investigations of redshifts in nearby clusters in which large numbers of individual galaxies have been measured show that the mean redshift is very closely equal to the redshift of the brightest galaxy in the cluster. This equivalence allows the redshifts of Fig. 2 to be considered a homogeneous sample although for a few of the nearby clusters the means are based on large numbers of individual redshifts (up to 50) while beyond values of log = -1.4 the plotted value for the most part is the single redshift of a cD type galaxy.

With regard to cluster richness, the sample is not homogeneous. Although all of the clusters are rich enough to appear in Abell's catalog, [5] some of the nearby clusters fall below the general richness level of the rest of the sample. Among the closest clusters (log of redshift <-1.47) eight members of the sample contain radio sources, while none of the clusters at greater distance contains a radio source. Observational selectivity of distant clusters on the basis of their being rich has resulted in exclusion of clusters with radio sources. This is consistent with the statistics concerning the association of radio sources in clusters with poorer clusters. [6]

In the cluster sample, the Abell richness of the cluster is correlated with the position of the cluster's redshift within a band. The richer clusters are found toward the center of a band, the sparser clusters toward the edges of a band. The variation of richness across the bands in this manner strongly reinforces an interpretation of the bands and gaps as reflecting an actual distribution of matter and not being due to random fluctuations or selectivity effects.

In Figure 3, the radio source and cluster redshift distributions are compared. The banded structure does not appear in either distribution until distances corresponding to redshifts whose logs are greater than (-1.4), i.e., distances of the order of 150 megaparsecs. An examination of the three dimensional distributions of the present samples of the radio sources and rich clusters suggest the possible existence of some sort of "local super-aggregate" some 200 megaparsecs in diameter which extends to the first gap in the redshift distribution. However, a far greater sample will be needed before the existence of any such super-aggregate can be confirmed or its extent delineated. Beyond the redshift value, log = -1.4, three corresponding bands are identifiable. From the respective positions of the bands, it appears that there is a displacement of the redshifts of the radio sources with respect to the redshifts of the clusters. The displacement only occurs when the distances are greater than the limits of the hypothetical local super-aggregate, somewhat analagous to the onset of systematic redshifts beyond the limits of the local group of galaxies.

If we compare the values of the redshifts at the lower and upper limits of the corresponding bands, we find the redshift displacement, Δz, of the radio sources with respect to the cluster redshifts is systematic following to good approximation the relation

$$\Delta z = (\text{constant}) \cdot (\bar{z}_c)^{3/2}$$

Assuming the bands have been placed into correct correspondence, this relation is shown in Figure 4. It must be noted that even though the sample is large enough to give statistical confirmation to the existence of the bands, the sample is not large enough to remove the considerable uncertainty as to the correct values of the upper and lower limits of the individual bands. Because of this the "three-halves relation" must be held as quite tentative. It is surprising, however, to find such a good fit to a simple relation considering the uncertainties.

In the ranges under consideration in Figure 3, we are in essence comparing the optical redshifts of the two types of D galaxies - those which are radio sources and those which are not. This is because at the larger distances the choice of object whose redshift is to be measured, whether from a list of radio sources or from a list of rich clusters, is subject to a selectivity factor which results in the selection of the brightest objects available - the super galaxies of Morgan's type D. [7] In addition, the type D galaxies which are radio sources are usually found in poorer clusters and D galaxies which are radio quiescent are found in the rich clusters. It follows that a sample of clusters selected on the basis of richness would result in a sample of radio quiescent D galaxies.

It is not understood why certain D galaxies are radio sources and others are not, nor why the radio D galaxies are found in the poorer clusters and the radio quiescent D galaxies in the richer clusters. Morgan suspects that the radio D galaxies are the largest and most massive single structures known. If this be so, we have a possible explanation for the differentiation between radio and non-radio D galaxies in the displacement of the radio source redshifts with respect to the non-radio redshifts: the displacement is due to an Einstein shift. If the radio sources are more massive than the ordinary D galaxies, part of the observed redshift may be a gravitational shift.

It is reasonable to assume that radio sources and clusters are cosmically distributed in the same manner. This means that the observed bands and gaps for both radio sources and clusters must be identically distributed in distance requiring that the differences in redshifts between the radio source and cluster bands be attributed to some other cause than cosmic distance operating in accord with Hubble's Law. In other words, the cosmic or Hubble components of the observed redshifts of radio sources and clusters at the same distance must be the same, but superimposed on the cosmic redshifts is a second component of the observed redshift which is different for the radio sources and cluster D galaxies. The differences in this second component are manifested as the redshift displacement. Aside from the cosmic or Hubble redshift, which is presumably a doppler shift, the only other established source of a redshift is an Einstein shift. It seems reasonable to assume, then, that the redshift displacement between radio sources and clusters is attributable to an Einstein shift and that the emission lines in the optical spectra of radio galaxies come from sources which are located in regions of higher potential.

If the displacement is a gravitational redshift,then for a radio source and cluster having the same cosmic redshifts,

$$\Delta z = \frac{G}{c^2}\left[\frac{M_r}{R_r} - \frac{M_g}{R_g}\right]$$

where the subscript r designates a radio source and g a cluster galaxy.

If we designate nearer objects with a "second" and more distant objects with a "prime" then since $\Delta z' < \Delta z''$, (the three-halves law is not essential to the argument, only that Δz increase with distance) we have

$$\frac{M_r'}{R_r'} - \frac{M_r''}{R_r''} \quad \frac{M_g'}{R_g'} - \frac{M_g''}{R_g''}$$

This equation states that changes in the potentials of radio sources over equal intervals of time exceed any changes in potentials of normal cluster galaxies (the right member may be zero). Taking any change in potential of cluster D galaxies as a standard of reference with value unity, we have for all values of 2 beyond the local (-1.4) boundary that

$$\frac{M_r'}{R_r'} > 1 + \frac{M_r''}{R_r''}$$

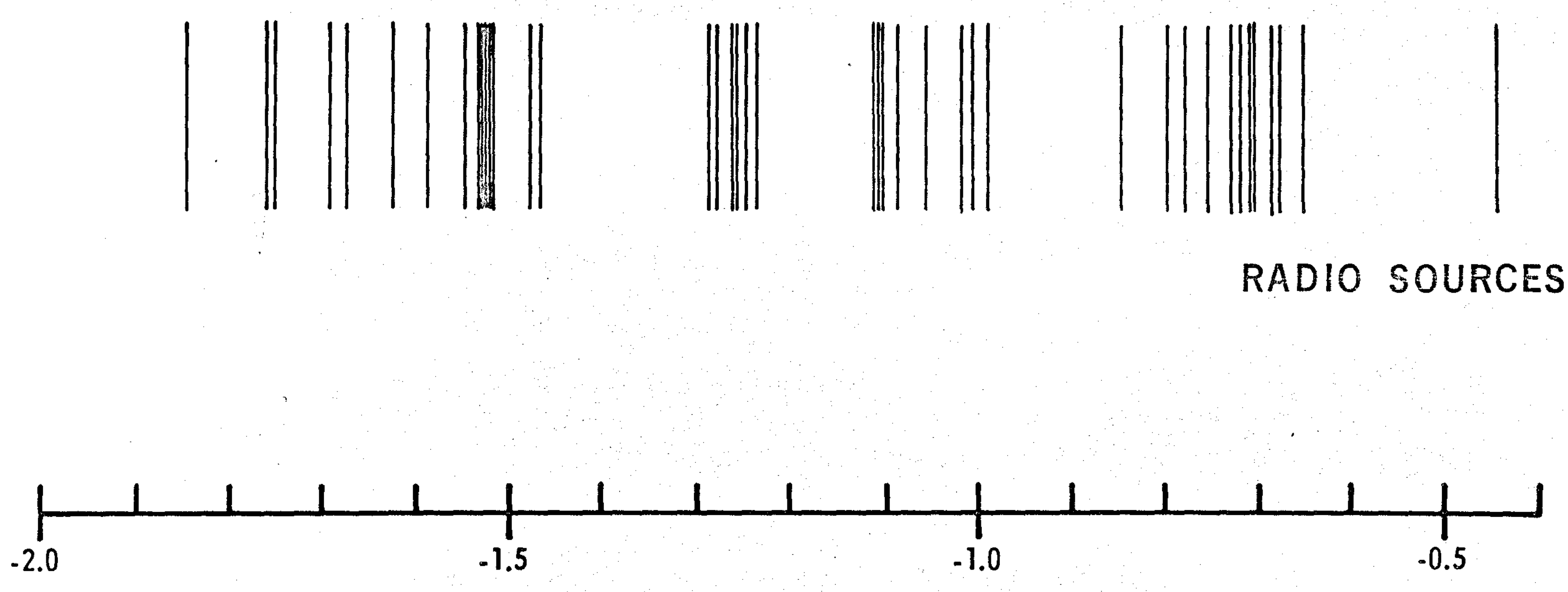
Since the right member is smaller, Mr/Rr is decreasing with time. Hence, the redshift displacements between the radio sources and clusters may be interpreted as resulting from expansion and/or mass loss in the radio sources. However, only that portion of expansion in excess of any cluster D galaxy expansion is reflected in the displacement.

The observational conclusion that the radio sources are expanding is not only consistent with theoretical models, but also is in accord with explanations of the radio counts based on secular power decrease.

In conclusion, the interpretation of the redshift displacement as an Einstein shift leads to a consistent accounting for the differences between radio and non-radio D galaxies, expansion and mass loss of radio sources, and the radio source counts. In addition, the resulting superposition of the bands in the two distributions, strengthens the evidence for the non-uniform distribution of matter over distances greatly exceeding the sizes of any presently recognized clusterings, a matter with important cosmological implications.

Although this interpretation affords satisfying qualitative consistency, quantitative conclusions must await a substantial increase in the size of the redshift samples.

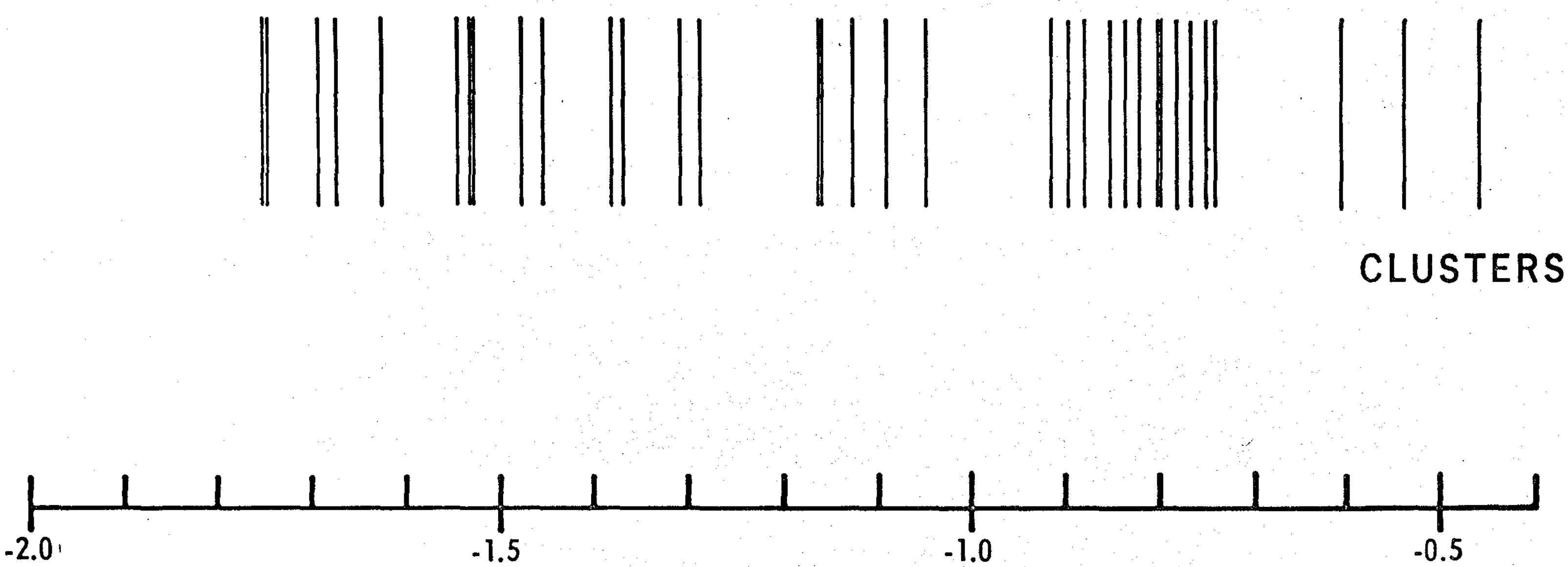
REDSHIFTS



$$\mathrm{LOG}\left[\frac{(1+Z)^2-1}{(1+Z)^2+1}\right]$$

FIG 1

REDSHIFTS



$$\text{LOG}\left[\frac{(1+Z)^2-1}{(1+Z)^2+1}\right]$$

FIG 2

REDSHIFTS

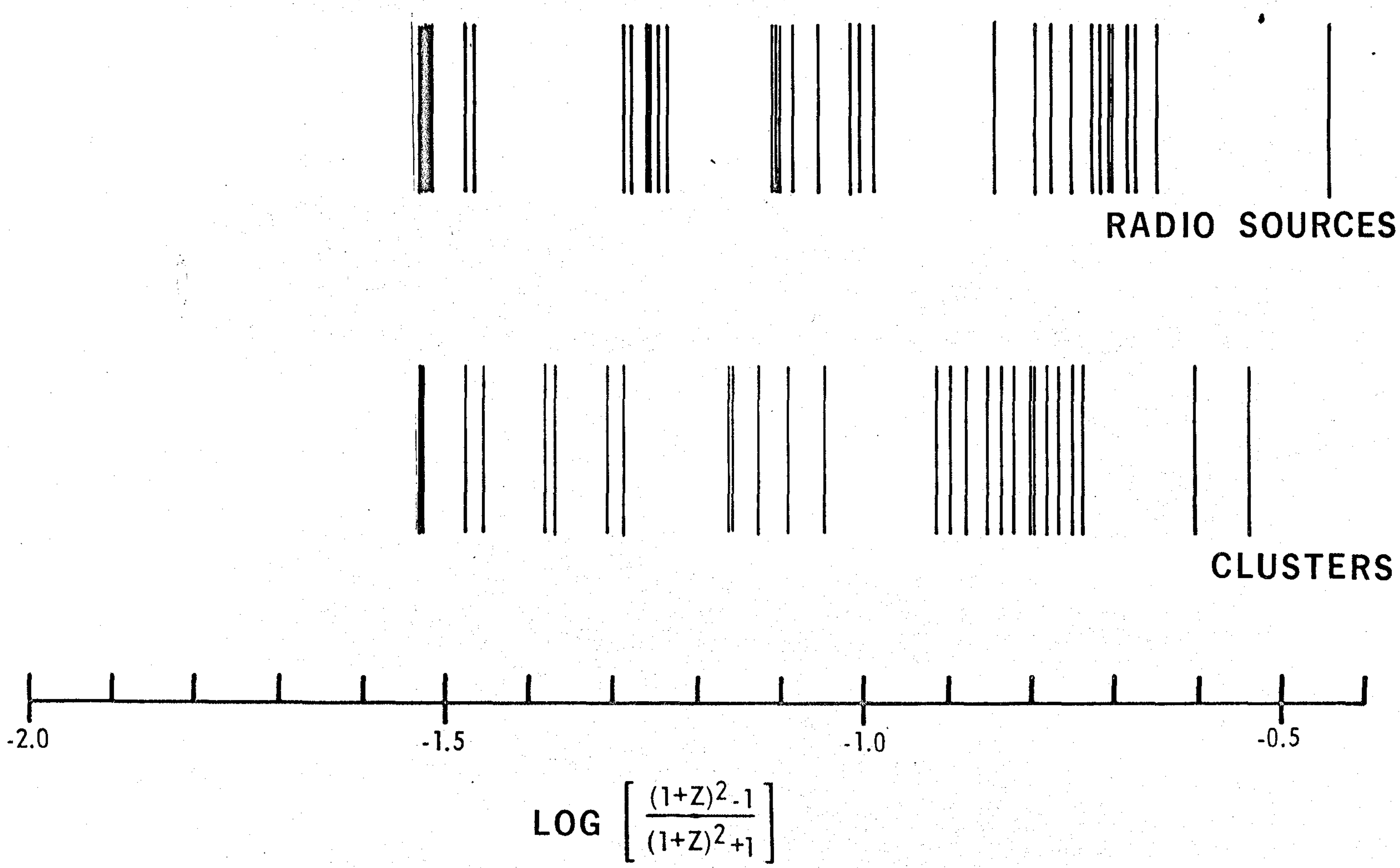
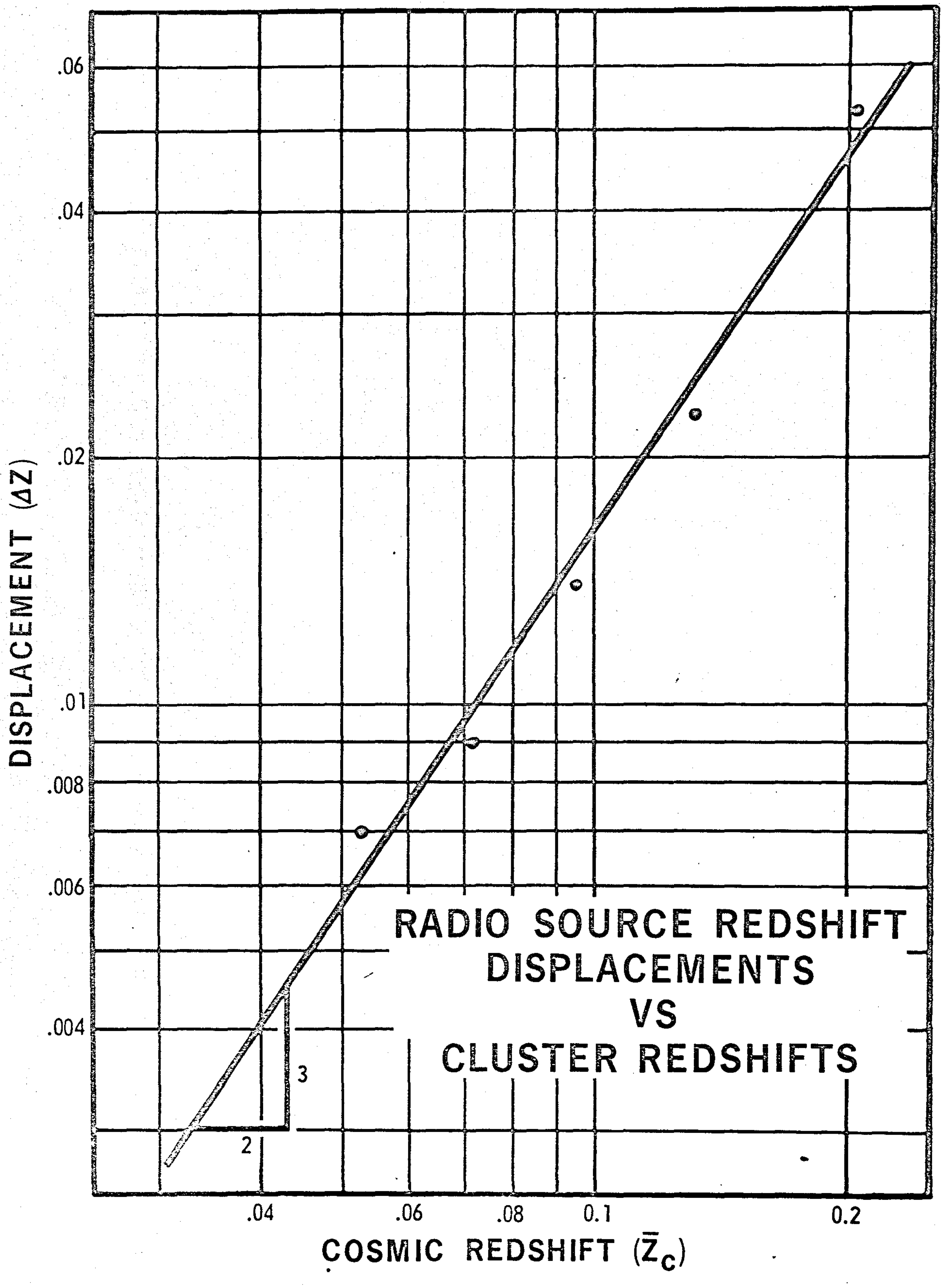


FIG 3



RADIO SOURCE REDSHIFT
DISPLACEMENTS
VS
CLUSTER REDSHIFTS
DISPLACEMENT (ΔZ)
.06
.04
.02
.01
.008
.006
.004
3
2
.04
.06
.08
0.1
0.2
COSMIC REDSHIFT ($\bar{Z}_c$)

CAPTIONS

Figure 1

Distribution of sample of 42 optical redshifts given in terms of the logarithm of the doppler function of special relativity. [1, 2, 3] Sample contains no quasars.

Figure 2

Distribution of mean redshifts of rich clusters of galaxies given in terms of the logarithm of the doppler function. [4]

Figure 3

Comparison of distributions of optical redshifts greater than 0.035. Upper distribution redshifts of radio galaxies, lower distribution primarily redshifts of cD type galaxies in clusters.

Figure 4

The differences in the redshifts of radio sources and cD type galaxies in clusters vs. cluster redshifts. Derived from the bands of Figure 3.

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COSMIC REPLICATION OF ATOMIC PARAMETERS

An upper bound to the ratio of gravitational energy to ~~xxxx~~ total energy of non-degenerate cosmic bodies has been observationally established. The ratio of the non-degenerate bound to the relativistic bound predicted by Schwarzschild is equal, to within observational uncertainties, to the basic atomic structural ratio α^2 (where α is Sommerfeld's fine structure constant). While the occurence of this ratio between the non-degenerate and totally degenerate states may be readily explained in the case of stars, (since stellar degeneracy is defined on the basis of atomic structure), it is difficult to account for the appearance of the same ratio in larger aggregates -- galaxies, clusters, second-order clusters.

Either some process is operative in the formation of higher order aggregates which reflects atomic constants, or there exists some basic universal property of all structures which relates them to the dimensionless constants observed in both atomic and cosmic physics. In the second case, the constants may be of "trans-physical" ~~origin~~, possibly of number theoretic ~~genesis.~~ ~~gensis.~~ origin

draft: AWilson:3/12/66 mailed

Freiburg Abstract

10-22-68

COSMIC REPLICATION OF ATOMIC PARAMETERS

Albert G. Wilson

(Paper read A. I. P., Ober Wolfach, July, 1966)

ABSTRACT

An upper bound to the ratio of gravitational energy to total energy of non-degenerate cosmic bodies has been observationally established. The ratio of the non-degenerate bound to the relativistic bound predicted by Schwarzschild is equal, to within observational uncertainties, to the basic atomic structural ratio, α^2, with α being the Sommerfeld fine structure constant. While the occurrence of this ratio between non-degenerate and totally degenerate states may be readily explained in the case of stars (since stellar degeneracy is defined on the basis of atomic structure), it is difficult to account for the appearance of the same ratio in larger aggregates - galaxies, clusters, second-order clusters.

Either some process is operative in the formation of higher order aggregates which reflects atomic constants, or there exists some basic universal property of all structures which relates them to the dimensionless constants observed in both atomic and cosmic physics. In the second case, the constants may be of "trans-physical", possibly of number theoretic origin.

Being neither a physicist nor a philosopher, but speaking as an observer, I want to re-emphasize Prof. Flugge's remarks that our goal is not simply the accumulation of data, but achieving an organization of the emerging basic relationships. This is sometimes lost sight of in certain quarters and we view with alarm the warehouses full of magnetic tapes of data - all unreduced.

Speaking as an astronomer, I would like to insert a modification into Prof. Noll's trilogy of

experience - theory - experiment

observation → theory → (theory directed observation)

ab initio observation being all too often neglected.

And I also want to acknowledge that observational astronomers know all to well what Prof. Tornebohm means by <u>low grade knowledge</u>.

COSMIC NUMBERS

Albert Wilson

INTRODUCTION

The purpose of this paper is to make two observations concerning the so-called cosmic numbers and to discuss briefly some of their philosophical implications. The first observation is the occurrence of the cosmic numbers in the structure of physical aggregates ranging in scale from atoms to clusters of galaxies. The second is that the numbers are representable by simple expressions containing only basic mathematical constants.

PART I

A feature of the physical world that is repeatedly observed in the microcosmos, the mesocosmos, and the macrocosmos is likely to be a manifestation of the basic structure of the universe. Such a feature holds possible clues to the foundations of the natural order. The so-called cosmic numbers, or dimensionless constants of physics, such as the Sommerfeld Fine Structure Constant, $\alpha = \frac{2\pi e^2}{hc}$, and the ratio of Coulomb to gravitational forces, $S = \frac{e^2}{Gm_p m_c}$, have numerical values that occur frequently in dimensionless combinations of observables measured not only in atoms but in material aggregates of all sizes. If the numerical reoccurrence of these values may be taken simply as an observed phenomenon, their frequency of occurrence implies their fundamental significance and any ultimate construct or cosmological model which successfully represents the physical world will have to contain and account for these numbers.

In this paper, I shall not give a history of the numbers nor go into the interpretations which have been given to them by Eddington, Dirac, and others. I plan to limit myself as much as possible to the empirical aspects of the numbers. The experimental values adopted by DuMond and Cohen (1965) for $\alpha^{-1} = 137.0388$ and $\log_{10} S = 39.356$, the latter number possesses uncertainty in the last place because of their relatively inaccurate knowledge of the gravitational coupling constant, G.

It is well known that numbers of the order of 10^{40} occur in cosmology. For example, the ratio of the "Hubble Radius of the Universe" c/H to the radius of the electron $e^2/m_e c^2$ is $10^{40.5}$. Sometimes the square of this quantity occurs. Eddington's "number of heavy particles in the universe" is observationally

$$\frac{\rho_o (c/H)^3}{m_p} = 10^{78} = (10^{39})^2$$

These instances of the numbers have been speculated over for some four decades and have been widely discussed without any conclusions being reached. I would like to point to some additional occurrences of these numbers that have not been reported until recently (Wilson, 1966). The first table gives the maximum observed values of the potentials of four species of cosmic aggregate - stars, galaxies, clusters, 2! clusters.

SUMMARY OF OBSERVATIONS

SYSTEM	$\log_{10}[R]$ (c.g.s.)	$\log_{10}[M]$ (c.g.s.)	$\log_{10}[M/R]$ (c.g.s.)	$\log_{10}[M/R]$ (dimensionless)
HYDROGEN ATOM	−8.27640	−23.77642	−15.50002	
STARS				
V444 CYG A	11.185	34.457	23.272	38.8
40 ECL. BINARIES	11.541	34.205	22.664	
SUN	10.843	33.299	22.456	
GALAXIES				
M87	22.3	45.9	23.6	39.1
M31	22.2	44.8	22.6	
7 GALAXIES	--	--	22.6	
MILKY WAY	22.26	44.30	22.04	
CLUSTERS				
COMA	25.95	49.40	23.45	39.0
7 CLUSTERS	--	--	22.59	
4 CLUSTERS	25.54	48.08	22.54	
SECOND-ORDER CLUSTERS				
ABELLIAN CELL	26.0	49.2	23.2	38.7
LOCAL SUPER-CLUSTER	25.7	--	--	

It is seen that each of these potentials, when expressed in dimensionless form, i.e., with respect to M_H/a_o, is again a number of the order of 10^{39}. This is true for stars ($R = 10^{11}$cm), Galaxies ($R = 10^{22}$cm), Clusters ($R = 10^{25}$cm), 2° order clusters ($R = 10^{26}$cm). (It may also be true for Quasars if Smith's values for the periodicities in light fluctuation have a conventional interpretation.) This result is especially interesting since the technique of measuring the potentials is different in each case and does not depend on a distance scale. Dirac held that that the repeated occurrence of a number of this magnitude can hardly be attributable to chance. If we spin an epistemological roulette wheel and come up with this number six times, the probability of this is, say, $1/n^5$, where n is the number of numbers on the wheel. If there are a large number of numbers, i.e., if n is large - then this is not a chance coincidence. If n is small, then this itself would be an even more remarkable fact about the universe.

Dirac postulated as a "principle" that all of these large dimensionless numbers which occur in physics are the same, or differ from each other at most by some simple factor of the order of unity such as 2 or π, etc. Let us assume that this is a valid principle and that these numbers are the same if we but knew the proper factor of the order of unity to insert. (Our errors are of the order of 2 or π anyway.) If then we say these numbers are equal to S, with $\log_{10} S = 39.356$, we have $\log_{10} \frac{M_N}{R_N} = 23.856$ and

$$\log_{10} \frac{GM_N}{c^2R_N} = -4.274 = \log_{10} \alpha^2.$$

That is to say that the observed bound on the value of the ratio of the gravitational radius to the linear radius for all observed non-degenerate cosmic aggregates is α^2, which is the same as the ratio of the first Bohr radius to the electron radius. So it appears to within the relatively small errors of measurement that both α and S occur at the scales of all bodies observed in the cosmic hierarchy.

The Schwarzschild Limit states $\frac{GM}{c^2R} < \frac{1}{2}$

The observed limit is $\frac{GM}{c^2R} \leq \alpha^2$ or $\frac{\alpha^2}{2}$, etc.

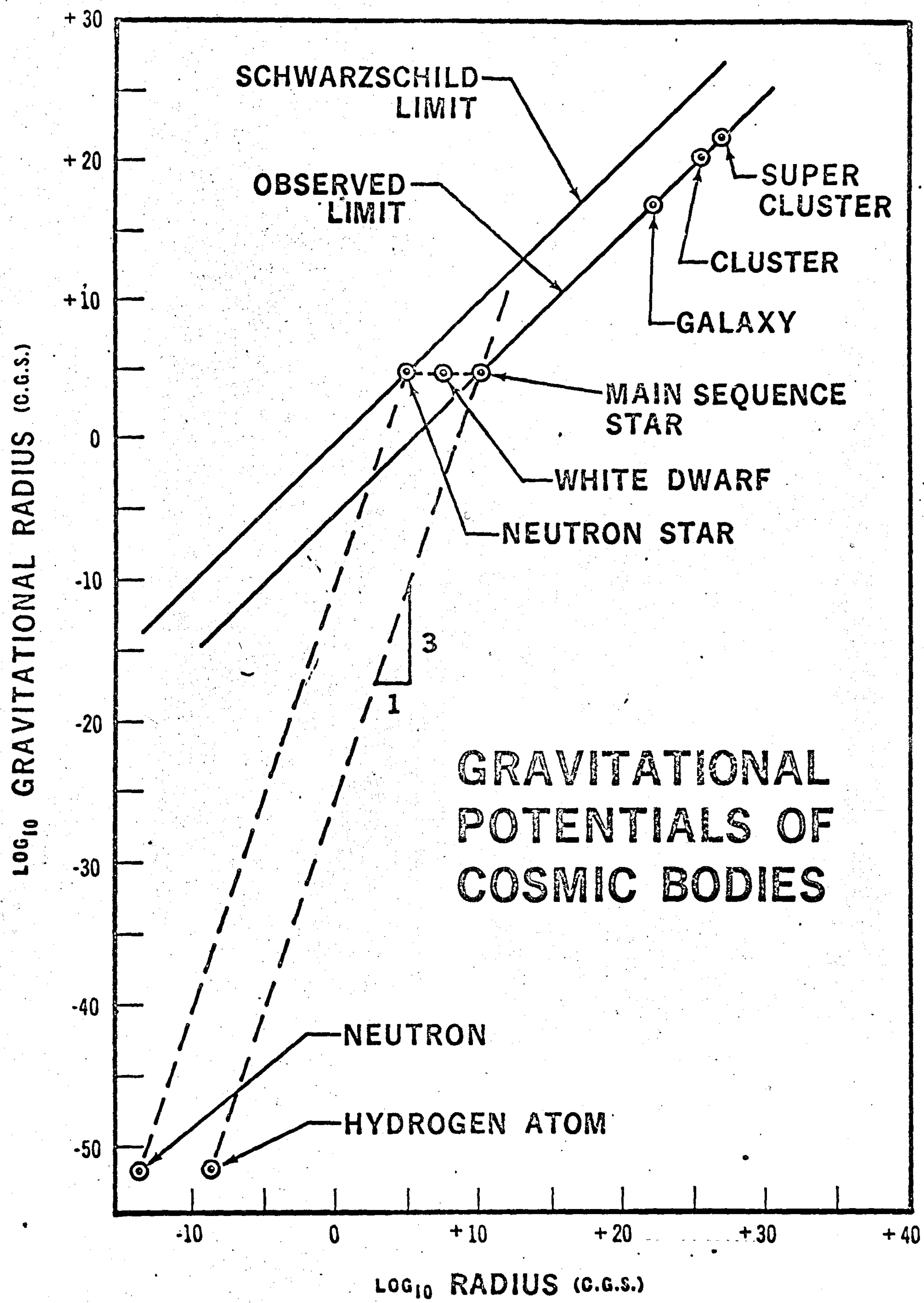
This may be alternatively interpreted that the highest velocity any bound or attached material body may have is αc, whether this is the speed of an electron in the first Bohr orbit or the escape velocity from a star, galaxy, cluster, or whatever.

I do not intend to discuss here the physics or astrophysics of this ratio which states that

$$\frac{\text{gravitational radius}_N}{\text{linear radius}_N} = \frac{\text{nuclear dimensions}}{\text{atomic dimensions}}$$

I only want to draw attention to the reoccurrence of the quantities α and S.

We may portray this graphically in Figure 2.


SCHWARZSCHILD LIMIT
OBSERVED LIMIT
SUPER CLUSTER
CLUSTER
GALAXY
MAIN SEQUENCE STAR
WHITE DWARF
NEUTRON STAR
3
1
GRAVITATIONAL POTENTIALS OF COSMIC BODIES
NEUTRON
HYDROGEN ATOM
+30
+20
+10
0
-10
-20
-30
-40
-50
-10
0
+10
+20
+30
+40
LOG10 GRAVITATIONAL RADIUS (C.G.S.)
LOG10 RADIUS (C.G.S.)

Now, we are faced with what may be interpreted as a set of numerical coincidences or numerical curiosities which like other such curiosities, e.g., the Titius-Bode Law, are to be filed away until some future time when a theoretical construct can be built out from existing knowledge to encompass these oddities. If we hold the existing body of knowledge as that which is interpretable in terms of The Theory (in the sense of Max Born), these oddities lie definitely outside the pale.

However, there seem to be enough of these detached pieces which fit together that it may be possible to build the bridge in both directions. Before this we must be concerned with two things: first, are these detached pieces part of the real puzzle - and it seems likely (by paradigmatic inference) that they do belong to the same picture that The Theory is developing. If we are reasonably certain of this, then secondly, can we synthesize from the "low grade" knowledge which these detached oddities provide and actually begin to construct on them, i.e., make predictions from them. In other words, may we develop hypotheses spanning inward.

What, if anything, can be said at this time which will allow us to develop testable hypotheses. We might re-examine the "Conjecture of Eddington" - that the cosmic numbers and other constants are expressible in mathematical constants; in view of the fact that as of now most of the fundamental constants of physics have been measured with sufficient accuracy to make a re-examination worthwhile.

PART II

Eddington held that the dimensionless physical constants could be evaluated as simple mathematical expressions. His approach to this conjecture was through a construct established by purely rational arguments from which the values of the dimensionless physical constants could be derived solely by mathematical inference (1). His success in proving his conjecture by means of the fundamental theory has been questioned. The difference, for example, between the derived 137 (2) and the observed 137.0388 (3) is considered by some to be unsatisfactory in view of the essential claim to derive the observed world from first principles. However, because of the philosophical implications which Eddington's conjecture has for the foundations of physics, it is important to know, regardless of the validity of Eddington's fundamental theory - or other theory - whether the conjecture is true. Is there a simple mathematical expression for these numbers. But apart from the context of a theory can the conjecture have a meaning?

Meaning may be given to the conjecture, without an explicit theory, if two specifications are agreed to. (1) A specification as to degree of fit between the observed value and the mathematical value, and (2) a definition of simple. The form of specification (No. 1) which most physicists would insist on is that the fit be such that the difference between the mathematical and observed values be less than the experimental uncertainty in the observed value. As subsequent experiments improve the observed value, the difference must remain less than the new observational uncertainties. In this sense the mathematical value legitimately plays the role of a hypothesis, i.e., the hypothesis that a purely mathematical expression, M = the value of the dimensionless physical constant. If refined observation shows the observed value does not converge to M, the hypothesis fails to make valid predictions and is discarded. So long as the observed value continues to converge to M, the hypothesis may be used as any conventional hypothesis derived from theory. This is standard procedure.

A satisfactory convention for specification No. (2) is more difficult to formulate. Any numerical quantity can be approximated to any degree of accuracy by sophisticated combinations of basic mathematical quantities. What one considers to be a simple expression is ultimately a matter of personal taste. To avoid these difficulties, we propose as a possible approach to specification No. (2), the introduction of the requirement that the same mathematical expression occurs in at least two of the dimensionless physical constants. By this demand the aspects of simplicity and improbability of occurrence serve as checks on one another; i.e., an expression which begins to reach a level of complexity which exceeds the threshold of permissibility as simple, and therefore appears to be ad hoc, is at the same time reaching a level of improbability of simultaneous occurrence by chance in two or more cases. Hence, involvement in two or more instances restores the expression to continued interest as arising from real, albeit unknown, relationships. The essential feature of meaningfulness - interpretability through theory - is deferred. The existence of sufficiently accurate replication of a phenomenological feature together with a sufficiently large improbability of this being a chance occurrence combine to create confidence in significance and ultimate interpretability by theory. Reasoning such as this has been implicit in the rationale for continuing interest by astronomers and physicists in observed, but inexplicable features, such as the Titius-Bode Law.

In this epistomological context, the following hypothesis "M" is proposed: in the usual notations, three dimensionless physical constants, the Sommerfeld fine structure constant,

$$\alpha = \frac{2\pi e^2}{hc}$$

and the ratio of Coulomb to gravitational forces,

$$S = \frac{e^2}{Gm_p m_e}$$

and the ratio of proton to electron mass,

$$\mu = \frac{m_p}{m_e}$$

are given by the following purely mathematical quantities.

$$\alpha = \frac{1}{2 + \omega} \text{ ; and } S = \frac{2^{\omega}}{2\pi^2} \text{ ; and } \mu = 6\pi^5 .$$

where $\omega = \pi^4 \ln 4$ (natural logarithm). The <u>mathematical</u> value of α^{-1} to nine significant digits is 137.037664. The present but <u>observed</u> values for α^{-1} are between 137.0352 and 137.0387 with a minimum error adopted value of 137.0378 (Cohen, E. R., NASC, M384, p. 6). For specification No. (1) mean values and "adopted values" are of less interest that the range in recent determinations.

The logarithm to the base 10 of the <u>mathematical</u> value of S is 39.355058, while the present <u>observed</u> value is close to 39.356. A more accurate observed value cannot be given until better determinations of the gravitational coupling constant G have been made.

The mathematical value of $6\pi^5$ = 1836.118101, while the best present observational value of the ratio mp/me is 1836.12.

The quantity $\omega = \pi^4 \ln 4$, appearing in the mathematical values of both α and S thus satisfies specifications No. (1) and No. (2). The occurrence of ω in both numbers reduces the likelihood of its being ad hoc, yet it is still a "simple expression" involving only integers and the basic mathematical constants π and e. The quantity $6\pi^5$ meets even more satisfactorily specifications Nos. (1) and (2). Granting the epistemological rationale of the two specifications, we conclude - until more refined observations contradict the mathematical values - that

Eddington's Conjecture appears to be true.

The exhibiting of a simple mathematical expression whose value lies within the measurement uncertainties of the physical quantities does not constitute a proof of Eddington's Conjecture. However, since present experimental accuracies allow for a test to six significant figures, the sieve for isolating "simple" expressions is becoming fine, and the ability to pass the sieve in three cases certainly is reasonable grounds for the "M" hypothesis.

The question here is, since proof is lacking, and can probably only be given in terms of a physical theory, can the "M" hypothesis be put to any use.

I think the answer is yes. 1) The properties of the mathematical expression can be studied. These may give clues to physical relations. ~~but~~ 2) Several interesting inferences can be drawn.

CONCLUSIONS

What are the implications of the expressibility of the fundamental dimensionless physical quantities in terms of purely mathematical constants?

First, there is the inference that local conditions are not atypical, i.e., the "universal constants" are really universal. A second consequence of the truth of the conjecture would be that the dimensionless constants, μ, α, and S do not vary with time.

This does not preclude the separate variation of G, h, etc., but requires any variation of fundamental constants with time to be such that

$$\frac{d}{dt}\left(\frac{2\pi e^2}{hc}\right) = 0; \quad \frac{d}{dt}\left(\frac{e^2}{Gm_p m_e}\right) = 0; \quad \text{and} \quad \frac{d}{dt}\left(\frac{m_p}{m_e}\right) = 0.$$

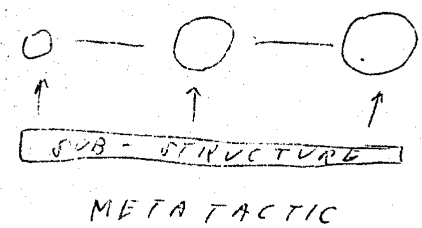
Third, there is no known theoretical relation between G and the other fundamental constants of physics. Hence, a second interesting consequence of the mathematical formulae is a possible relation linking G and the charge to mass ratio of the electron:

$$G = \frac{8\pi^2}{2^{1/\alpha}} \frac{m_e}{m_p} \left(\frac{e}{m_e}\right)^2 = \frac{4}{3\pi^3 2^{1/\alpha}} \left(\frac{e}{m_e}\right)^2$$

This equation may have interesting implications for relativistic electrons. If mass is velocity dependent and charge is not, then G must also be velocity dependent.

Fourth is the matter of Repitaxis and Metataxis. The basic values discussed above are fundamental to the structure of the atom, but they also occur in higher order aggregates like stars. Since the stars are made of atoms it is likely that they would reflect in their own structure the structure of the atom, just as the macroscopic shape of a crystal replicates the molecular structure of the molecules composing the crystal. We shall call this view - the repitactic view - the large deriving its properties from the small. Or inversely the small deriving its properties from the large - The Machian repitactic view.

The second point of view is that the atom, the star, the galaxy, etc., derive their structural limitations, not from one another, but from underlying structural laws which independently govern all aggregates whatever their scales. This point of view we may name metatactic. Our question then becomes: Is the universe repitactic or metatactic and can we discover the answer in the nature of the cosmic numbers?



If it proves that the dimensionless physical constants indeed are determined by certain geometrical or combinatorial theorems - or even number theoretic relations - accounting for the presence of the basic mathematical constants, π, e, etc., and being independent of physical scale, then the surmise that physical structure derives directly from a more basic non-physical structure leads to a metatactic view of the universe. If π's, e's, etc., appear as the result of properties of quantities with physical dimensionality, then either a reductionist or Machian repitactic view is supported. The present findings are supportive of the metatactic view but this is not surprising for the Einstein Field equations,

$$R_{AB} - \frac{1}{2} R g_{AB} = \kappa T_{AB},$$

have already equated geometry and physics.

The primary importance of the repitactic vis-a-vis metatactic views is in the process of development of our theories. If the substructure implied by metataxis exists, then theoretical attempts to explain the phenomenological world without it, even if successful, may become quite complex. Further a metatactic universe, allows for an explanation of human understanding and a resolution of the subjective vs. objective problem. In a metatactic universe, the substructure maps not only onto the physical world but onto the mental patterns by which the world is understood.

G8-52-ARL-553
June 20, 1966

The Editors, NATURE
MacMillan (Journals) Limited
4 Little Essex Street
London W.C. 2, England

Gentlemen:

Enclosed is a manuscript entitled, "The Dimensionless Physical Constants and Basic Mathematical Constants" which I would like to submit for publication in NATURE. The manuscript reports the determination of an expression giving the fine structure constant and the ratio of the Coulomb to gravitational forces in terms of integers, π and e, which is of sufficient precision and simplicity to play a possible role in questions concerning the nature of the fundamental physical constants.

Sincerely yours,

Albert G. Wilson
Associate Director
Advanced Research Laboratory

AGW/jdu
Enclosure

C.1 - Anna
C.2-3 here
C.4 - home.

THE DIMENSIONLESS PHYSICAL CONSTANTS AND BASIC MATHEMATICAL CONSTANTS

A. G. Wilson

June 17, 1966

Eddington held that the dimensionless physical constants could be evaluated as simple mathematical expressions. His approach to this conjecture was through a construct established by purely rational arguments from which the values of the dimensionless physical constants could be derived solely by mathematical inference [1]. His success in proving his conjecture by means of the fundamental theory has been generally questioned. The difference, for example, between the derived 137 [2] and the observed 137.0388 [3] is considered bymany to be unsatisfactory in view of the essential claim to derive the observed world from first principles. However, because of the philosophical implications which Eddington's conjecture has for the foundations of physics, it is important to know, regardless of the validity of Eddington's fundamental theory—or other theory— whether the conjecture is true. But apart from the context of a theory can the conjecture have a meaning?

Meaning may be given to the conjecture, without an explicit theory, if two specifications are agreed to. (1) A specification as to degree of fit between the observed value and the mathematical value, and (2) a definition of <u>simple</u>. The form of specification No. (1) which most physicists would insist on is that the fit be such that the difference between the mathematical and observed values less than the experimental uncertainty in the observed value. As subsequent experiments improve the observed value, the difference must <u>remain</u> less than the new observational uncertainties. In this sense the mathematical value legitimately plays the role of a hypothesis, i.e., the hypothesis that a purely mathematical expression, M ≡ the value of the dimensionless physical constant. If refined observation shows the observed value does not converge to M, the hypothesis fails to make valid predictions and is discarded. So long as the observed value continues to converge to M, the hypothesis may be used as any conventional hypothesis derived from theory. This is quite conventional.

A satisfactory convention for specification No. (2) is more difficult to formulate. Any numerical quantity can be approximated to any degree of accuracy by sophisticated combinations of basic mathematical quantities. What one considers to be a simple expression is ultimately a matter of personal taste. To avoid these difficulties, we propose as a possible approach, to specification No. (2), the introduction of the requirement that the same mathematical expression occurs in at least two of the dimensionless physical constants. By this demand the aspects of simplicity and improbability of occurrence serve as checks on one another. An expression which begins to reach a level of complexity which exceeds the threshold of permissibility as <u>simple</u>, and therefore appears to be ad hoc, is at the same time reaching a level of improbability of simultaneous occurrence by chance in two or more cases. Hence involvement in two or more instances restores the expression to continued interest as arising from real, albeit unknown, relationships. The essential feature of meaningfulness—interpretability through theory—is deferred. The existence of sufficiently accurate replication of a phenomenological feature together with a sufficiently large improbability of this being a chance occurrence combine to create confidence in significance and ultimate interpretability by theory. Reasoning such as this has been implicit in the rationale for continuing interest by astronomers and physicists in observed, but inexplicable features, such as the Titius-Bode Law and the numerical coincidences which occur between certain atomic and cosmic measurements.

In this epistomological context, the following hypothesis "M" is proposed: In the usual notations, two dimensionless physical constants, the Sommerfeld fine structure constant

$$\alpha = \frac{2\pi e^2}{hc}$$

and the ratio of Coulomb to gravitational forces.

$$S = \frac{e^2}{Gm_p m_e}$$

are given by the following purely mathematical quantities

$$\alpha = \frac{1}{2 + \omega}$$

and

$$S = \frac{2^{\omega}}{2\pi^2}$$

where $\omega = \pi^4 \ln 4$ (natural logarithm). The mathematical value of α^{-1} to nine significant digits is 137.037664. The present observed values [3] for α^{-1} are given in the table.

137.0388 $\pm$ 0.0006	Triebwasser, Dayhoff, Lamb
137.0370	Robiscoe
137.0352	Hyperfine splitting in Hydrogen
137.0388 $\pm$ 0.0013	Hyperfine splitting in Muonium
137.0381 $\pm$ 0.0032	Electron magnetic moment anomaly
137.0361	Hughes

For specification No. (1) mean values and "adopted values" are of less interest than the array of recent determinations given in the table.

The logarithm to the base 10 of the mathematical value of S is 39.355058, while the present observed value is close to 39.356. A more accurate observed value cannot be given until better determinations of the gravitational coupling constant G have been made.

The quantity $\omega = \pi^4 \ln 4$, appearing in the mathematical values of both α and S thus satisfies specifications No. (1) and No. (2). The occurrence of ω in both numbers reduces the likelihood of its being ad hoc, yet it is still a "simple expression involving only integers and the basic mathematical constants π and e. Granting the epistemological rationale of the two specifications, we conclude—until more refined observations contradict the mathematical values—that Eddington's Conjecture is true.

An immediate consequence of the truth of the conjecture is that the dimensionless constants α and S do not vary with time. This does not preclude the separate variation of G, h, etc., but requires any variation of fundamental constants with time to be such that

$$\frac{d}{dt}\left(\frac{2\pi e^2}{hc}\right) = 0 \text{ and } \frac{d}{dt}\left(\frac{e^2}{Gm_p m_e}\right) = 0 \ .$$

There is no known theoretical relation between G and the other fundamental constants of physics. ~~Consequently~~ Hence, a second interesting consequence of the mathematical formulae is a possible relation linking G and the charge to mass ratio of the electron:

$$G = \frac{8\pi^2}{2^{1/\alpha}} \frac{m_e}{m_p} \left(\frac{e}{m_e}\right)^2$$

This equation may have interesting implications for relativistic electrons.

The author wishes to thank the RAND Corporation for its support and for making available "JOSS," a computer system invaluable for the derivation of numerical congruences such as those reported here.

Albert G. Wilson
Douglas Advanced Research Laboratory
Huntington Beach, California

Bibliography

1. Eddington, Fundamental Theory, Cambridge University Press, 1948.

2. Eddington, Relativity Theory of Protons and Electrons, Cambridge University Press, 1936.

3. Cohen and DuMond, Phys. Rev. Vol. 37, Oct. 1965, Pp. 537-594.

To Nature Aug 4

DIMENSIONLESS PHYSICAL CONSTANTS IN TERMS OF MATHEMATICAL CONSTANTS

It is of interest to note that the present empirical values of two basic dimensionless physical constants may be approximated to within experimental uncertainties by a simple logarithmic expression involving π. With the usual notations, the Sommerfeld fine structure constant $= 2\pi e^2/hc$, and the ratio of Coulomb to gravitational forces $S = e^2/Gm_p m_e$ are given by

$$\alpha = \frac{1}{2+w} \quad \text{and} \quad S = \frac{2^w}{2\pi^2}$$

Where $w = \pi^4 \log 4$ (natural logarithm), the numerical value of 2+w to nine digits is 137.037664. The present measured values [1] for α^{-1} are:

137.0388 + 0.0006	Triebwasser, Dayhoff, Lamb
137.0370	Robiscoe
137.0352	Hyperfine splitting in Hydrogen
137.0388 ± 0.0013	Hyperfine splitting in Muonium
137.0381 ± 0.0032	Electron magnetic moment anomaly
137.0361	Hughes

The numerical value of $\log_{10}(2^w/2\pi^2)$ is 39.355058. The present indicated empirical value of $\log_{10} S$ lies between the three σ limits 39.357 and 39.355, the largest part of the uncertainty being in the value of G. The three σ limits of S are $2.27(01)\times10^{39}$ and $2.25(46)\times10^{39}$, whereas $2^w/2\pi^2 = 2.264947 \times 10^{39}$.

From these two relations a third numerical relation

$$G = \frac{8\pi^2}{2^{1/\alpha}} \frac{e^2}{m_p m_e}$$

may be derived. This equation, giving G in terms of other fundamental physical constants, is independent of w.

Although one may be reminded of relationships derived by the late Sir Arthur Eddington, the quantity w used here has no known physical basis and the approximations are quite possibly all fortuitous.

[1] Cohen and DuMond, Phys. Rev. Vol. 37, Oct. 1965, Pp. 537-594.

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Dimensionless Physical Constants in Terms of Mathematical Constants

It is of interest to note that a simple logarithmic expression involving π may be used to obtain the values of two basic dimensionless physical constants within the experimental uncertainty. The Sommerfeld fine structure constant, $\alpha_0 = 2\pi e^2/hc$, and the ratio of Coulomb to gravitational forces $S = e^2/Gm_p m_e$, where m_p, m_e are the masses of the ~~particle~~ and the ~~Earth~~, respectively, are given by $\alpha = 1/(2+w)$ and $S = 2^w/2\pi^2$.

When $w = \pi^4 \ln 4$, the numerical value of $(2+w)$ to nine significant figures is 137·037664. The present measured values[1] for α^{-1} are

137·0388 + 0·0006	Triebwasser, Dayhoff, Lamb
137·0370	Robiscoe
137·0352	Hyperfine splitting in hydrogen
137·0388 ± 0·0013	Hyperfine splitting in muonium
137·0381 ± 0·0032	Electron magnetic moment anomaly
137·0361	Hughes

The numerical value of $\log_{10}(2^w/2\pi^2)$ is 39·355058. The present indicated empirical value of $\log_{10} S$ lies between the three standard deviations of the mean, that is, between 39·357 and 39·355, the largest part of the uncertainty being in the value of G. The three standard deviations of the mean of S are $2{\cdot}27\ (01) \times 10^{39}$ and $2{\cdot}25\ (46) \times 10^{39}$, whereas $2^w/2\pi^2 = 2{\cdot}264947 \times 10^{39}$.

From these two relations a third numerical relation

$$G = \frac{8\pi^2}{2^{1/\alpha}} \cdot \frac{e^2}{m_p m_e}$$

may be derived. This equation, giving G in terms of other fundamental physical constants, is independent of w.

Although one may be reminded of relationships derived by the late Sir Arthur Eddington, the quantity w used here has no known physical basis and the approximations are quite possibly fortuitous.

ALBERT WILSON

5251 Bolsa Avenue,
Hungtington Beach, California.

[1] Cohen, ——, and DuMond, ——, *Phys. Rev.*, **37**, 537 (1965).

1966 Sept. [RAD

Dr. A. G. Wilson

INVESTIGATION TITLE:

Gravitational Potentials of Cosmic Bodies

OBJECTIVES AND APPROACH:

The dimensionless gravitational potential $\phi = GM/c^2R$ of a cosmic body (star, galaxy, cluster, etc.) is of cosmological importance, 1) because it can often be evaluated independently of any knowledge of the distance to the body and is therefore free of the problems besetting distance scales and their calibration, and 2) because it can be used in lieu of the classical Hubble-Tolman tests for verification of cosmological models. [1]

The value of ϕ for all cosmic bodies so far measured appears to be bounded by a value about 10^{-4} of the Schwarzschild Bound [2]. The measure of potentials of additional clusters of galaxies of various richnesses, both nearby and remote, is of importance to determine in what way this maximum value of ϕ may change with time. An observational program for measuring redshifts in various clusters using the B spectrograph of the 100 inch telescope has been set up through the courtesy of the Mt. Wilson and Palomar Observatories. Cluster richness and dimensions are being measured on 48 inch Schmidt plates. Masses are derived from redshift dispersions by means of the Virial Theorem.

Continuation of effort - new
Principal Investigator - A. G. Wilson, Ph.D, Astronomer
Estimated Profession Man Years - 0.6
Established Other Expenditures - $1500

SUMMARY OF PAST ACHIEVEMENTS:

[1] D. G. B. Edelen and A. G. Wilson (to be submitted)

[2] A. G. Wilson, A. J. ~~(in press)~~ vol 71, Aug 1966, p 402

INVESTIGATION TITLE:

Gravitational Potentials of Cosmic Bodies

INTRODUCTION

Empirical studies of the properties of gravitation continue to be a most important subject for physical research. This is especially so because many of the recent contributions to gravitational theory are in need of experimental verification. Since gravitational forces are less than 10^{-39} the magnitude of coulomb and other forces, gravitational effects are difficult to study on the scale of the laboratory. For this reason the observation of the structure and motions of cosmic bodies continues to be/one of the most useful sources of data for the study of gravitation.

A quantity of particular usefulness in cosmic gravitational studies is the dimensionless gravitational potential of a body,

$$\phi = \frac{2GM}{c^2R}$$

where M is the mass, R is the geometric radius, G the Newtonian coupling constant, and c the velocity of light. This potential can often be evaluated independently of knowledge of the distance to the body and is consequently free of the many problems besetting astronomical distance scales and their calibration.

Two applications of ϕ are of current interest:

1. The maximum numerical value ϕ_o of the potential observed for stable non-degenerate cosmic bodies - stars, galaxies, clusters of galaxies, second order clusters of galaxies - appears to be of the order of 10^{-4}, [1]. Chandrasekhar [2] has recently shown that under the Schwarzschild conditions, general

relativity predicts any mass becomes unstable when the potential ϕ exceeds a value of $(\gamma - \frac{4}{3})/K$, where γ is the specific heat ratio and K is a constant of the order of unity depending on the polytropic index of the body. Demonstration that $K\phi_o = \gamma - \frac{4}{3}$ for stable cosmic bodies will constitute a fourth observational verification of the general theory of relativity.

2. Observational discrimination between cosmological models is usually made on the basis of one or more of the three classical Hubble-Tolman [3] tests for distant cosmic bodies: magnitude-redshift relation, diameter-redshift relation, count-redshift relation. The class of homogeneous, isotropic models under test are characterized by the values of the curvature parameter, the cosmological constant, the Hubble parameter, and the deceleration parameter. Knowledge of these four parameters allows selection of the model which most closely represents the observable sample of the universe: open, closed, monotone expanding, oscillating, steady state, etc. [4].

   Edelen and Wilson [5] have shown that the dimensionless gravitational potential ϕ may be used to discriminate between homogeneous models and used to derive values, or bounds on values, of the characterizing parameters.

OBJECTIVES AND APPROACH:

Theoretical and observational investigations are under way by means of which it is hoped to develop data which will make contributions toward solutions of the above two problems. The measure of potentials of additional clusters of galaxies of various richnesses, both nearby and remote, is of importance to determine whether the bound on ϕ changes with time.

Through the courtesy of the Mt. Wilson and Palomar Observatories, use is being made of the B spectrograph with the 100 inch telescope to obtain redshifts in several clusters of galaxies. The potentials of the clusters are readily derived from the redshift dispersions by means of the virial theorem [6] [7]. Cluster richnesses and dimensions are being obtained from plates taken on a supplementary observing program with the Palomar 48 inch Schmidt telescope. The observing program has been underway for several months.

REFERENCES:

1. A. G. Wilson, "Structural Parallels in Non-degenerate Cosmic Bodies", A. J., Vol. 71, p. 402, 1966.

2. S. Chandrasekhar, "The Dynamical Instability of Gaseous Masses Approaching the Schwarzschild Limit in General Relativity", Ap. J., Vol. 140, p. 417, 1964.

3. R. Tolman, "Relativity, Thermodynamics, and Cosmology", Oxford U. P., 1934.

4. A. Sandage, "The Ability of the 200 inch telescope to discriminate between selected world models" Ap. J., Vol. 133, p. 355, 1961

5. D. G. B. Edelen and A. G. Wilson, "Homogeneous Cosmological Models with Bounded Potential" (to be submitted)

6. S. Chandrasekhar, "An Introduction to the Study of Stellar Structure", Univ. of Chic. Press, 1939, p. 49

7. F. Zwicky, "Morphological Astronomy", Springer-Verlag, p. 129, 1957.

FORM L-M-214 (REV. 2-64)

DOUGLAS AIRCRAFT COMPANY

NO.

DOUGLAS

INDEPENDENT R & D LINE ITEM DESCRIPTION

FISCAL YEAR 1967
TYPE OF R & D
BASIC ☒
APPLIED ☐
DEVELOPMENT ☐

INVESTIGATION TITLE:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

OBJECTIVES AND APPROACH

Empirical studies of the properties of gravitation continue to be a most important subject for physical research. This is especially so because many of the recent contributions to gravitational theory are in need of experimental verification. Since gravitational forces are less than 10^{-39} the magnitude of coulomb and other forces, gravitational effects are difficult to study on the scale of the laboratory. For this reason the observation of the structure and motions of cosmic bodies continues to be one of the most useful sources of data for the study of gravitation.

A quantity of particular usefulness in cosmic gravitational studies is the dimensionless gravitational potential of a body,

$$\Phi = \frac{2GM}{c^2R}$$

where M is the mass, R is the geometric radius, G the Newtonian coupling constant, and c the velocity of light. This potential can often be evaluated independently of knowledge of the distance to the body and is consequently free of the many problems besetting astronomical distance scales and their calibration.

Two applications of Φ are of current interest:

1. The maximum numerical value Φ_o of the potential observed for stable non-degenerate cosmic bodies - stars, galaxies, clusters of galaxies, second order clusters of galaxies - appears to be of the order of 10^{-4}, (1). Chandrasekhar (2) has recently shown that under the Schwarzschild

(Continued)

Continuation of research efforts described in: New (Initiated 1/66)

Report ______ Fiscal Year ______ Page ______

PRINCIPAL INVESTIGATOR QUALIFICATIONS

A. G. Wilson, Ph.D., Astronomer, Assoc. Director, Environmental Sciences
D. G. B. Edelen, Ph.D., Professor of Mathematics, Purdue Univ., Consultant

ASSISTING PERSONNEL QUALIFICATIONS

Research Associate

	THIS FISCAL YEAR	PREVIOUS YEAR/YEARS
ESTIMATED PROFESSIONAL MAN YEARS	.6	
ESTIMATED OTHER EXPENDITURES	\$24,980	
ESTIMATED TOTAL	\$54,560	\$99,952

PROJECT START DATE 1/66
ESTIMATED COMPLETION DATE Continuous
ESTIMATED PERCENT COMPLETION THIS DATE Continuous

R & D ADMINISTRATOR

DOUGLAS INDEPENDENT R & D LINE ITEM DESCRIPTION

INVESTIGATION TITLE:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

OBJECTIVES AND APPROACH

conditions, general relativity predicts any mass becomes unstable when the potential Φ exceeds a value of $(\gamma - \frac{4}{3})/K$, where γ is the specific heat ratio and K is a constant of the order of unity depending on the polytropic index of the body. Demonstration that $K\Phi_o = \gamma - \frac{4}{3}$ for stable cosmic bodies will constitute a fourth observational verification of the general theory of relativity.

2. Observational discrimination between cosmological models is usually made on the basis of one or more of the three classical Hubble-Tolman (3) tests for distant cosmic bodies: magnitude-redshift relation, diameter-redshift relation, count-redshift relation. The class of homogeneous, isotropic models under test are characterized by the values of the curvature parameter, the cosmological constant, the Hubble parameter, and the deceleration parameter. Knowledge of these four parameters allows selection of the model which most closely represents the observable sample of the universe: open, closed, monotone expanding, oscillating, steady state, etc., (4).

   Edelen and Wilson (5) have shown that the dimensionless gravitational potential Φ may be used to discriminate between homogeneous models and used to derive values, or bounds on values, of the characterizing parameters.

Theoretical and observational investigations are under way by means of which it is hoped to develop data which will make contributions toward solutions of the above two problems. The measure of potentials of additional clusters of galaxies of various richnesses, both nearby and remote, is of importance to determine whether the bound Φ_o changes with time.

Through the courtesy of the Mt. Wilson and Palomar Observatories, use is being made of the B spectrograph with the 100 inch telescope to obtain redshifts in several clusters of galaxies. The potentials of the clusters are readily derived from the redshift dispersions by means of the virial theorem (6) (7). Cluster richnesses and dimensions are being obtained from plates taken on a supplementary observing program with the Palomar 48 inch Schmidt telescope. The observing program has been underway for several months.

Applicable Government Technical Requirements: RTD 68-11, RTD 68-23, RTD 68-24, RTD 68-31, and RTD 68-41.

Facilities used in the furtherance of this program includ: Telescopes, Loci 2ab computer, Isodensitracer.

DOUGLAS | INDEPENDENT R & D LINE ITEM DESCRIPTION

INVESTIGATION TITLE:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

SUMMARY OF PAST ACHIEVEMENTS ON THIS &/OR RELATED INVESTIGATIONS

REFERENCES:

1. A. G. Wilson, "Structural Parallels in Non-degenerate Cosmic Bodies", A. J., Vol. 71, p. 402, 1966.

2. S. Chandrasekhar, "The Dynamical Instability of Gaseous Masses Approaching the Schwarzschild Limit in General Relativity", Ap. J., Vol. 140, p. 417, 1964.

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5. D. G. B. Edelen and A. G. Wilson, "Homogeneous Cosmological Models with Bounded Potential" (to be submitted).

6. S. Chandrasekhar, "An Introduction to the Study of Stellar Structure", Univ. of Chic. Press, 1939, p. 49.

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I. A. U.

A. G. Wilson has been studying the distribution of rich clusters of galaxies. Regularities in their distances (1) and angular separations (2) suggest the existence of larger scale structures than any presently recognized. Radio galaxies also appear to share cluster distributions (3) in the sense of indicating the existence of large scaled structures. The redshift distributions may be interpreted as indicative of a continuation of a Charlier type hierarchy up to organizations of the order of 500 megaparsecs.

Observed gravitational potentials of cosmic bodies suggest a universal upper limit with a value of about 10^{23} g/cm (4). This observed bound is consistent with the bound predicted by Chandrasekhar (5) and under certain conditions constitutes a fourth observational test of the general theory of relativity. Edelen and Wilson have applied the values of bounds on potentials to discriminate between homogeneous cosmological models. They conclude that the only nonempty, homogeneous, isotropic cosmological models with a bounded potential at the present epoch are those with $k = +1, \lambda > 0, q_0 < -1$ and with negative first derivative of potential.

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To Minkowski

October 31, 1966

Mailed to AAS
Nov. 8, 1966

A Charlier-Type Cosmological Model

Albert Wilson
Douglas Advanced Research Laboratories
5251 Bolsa Avenue, Huntington Beach, California, 92646

If the observed upper limit to the gravitational potential of stars, galaxies, and clusters (Wilson, A.G.; Astron. J. 71, p402, 1966) may be extrapolated to represent a global bound on the stability of gravitating systems, then the critical limiting size, R_c, for any stable aggregate in terms of its mean density $\bar{\rho}$, will be given by

$$\log_{10}(\bar{\rho} R_c^2) = 22.9 \quad \text{c.g.s. units.}$$

In assuming the extended validity of this relation, the formation of a Charlier-type hierarchal sequence of aggregates is predicted. The stability limit prohibits the existence of uniformly distributed matter of indefinite extent. Large masses may only be accommodated hierarchically, radii of successive aggregations depending on successive mean densities.

If the observed limit corresponds to the Chandrasekhar limit for dynamic stability of polytropes (Astrophys. J. 140, p417, 1964), then the hierarchal sequence would terminate when the assumptions governing the gas models no longer were valid for the larger aggregates. The potential bound would then be expected to be closer to the Schwarzschild value.

The existence of the banded structures in the distributions of the redshifts of rich clusters of galaxies and radio sources (Wilson, A.G.; Proc. 14th Int. Astrophys. Symp., Liege, 1966 (in press)) may be explained on the basis of the operation of the same potential bound as observed for stars, galaxies, and clusters. The Charlier hierarchy appears to continue to higher orders of clustering up to aggregates with R_c = 500 megaparsecs.

Superceded

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G8-52-ARL-991
November 10, 1966

Dr. Warren D. Rayle
Chief, Advanced Concepts Branch
National Aeronautics
and Space Administration
21000 Brookpark Road
Cleveland, Ohio 44135

Dear Dr. Rayles: Reference: 2260

Thank you for your interesting letter regarding my note on "Structural Parallels", in the Astronomical Journal.

I have been observing clusters of galaxies for some years to obtain data relevant to their structure and dynamics. At the present time there are only seven clusters for which we have adequate data for deriving potentials. These average about 0.37×10^{23}gm/cm, with the Coma cluster having the largest potential, approximately $10^{23.45}$. These potentials are based on the virial theorem and are independent of any assumptions regarding distance scales or cluster size.

It is quite interesting that the largest observed potential for stars, based on the available sample of eclipsing binaries, has a value of the same order $10^{23.27}$. Determinations of masses and radii of eclipsing binaries also have the advantage of being independent of distance calibrations.

The situation for galaxies is somewhat more complex. Masses and radii are determined from rotational dynamics. A mean potential of galaxies for the sample of some 20 for which data are available gives a value of $10^{21.9}$, somewhat lower than for the spherical stars and clusters. A few galaxies, e.g., M104, have potentials as high as $10^{22.4}$. Poveda has given data for M87 (quoted by Reddish, V. C., in Observatory, V.81, p.19, 1961) of a mass for NGC4486 of 3.6×10^{12}. This is a spherical system with an estimated linear radius of 6.5 kpc. The potential corresponding to these values is $10^{23.6}$gm/cm. This value, I believe, is derived from an assumed mass to luminosity ratio of 100 and therefore depends on distance scales, a defect not shared by some of the dynamical determinations. In any event, the maximum potential is again of the same order of magnitude, 10^{23}. A more detailed discussion of these values is in preparation. All of the above potentials are values for the surfaces and in general will be the maximum values.

Ordinarily, a sample of only three objects cannot be used as a basis for meaningful generalization. But when we observe that stars, galaxies, and clusters all have essentially the same maximum surface potentials - determined by distinct and independent techniques - we rightly question whether something general is in operation.

The 10^{23} bound is exceeded by white dwarfs (up to 10^{26}). This, and the fact that a neutron star would lie essentially on the Schwarzschild limit, are the reasons for specifying non-degeneracy. It appears, however, that stability has more to do with the 10^{23} bound than does degeneracy. A paper by Chandrasekhar (Ap. J. 140, 417) gives conditions for relatavistic dynamic stability of polytropic spheres, namely $2GM/c^2R < (\gamma - \frac{4}{3})/K$, for γ slightly larger than 4/3. K is of the order of unity and depends on the polytropic index. This condition closely fits the observed bound and the non-existence of bodies between the 10^{23} limit and the Schwarzschild limit may be explained by mass loss, fragmentation, expansion toward the stability limit, or collapse to the Schwarzschild limit resulting from dynamic instability. This, incidentally, provides a fourth test of Schwarzschild general relativity.

We are considering the cosmological implications of all of this and have in preparation two papers, which we will send to you when pre-prints are ready.

I would be most interested in your and Harrison's ideas of this bound and welcome hearing from both of you.

Sincerely yours,

Albert G. Wilson
Associate Director

AGW/jbg
cc: Prof. E. R. Harrison

For I.A.F.
Madrid 1966 - Oct.

20TH ANNIVERSARY

At this 1966 meeting of the International Astronautical Federation it is appropriate to take note of an event of historical importance to astronautics. This year marks the 20th anniversary of the first attempt to launch matter into space with escape velocity.

Ten years before Sputnik and the official beginning of the Space Age, Dr. Fritz Zwicky, using the hardware on hand in 1946, a V-2 rocket and augmented shaped charges - devised a method of sending matter away from the earth with sufficient velocity to assure its never returning. The historic initial attempt to achieve orbital or escape velocity was made on December 17, 1946 at the White Sands Proving Ground in New Mexico.

A V-2 rocket was equipped with six 150 gram penolite shaped charges with 30 gram steel inserts. These were set to fire at times after launching that would eject the slugs of molten steel at elevations of approximately 50, 65, and 75 kilometers. At these elevations the ejection velocities in the neighborhood of 10 to 15 km/sec would place the slugs either in orbit or on escape trajectories depending on the firing orientation and the actual charge ejection velocity.

The launching was set at night so that the so called "artificial meteors" ejected by the charges could be photographed. At $22^h\ 12^m$ Mountain Standard Time, the V-2 was launched on a trajectory which took it to a new altitude record of 114 miles above the surface of the earth.

Although the rocket flight was successful and although tests of the charges made on previous evenings confirmed the expectancy of achieving escape velocity, no particles were detected either visually or photographically. Just as man's first attempts at flight in the atmosphere failed, this first attempt to reach space failed. It is significant, however, that whereas the span between the first attempts to fly and the first successful flight is measured in centuries, the span between the first attempt to achieve orbital velocity and the successful orbiting of Sputnik was only one decade.

Those who participated directly in this event Fritz Zwicky, J. A. Van Allan, the groups from the California Institute of Technology, the Johns Hopkins Applied Physics Laboratory, the Harvard College Observatory, the New Mexico School of Mines, and Army Ordnance; as well as those who participated indirectly through the design and development of the V-2 rocket, — though failing to inaugurate the space age on the night of December 17, 1946, all have to their credit an important experiment in the preparation of the Space Age.

Zwicky's idea was ultimately vindicated, when success crowned the <u>second</u> experimental firing of shaped charges from a rocket on October 16, 1957 - 12 days after Sputnik.

A. G. Wilson

"NICHT JEDER GEWORFENER STEIN MUSS FALLEN" (1)

Nov. 14, 1966
→ Eng & Sci

The pages of Engineering and Science provide an historical record of the achievements and successes of CalTech's alumni and staff in their work to advance the frontiers of science. The dead-ends and failures rarely reach these pages. If, in the spirit of fairness, space for documenting the year's research failures were made available to all, the result would be a paginal explosion. Fortunately for publication costs, among other things, few want the failures recorded. However, now and then, certain types of failure become historic and should receive mention in the record. First attempts are types of failures that are usually historic. (And first attempts in typical history are usually failures.)

The 17th of December this year marks the 20th anniversary of an historic first attempt--and a failure--to launch particles into space with escape velocity. A team from CalTech in cooperation with Army Ordnance, the Johns Hopkins Applied Physics Laboratory, the Harvard College Observatory, and the New Mexico School of Mines, put together a project combining the hardware components available in 1946 in a way which could theoretically launch a few pellets in orbit about the earth or throw them off into interplanetary space. The then state of the art was pushed to the limit. Two marginal devices and one valid motivation made the attempt worthwhile. The devices were the V-2 rocket and the Monroe rifle grenade or "shaped charge". The motivation was to generate a shower of artificial meteors in order to calibrate the luminous efficiency of natural meteors.

A V-2 rocket was equipped with six 150 gram penolite shaped charges with 30 gram steel inserts. These were set to fire at times after launching that would eject the slugs of molten steel at heights of approximately 50,65, and 75 kilometers. At these heights the ejection velocities of from 10 to 15 km/sec would place the slugs either in orbit or on escape trajectory. The ultimate fate of a slug would depend on its mass and velocity, most would be meteors, but some might not be consumed.

To determine the destinies of the meteors a battery of K 4 aerial cameras equipped with rotating shutters was scattered over the White Sands Proving Range. The sites were selected to acquire optimal triangulation data. In addition the CalTech eight inch Schmidt camera was removed from its usual house at Palomar and set up a few miles south of the launch site to obtain spectra with an objective grating. Astronomers at near by observatories who possessed wide angle telescopes also focused in on the firing.

The news coverage of the event was as complete as the photographic coverage. The possibility of throwing something up that would not come down again fired the imagination. This was to be the first night firing of a V-2 in the United States. In those days the launching of a V-2 with or without an instrument on board was as much news as the launching of a Gemini today. Professor Fritz Zwicky, the designer of the experiment and the, what has now become to be called, xxx principal investigator, placed the event in context: "We first throw a little something into the skies, then a little more, then a shipload of instruments--then ourselves." E. B. White in the New Yorker (2) saw the context somewhat differently:

> The year ends on a note of pure experimentation. Dr. Fritz Zwicky last week tried to hurl some metal slugs out into space free of the Earth's gravitational pull. Dr. Zwicky stood in New Mexico and tossed from there. He was well equipped: he had a rocket that took the slugs for the first forty mile leg of the journey and then discharged them at high velocity to continue on their own. The desire to toss something in a new way, or to toss it at a greater distance, is fairly steady in men and boys. Boys stand on high bridges, chucking chips down wind, or they stand on the shore of a pond, tossing rocks endlessly at a floating bottle, or at a dead cat, observing closely every detail of their experiment, trying to make every stone sail free of the pull of past experience. Then the boys grow older, stand in the desert, still chucking, observing, wondering. They have almost exhausted the Earth's possibilities and are going on into the empyrean to throw at the stars, leaving the Earth's people frightened and joyless, and leaving some fellow scientists switching over from science to politics and hoping they have made the switch in time.

As the 17th post war V-2 left the pad at 22^h 12^m 49^s mountain standard time, expectations were high. There was a feeling that history was being made. There was also the anxiety that has become as much part of every launching as the count-down. (The 16th rocket had tilted on lift off and travelled 131 miles horizontally.) Lifting slowly, xxxxxx No. 17 filled the whole range with sound and falling upward held true to its course-- 5^o tilt north. The shutters clicked and telescopes tracked, then burn out; but the rocket could still be followed by the red glow from its exhaust vanes. The time came and passed for the three pair of charge detonations. Nothing was seen. The rocket mounted to a new record of 114 miles then returned to earth. Films were hastily developed in hope of seeing on the emulsion what could not be seen in the sky. There were no trails. Tests of the charges made on previous evenings had been in every way successful. Had the charges fired, but been undetected? Subsequent investigations have not solved the mystery.

Just as man's first attempts at flight in the atmosphere failed, the first attempt to reach space with a chance of succeeding, also failed. It is significant, however, that whereas the span between the first attempts to fly and the first successful flight is measured in centuries, the span between the first attempt to achieve orbital velocity and the successful orbiting of Sputnik was only one decade. Those who participated directly xxxxxxxxxxxxxxxx and indirectly in this experiment, though failing to launch the space age on the night of December 17, 1946, have to their credit an important contribution leading to later triumphs. Zwicky's idea was ultimately vindicated, when success crowned the second experimental firing of shaped charges from a rocket on October 16, 1957 -- twelve days after Sputnik.

A. G. Wilson

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Table 1. COMT ACTIVITY IN THE VARIOUS ORGANS EXAMINED

	C.p.m. in ethyl acetate extract	Specific activity (mμmoles/mg protein/h)
Liver (supernatant fraction)	7,505	50·6
Skeletal muscle (rat)	955	1·33
Skeletal muscle (mouse)	545	0·82
Boiled rat skeletal muscle	5	—
Lung	1,265	1·65
Heart	885	1·34

After incubation, 10 μl. of the mixture was transferred from each tube to 0·5 ml. of 0·13 molar borate buffer, *p*H 10. After extraction by 5 ml. ethyl acetate and centrifugation, 2 ml. of the upper phase was used to determine the radioactivity in a liquid scintillation counter ('Tricarb', type 314 *X*). The 2 ml. are mixed with 10 ml. of scintillating fluid (4 g PPO, 0·1 g POPOP, toluene q.s.p. 1,000, 400 ml. ethanol).

The results of the various determinations are given in Table 1.

Under the conditions used here, ethyl acetate extracted half the methoxyadrenaline present in the sample. In the control preparations, ethyl acetate extracts contained little radioactivity. The small radioactive fraction obtained in controls can be explained by the untransformed *S*-adenosyl methionine and by degradation products. This radioactivity does not interfere significantly in determinations (0·3 per cent).

To test whether the enzyme transformation actually occurred, 100 μl. was spotted after each incubation on Whatman No. 3 *MM* paper with 3 μg methoxyadrenaline as carrier. The sample was chromatographed in an ascending system of butanol, glacial acetic acid and water (80 : 20 : 20). The bands were then dried and their radioactivity analysed (Packard chromatogram scanner). An important radioactive peak was found in all chromatograms of incubation residues from supernatant organ fractions (Fig. 1 *A*; rat muscle). The R_F of the radioactive zone corresponds with that of methoxyadrenaline. In chromatograms effected with controls, however, no radioactivity appeared in this zone (Fig. 1 *B*). A peak of radioactivity of intermediate R_F can also be seen on all chromatograms. It has been ascribed to methionine resulting from a slight degradation of *S*-adenosylmethionine at 37° C.

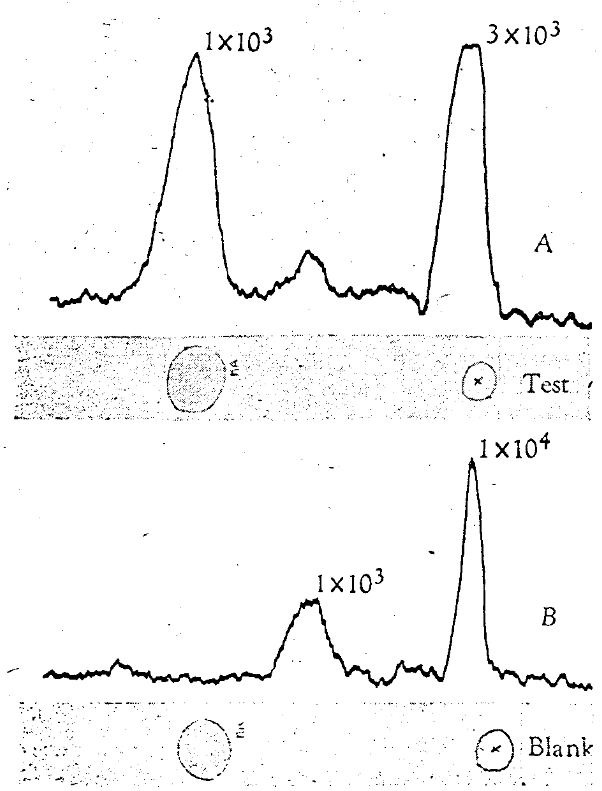

Fig. 1.

We were careful to ensure that the activity found was not caused by contamination from the blood. No COMT was found in rat serum. The activity of COMT in muscles was not modified by partial inhibition, but we were unable to show any inhibitory effect of muscle preparations on the enzyme extracted from rat liver.

These results show that skeletal muscles in rats and mice contain a considerable quantity of COMT. The results are indirectly confirmed by the results of Tomita *et al.*[2], who found COMT activity in rabbit skeletal muscle. A similar enzyme, but with a more limited specificity, GEP-*o*-methyltransferase (GEP, guanidoethyl phosphate) has been found in an Annelid (*Ophelia neglecta* Schneider) and localized only in muscle tissue[3].

We have also attempted to demonstrate MAO activity. This enzyme, too, is present in skeletal muscle tissue. A determination using the method of Weissbach *et al.*[4] showed a mean value of 7·7 mμmoles cynuramine transformed per mg protein per hour. The origin of the COMT and MAO in skeletal muscle, and the eventual action of these enzymes in muscular function, remains to be explained.

M. ASSICOT
C. BOHUON

Institut Gustave Roussy, Villejuif, France.

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GENERAL

Dimensionless Physical Constants in Terms of Mathematical Constants

IT is of interest to note that a simple logarithmic expression involving π may be used to obtain the values of two basic dimensionless physical constants within the experimental uncertainty. The Sommerfeld fine structure constant, $\alpha_0 = 2\pi e^2/hc$, and the ratio of Coulomb to gravitational forces $S = e^2/Gm_p m_e$, where m_p, m_e are the masses of the proton and the electron, respectively, are given by $\alpha = 1/(2+w)$ and $S = 2^w/2\pi^2$.

When $w = \pi^4 \ln 4$, the numerical value of $(2+w)$ to nine significant figures is 137·037664. The present measured values[1] for α^{-1} are

137·0388 ± 0·0006	Triebwasser, Dayhoff, Lamb
137·0370	Robiscoe
137·0352	Hyperfine splitting in hydrogen
137·0388 ± 0·0013	Hyperfine splitting in muonium
137·0381 ± 0·0032	Electron magnetic moment anomaly
137·0361	Hughes

The numerical value of $\log_{10}(2^w/2\pi^2)$ is 39·355058. The present indicated empirical value of $\log_{10} S$ lies between the three standard deviations of the mean, that is, between 39·357 and 39·355, the largest part of the uncertainty being in the value of G. The three standard deviations of the mean of S are 2·27 (01) × 10^{39} and 2·25 (46) × 10^{39}, whereas $2^w/2\pi^2$ = 2·264947 × 10^{39}.

From these two relations a third numerical relation

$$G = \frac{8\pi^2}{2^{1/\alpha}} \cdot \frac{e^2}{m_p m_e}$$

may be derived. This equation, giving G in terms of other fundamental physical constants, is independent of w.

Although one may be reminded of relationships derived by the late Sir Arthur Eddington, the quantity w used here has no known physical basis and the approximations are quite possibly fortuitous.

ALBERT WILSON

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[1] Cohen, E. R., and DuMond J. W. M., *Phys. Rev.*, **37**, 537 (1965).

(*Reprinted from Nature, Vol. 212, No. 5064, p. 862 only, November 19, 1966*)

Dimensionless Physical Constants in Terms of Mathematical Constants

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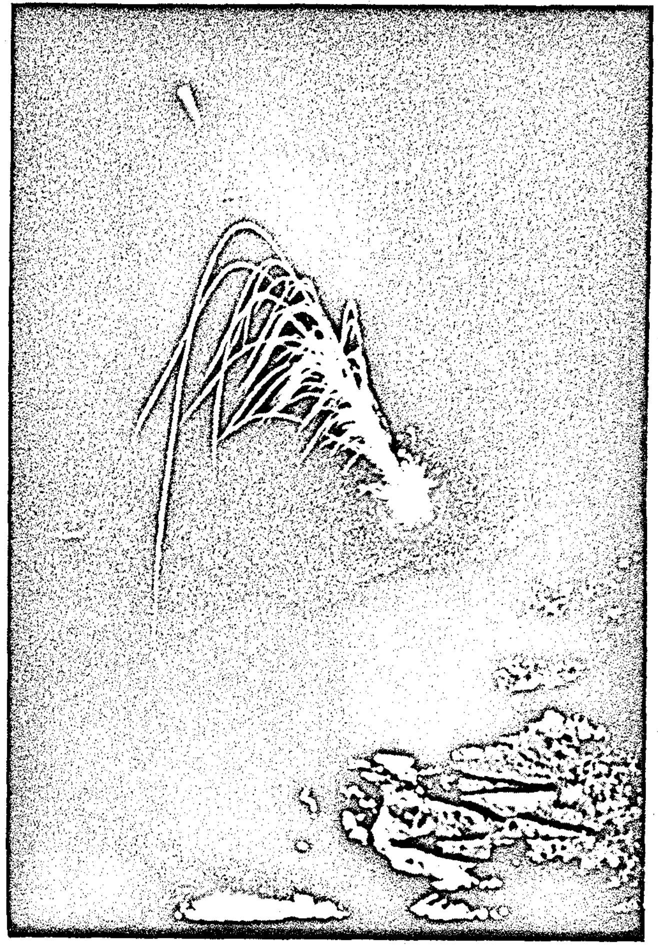
THE ANNIVERSARY OF A HISTORIC FAILURE

by Albert G. Wilson



A test of artificial meteors—December 16, 1946.

The pages of *Engineering and Science* magazine provide a historical record of many of the achievements and successes of Caltech researchers—alumni and staff. The dead ends and failures rarely appear in print. Fortunately for publication costs, few people want their failures recorded. However, now and then certain types of failures become historic and deserve a place in the record.

The 17th of December this year marks the 20th anniversary of such a historic failure—the first attempt to launch particles into space with escape velocity. A team of Caltech men headed by Fritz Zwicky, professor of astronomy, in cooperation with Army Ordnance, the Johns Hopkins Applied Physics Laboratory, the Harvard College Observatory, and the New Mexico School of Mines, put together a project in White Sands, New Mexico, combining the hardware components available in 1946 in a way which, theoretically, would launch a few pellets in orbit about the earth or throw them off into interplanetary space. Two marginal devices and one valid motivation made the attempt worthwhile. The devices were the V-2 rocket and the Monroe rifle grenade or "shaped charge." The motivation was to generate a shower of artificial meteors in order to calibrate the luminous efficiency of natural meteors.

The possibility of throwing something up that would not come down again fired the imagination. Although there had been 16 postwar V-2 rocket firings, this was to be first night firing of a V-2 in the United States. In those days the launching of a V-2, with or without an instrument on board, was as much news as the launching of a Gemini today. Dr. Zwicky, who designed the experiment, placed the event in historical context: "We first throw a little something into the skies, then a little more, then a shipload of instruments—then ourselves."

A V-2 rocket was equipped with six 150-gram penolite shaped charges with 30-gram steel inserts. These were set to fire at times after launching that would eject the slugs of molten steel at heights of approximately 50, 65, and 75 kilometers. At these heights the ejection velocities of from 10 to 15 km/sec would place the slugs either in orbit or on

Albert G. Wilson is an associate director at the Douglas Advanced Research Laboratory in Huntington Beach, California, where he is in charge of a laboratory for environmental science. A Caltech alumnus (MS '42, PhD '47) and a staff member of the Mt. Wilson and Palomar Observatories from 1947 to 1953, he was a member of the team which took part in this "historic failure."

escape trajectory. The ultimate fate of a slug would depend on its mass and velocity. Most would be meteors, but some might not be consumed.

To determine the destinies of the meteors, a battery of K4 aerial cameras equipped with rotating shutters was scattered over the White Sands Proving Range. One of these was equipped with a transparent objective grating to obtain spectra of the V-2 exhaust jet and the luminous artificial meteors launched. The sites were selected to acquire optimal triangulation data. In addition the Caltech eight-inch Schmidt camera was removed from its usual house at Palomar and set up a few miles south of the launch site to photograph the flight of the V-2 rocket and of the particles ejected from the shaped charges. Astronomers at nearby observatories with wide angle telescopes also focused in on the firing.

As this 17th postwar V-2 left the pad at 22^h 12^m 49^s mountain standard time, expectations were high. There was a feeling that history was being made. There was also the anxiety that has become as much part of every launching as the countdown. (The 16th rocket, fired a few days earlier, had tilted on lift-off and travelled 131 miles horizontally.) Lifting slowly, No. 17 filled the whole range with sound and, falling upward, held true to its course—5° tilt north. The shutters clicked and telescopes tracked—then burnout. But the rocket could still be followed by the red glow from its exhaust vanes. The time came and passed for the three pairs of charge detonations. Nothing was seen. The rocket mounted to a new record of 114 miles, then returned to earth.

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Just as man's first attempts at flight in the atmosphere failed, the first attempt to reach space with a chance of succeeding also failed. It is significant, however, that whereas the span between the first attempts to fly and the first successful flight is measured in centuries, the span between the first attempt to achieve orbital velocity and the successful orbiting of Sputnik was only one decade. Those who participated directly and indirectly in this experiment, though failing to launch the space age on the night of December 17, 1946, have to their credit an important contribution leading to later triumphs. Zwicky's idea was ultimately vindicated, when success crowned the *second* experimental firing of shaped charges from a rocket on October 16, 1957—twelve days after Sputnik.

A CHARLIER-TYPE COSMOLOGICAL MODEL

Albert Wilson
30 December 1966

Read before American Astronomical Society UCLA, Dec 30, 1966

For at least two centuries various forms of hierarchal cosmologies have interested philosophers and astronomers. The hierarchal models of Lambert and Swedenborg in the 18th Century were no more than speculative extrapolations based purely on analogy. The hierarchal cosmology proposed by Charlier in 1907 was designed to meet the difficulties raised by the Olbers-Seeliger Paradox. Charlier revived his hierarchal model in 1921 showing how a Universe full of matter could still have a mean density of zero through successive orders of clustering. World models based on General Relativity introduced about the same time provided alternate solutions to the Seeliger problem and provided for the observed expansion. Subsequently, hierarchal cosmologies have all but disappeared from current cosmological thinking. There seems at present no pressing theoretical necessity for reintroducing the concept of a hierarchal universe.

On Wednesday of this week we were privileged to witness an interesting episode in the continuing debate on the reality of second-order clustering on galaxies. We should remind ourselves, however, that the controversy over second-order clustering is only the <u>current</u> act of the play. Whether the title is "Reality of First-Order Galaxies" starring Curtis and Shapley or "The Reality of Third-Order Galaxies" starring Abell and Zwicky, this is one of Astronomy's best shows: and it becomes a very important matter to know whether we may expect to be treated to additional performances in the future.

There are also a couple of secondary reasons for taking another look at the hierarchal aspects of the universe. One of these is that in adopting homogenous or smoothed models we are thowing away the cosmological information contained in the masses and sizes of the various cosmic bodies we know to exist. My immediate motivation for looking into hierarchal models is the inference for hierarchaization implicit in the conecpt ~~ixgk~~ of bounded potentials.

At the 121st Meeting of the Society in March of this year, I presented the table shown in Slide 1 as evidence suggesting the existence of some sort of potential bound governing the potentials of stable, non-degenerate, cosmic bodies. The values tabulated are all $\log_{10}$ cgs values. In each case the maximum values are of the order of 10^{23} g/cm. The ~~xbx~~ observational determinations of these values for each species of cosmic body are independent. Further, the potentials may be determined independently of knowledge of the distance to the body. The observations of the same upper limit to the gravitational potentials of stars, ~~glax~~ galaxies, and clusters of galaxies suggests the hypothesis that this limit may be a universal stability bound for all gravitating systems.

Slide 2 shows a graphic representation of the data in a log M - log R plot. The observed lim it of 10^{23} g/cm is about 4 orders of magnitdue smaller than the Schwartzchild limit. The diagram is divided into three zones. Above and to the left of the Schwartzschild limit is terra incognito- or perhaps better, terra incommunicado. Below and to the right of the 10^{23} Limit is the region of the universe which is best observed. It is the region in which we are located and the region into which physics as we know it has been successfully ~~may be~~ extrapolated. In the

xx

region between the two bounds, we may expect new physical phenomena which might require a modification of our present concepts of gravitational fields. Objects whose dimensionless potentials lie ~~xxxx~~ in the $1 > \frac{2GM}{c^2R} > 10^{-4}$ range may be unstable, whatever their densities, and either expanding or fragmenting.

There is at the present no theoretical justification for the existence of a potential bound of this value. Chandarasekhar and ~~Rxxxxx~~ Fowler give a potential limit governing the stability of polytropic spheres under post-Newtonian conditions. Their stability limit depennds on the mass in a quite differnet way from the observed M/R = constant.

It is of supplementary interest, however, in connection with the 10^{23} limit that the simultaneous solution of Fowler's equation for the critical limiting radius for stability in terms of the gravitational radius and Harlan Smith's density relation for the quasar 3C273 leads to values for 3C273 of Mass = $10^7 \odot$ and R = .01 pcs giving M/R = $10^{23.8}$ g/cm. If Fowler's theoretical relation and Smith's density estimate are to be believed the potential of at least one quasar appears to be in aggrement with the observed 10^{23} limit.

At this stage there is not much more direct evidence which can be adduced either to support or refute the existence of such a bound. We therefore proceed by assuming there exists a univeral potential bound whose value is of the order of 10^{23} g/cm and look for possible inferential consistencies ~~or~~ and contradictions.

The immediate inference of a potential bound is the prohibition of uniformly distributed matter of indefinite extent. The maximum radius, $\hat{R}$, permitted to an aggregate with mean density, $\bar{\rho}$, will be given by

$$\log \hat{R} \simeq 11.6 - \frac{1}{2} \log \bar{\rho} \qquad (1) \qquad (c.g.s.)$$

Slide 3 shows the hierarchal structure which derives from a potential bound. We ~~assume the simplifications~~ make the simplifying assumptions of spherical symetry and uniform density for all successive bodies. If we proceed along a logarithmic size or distance scale outward from a center of a star, we find, so long as we are in the interior of the star (shown cross-hatched in slide) that the potential will increase until it reaches the limiting value determining the surface. The star can become no larger and still remain stable. We then encounter a region devoid of matter until, in the case of the solar neighborhood, we travel about 1 parsec. Then, within successive radii we begin to encounter stars and find ourselves again in a region occupied by matter, but at a much lower densit~~ies~~y. This situation continues until we reach the potential limit again at the surface of a galaxy. There is then another void to the nearest external galaxay , then a region of matter to the potential limit reached at the surface of a cluster, but with ~~even~~ still lower density.

There are two things to notice in this logarithmic representation. First, are the alternate bands and gaps of matter and no matter. Second, is the regular pattern to the distribution of the upper bounds. We shall ~~return to~~ examine this pattern in a minute.

Carpenter 1930

I would next like to show a slide of Dr. Schmidt's red shifts of radio sources. which are seen to display in a logarithmic representattion, a similar band-gap distribution. This distribution is suggestive of a continuation of hierarchization and is qualitatively, at least, consistent with the continued operation of ~~the~~ a potential bound. We shall discuss the question of ~~a~~ quantative consistentancy ~~of red shifts bands with~~ with redshifts of the 10^{23} limit after looking into the regularities displayed in the previous slide. [Lights]

Charlier constructed an infinite hierarchal universe on the basis of the nth order cluster being related to the (n - 1)st order cluster by the relation,

(2)

which is the limiting condition for convergence.

In the part of the universe accessible to observation we find a quite different relation between the successive orders of aggregates. We find that the nubbers of elements in successive orders of aggregates are closely ~~follows~~ approximated by a harmonic exponent ~~xxxxxx~~ law.

$$N_i \doteq N_0^{\frac{(5-i)!}{5!}}$$, where N_0 is the maximum number of baryons in a star. $\sim 10^{58-59}$

$i = 1$ N_1 = maximum number of stars in a galaxy $\doteq \sqrt[5]{N_0} \sim 10^{12}$

$N_2 = \sqrt[20]{N_0} \sim 10^3$

$N_3 = \sqrt[60]{N_0} \sim 10$

$N_4 = \sqrt[120]{N_0} \sim 3$ extrapolating

Here the sequence must stop. So if this empirical harmonic exponent sequence adequately represents the scale-wise distribution of matter, our play can have only one more act.

The relations $M/R \leq 10^{23}$ gm/cm and $N_i \leq N_o^{\frac{(5-i)!}{5!}}$ allow us to construct a hierarchal model. From the harmonic exponent law for numbers of elements in successive aggregates and the potential bound, we may derive a sequence of masses, radii, and densities for each body in the model. For N_o and for the precise value of M/R we shall take the values for V444 Cyg A, the main sequence star with the largest known potential: viz. $M = 10^{34.5}$ grams, $\Phi = 10^{23.3}$ gm/cm. This latter value may not actually represent the value of the potential bound but it represents the value of a body known to be stable and it is more precisely known that any potential values for galaxies or clusters. From V444 Cyg A, we derive Table II.

	Calculated Radii $\log_{10}$ c.g.s	Observed Radii $\log_{10}$ (c.g.s) (H=100)	Megaparsecs $\log_{10}$	Mean Density $\log_{10}$ c.g.s
Star	11.2	--	--	0.3
Galaxy	22.8	--	--	-23
Cluster	25.7	24.8	2	-29
2°	26.6	26	25 to 30	-30.6
3°	27.2	--	--	-31.7

Abell derives a mean density on the basis of second order clustering of 10^{-29} to 10^{-31} gm/cm^3. The existence of 3° order clustering would reduce the density still further to almost 10^{-32} gm/cm^3.

Using the V444 Cyg A value for the potential bound, we find that the observed radii for clusters and second order clusters (assuming H = 100) are smaller than the calculated values. This would make the observed potentials larger than the stability limit and place clusters and 2° clusters in the instability zone.

The calculated masses are derived from numbers of elements in successive aggregates. These masses are thus lower bounds and the calculated corresponding radii are accordingly lower bounds. If the masses of clusters and 2° clusters (and hence radii) are actually larger as suggested by the virial theorem, then the ratio of observed potentials to calculated potentials is even larger than the values suggested by Table II. In other words, if the observed potentials corresponding to the table lie in the instability zone, then calculated potentials corrected for intergalactic mass cause the observed potentials to lie more deeply in the instability zone.

We may look upon the potentials exceeding the proposed potential limit either as a contradiction of the assumption of a universal limit at $10^{23.3}$ gm/cm <u>or</u> we may take that the bound is valid and the fact that the potentials of clusters and higher order clusters exceed this bound accounts for their expansion. It readily follows that the general expansion of the universe itself may result from Mu/Ru having a value which places the universe in the instability zone.

Slide 1.

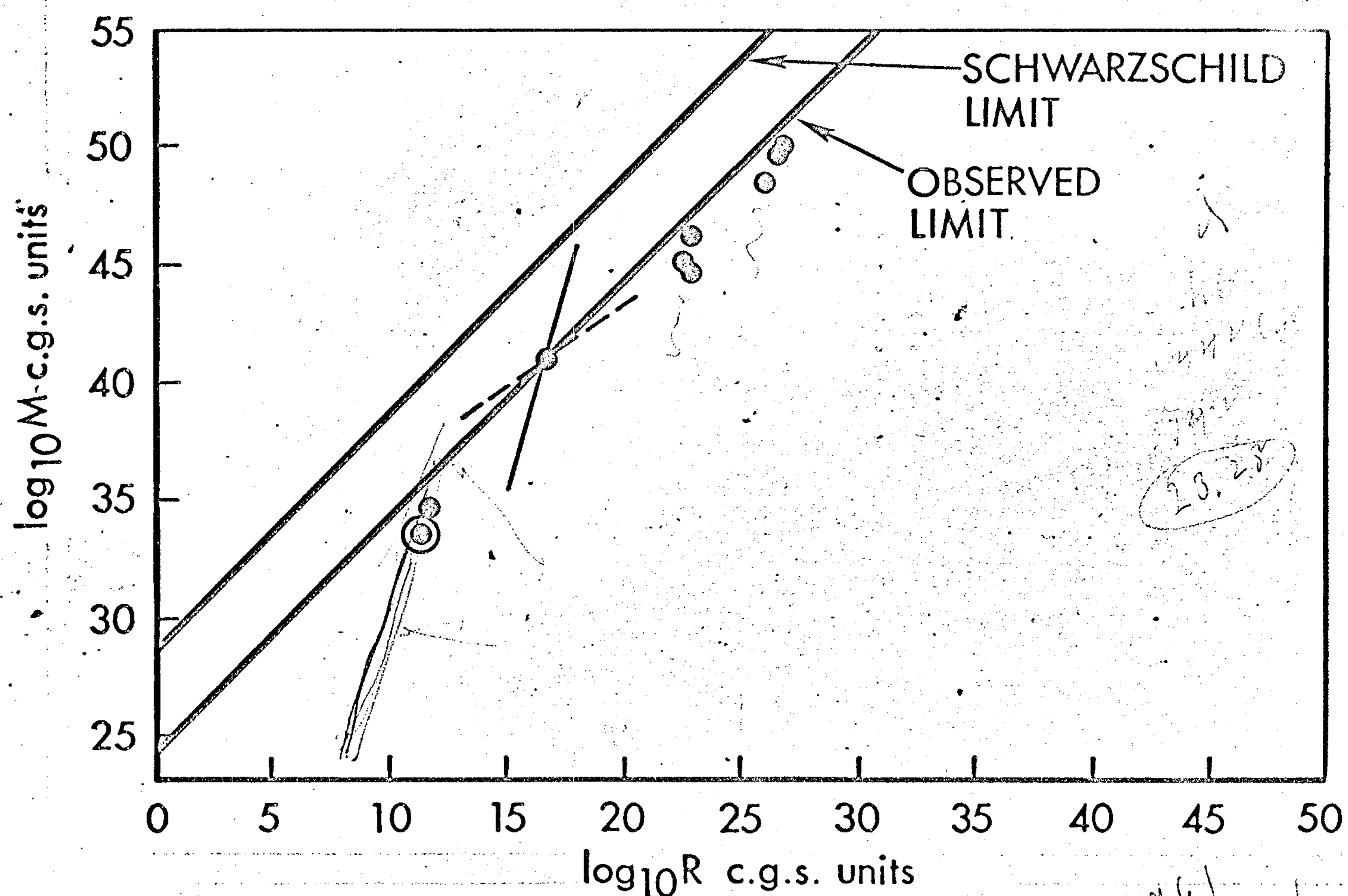
TABLE I

SUMMARY OF OBSERVED VALUES

SYSTEM	R	M	M/R
Stars			
V444 Cyg A	11.19	34.46	23.27
Mean of 40 Ecl.Bin.	11.54	34.21	22.66
Sun	10.843	33.299	22.456
Galaxies			
M87	22.3	45.9	23.6
M31	22.2	44.8	22.6
Mean of 7 Galaxies	--	--	22.6
Milky Way	22.3	44.3	22.0
Clusters			
Coma	25.95	49.40	23.45
Mean of 7 Clusters	--	--	22.6
Mean of 4 Clusters	25.5	48.1	22.5
2° Clusters			
Abellian Cell	26	49+	23
Local Super-cluster	25.7	--	--

All entries $\log_{10}$ c.g.s. units

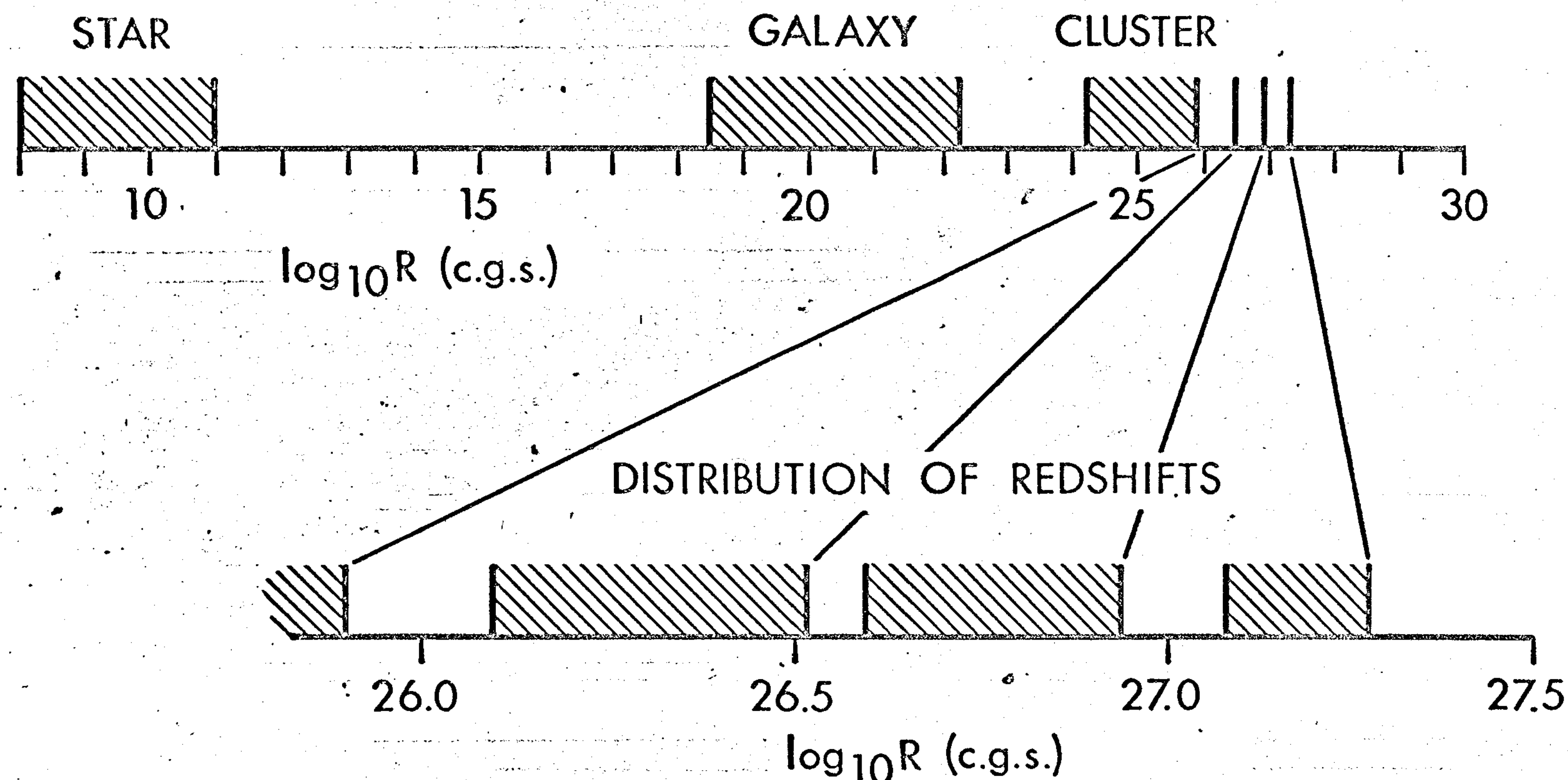
GRAVITATIONAL POTENTIALS OF COSMIC BODIES



A.J. Dec 1961
de Vau Col

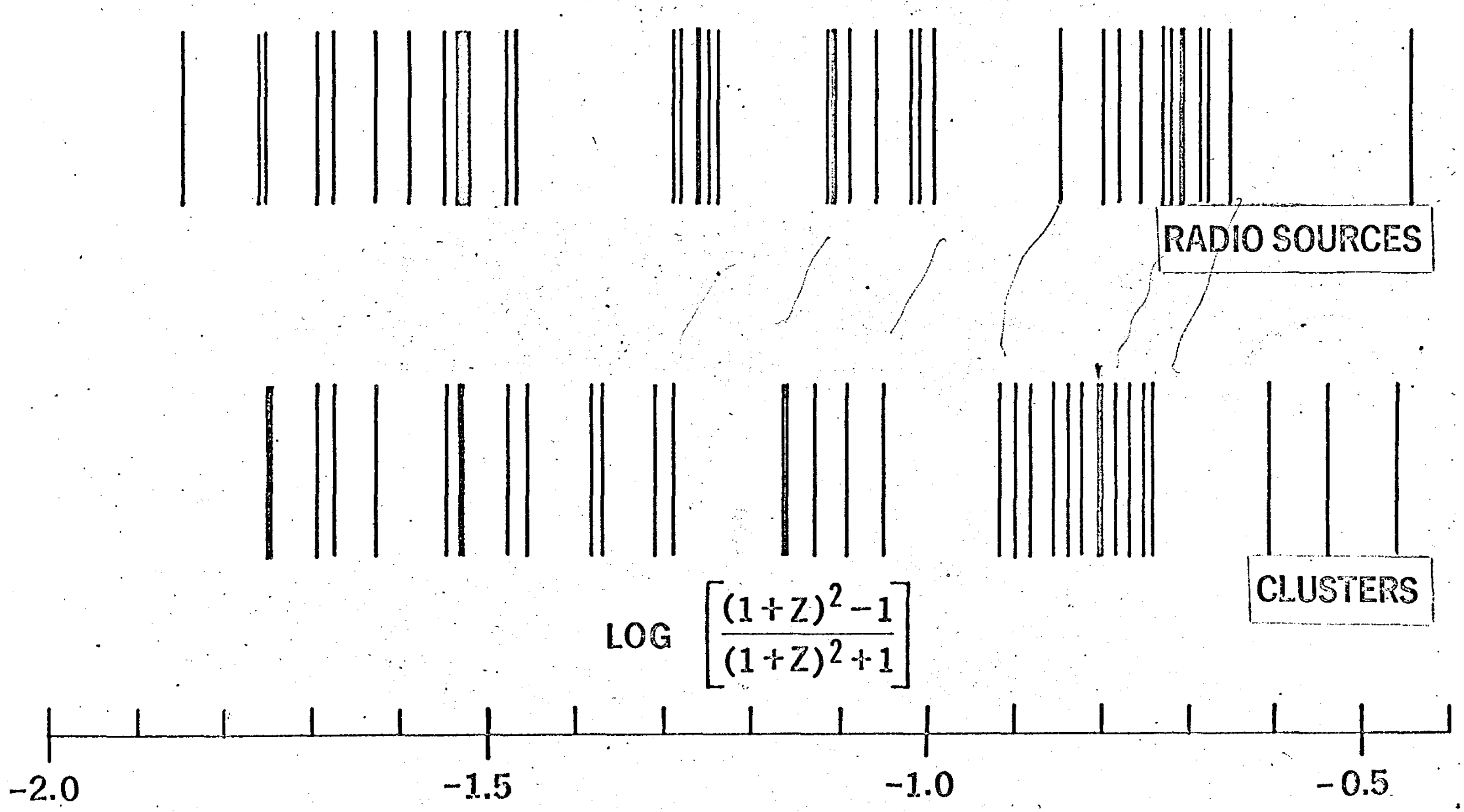
Slide 2

SCALEWISE DISTRIBUTION OF MATTER



Slide 3

REDSHIFTS



RADIO SOURCES
CLUSTERS
LOG $\left[\frac{(1+Z)^2-1}{(1+Z)^2+1}\right]$
-2.0
-1.5
-1.0
-0.5

Mailed to A.J.
Dec. 31, 1966

A Hierarchal Cosmological Model

The observation of equal maximum values of gravitational potential for stars, galaxies, and clusters of galaxies (A.G.Wilson, A.J. 71,402 1966) suggests the existence of a universal potential bound governing gravitational stability. The assumption that systems whose potentials lie in the zone between the observed maximum value and the Schwarzschild limit, ($10^{-4} < 2GM/c^2R < 1$), are unstable, whatever their densities or total energies, prohibits the stable existence of uniformly distributed matter of indefinite extent. Large masses in order to form stable systems must be structured hierarchically.

The existence of banded structures in the distributions of the redshifts of rich clusters of galaxies and radio sources (A.G.Wilson, Proc. 14th Int. Ap. Symp., Liege, 1966 (in press)) indicates the existence of one or more possible additional members of a hierarchal structure which would be expected as a consequence of the assumed universal stability bound. Estimates of the potentials of these indicated super systems place them within the instability zone, consistent with, and possibly causally related to, the observed general expansion.

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This paper read at the 123rd meeting of the American Astronomical Society UCLA, December 30, 1966. The above abstract and title to replace the abstract entitled, "A CHARLIER-TYPE COSMOLOGICAL MODEL".

Draft For 1966
Yearly report
Jan 4, 1966

A. G. Wilson

ABSTRACT

Gravitational Potentials of Cosmic Bodies

Observational evidence for the validity of the general theory of relativity rests on the three well known tests of the theory in its Schwarzschild form. (Advance in the perihelion of planetary orbits, curvature of geodesics near massive bodies, gravitational redshifts). An additional prediction of the Schwarzschild solutions for spherically symmetric gravitating systems is the existence of the potential limit, $2GM/c^2R < 1$, Where M and R are respectively the mass and radius of the system, G the gravitational constant and c the velocity of light.

The theory predicts the breakdown of the normal space-time characteristics of the metric at this limit where the gravitational radius equals the geometric radius.

Bodies whose potential equals the Schwarzschild limit cannot radiate and are thus invisible. Bodies whose potentials approach the Schwarzschild limit might be expected to manifest properties somewhat different from those of gravitating systems with the much smaller potentials which obtain for most bodies (e.g., $2GM/c^2R$ for the sun is $\sim 10^{-5}$). It has been shown, for example, that gravitating polytropes become unstable well before ~~this limit~~ the Schwarzschild limit is reached, and would be expected to collapse, expand, or disintegrate. Accordingly, a study of systems ~~with~~ having large potentials was

recommended ~~undertaken~~ as possibly providing additional evidence bearing on the validity of the general theory of relativity.

[handwritten margin note, partly legible: 1966] The values of potentials of various cosmic bodies - stars, galaxies, clusters of galaxies - were derived from old and new observational data. The search for systems with large potentials disclosed the remarkable fact that there was a common maximum value for observed potentials of stable cosmic bodies in the neighborhood of $2GM/c^2R = 10^{-4}$. This value, four orders of magnitude smaller than the Schwarzschild limit, appears to constitute some sort of stability limit itself; although neither the general theory of relativity, nor any other theory, predicts the existence of such a limit.

The investigation of large potentials ~~is now divided~~ divides itself into two phases:

1) Additional observational evidence confirming or refuting the existence of the 10^{-4} bound is being sought. Observational investigations are under way to derive the potentials of additional clusters of galaxies, and the potential of at least one ~~second~~ second order cluster.

2) Theoretical hypotheses as to possible causes of a potential bound with the value 10^{-4} are being examined. Theoretical inferences of such a bound are being compared with observations.

Observations which are consistent with the interpretation that there exists a transition from gravitational stability to instability at potentials of 10^{-4} include in addition to the commonality of the 10^{-4} upper limit to potentials of stable systems; 1) evidence for expansion of white dwarfs which have potentials exceeding 10^{-4}, 2) banded structure in the distribution of redshifts of radio sources and rich clusters of galaxies, hierarchal structure being demanded by a potential bound.

3) The general expansion itself. Current estimates of the mean density of matter in the observed sample of the universe require ~~for~~ that for a uniform distribution of matter ~~under~~ subject to a 10^{-4} bound, ~~the onset of~~ that instability onset at linear distances of the order of 25 megaparsecs, the observed size of second order clusters.

Applications of the hypothesis of the existence of a universal potential bound $\sim 10^{-4}$ to homogeneous cosmological models can be shown to lead to a positive cosmological constant, a positive curvature, and a deceleration parameter <u>less</u> than -1. Such a model universe would continue to expand for all ~~t > to the present~~ future time. The asymptotic values of such a model are the same as ~~in~~ for the Lemaitre-Eddington model.

Revision / Feb 17, 1967

Dr. A. G. Wilson, Associate Director, Environmental Sciences

Gravitational Potentials of Cosmic Bodies

Observational evidence for the validity of the general theory of relativity rests on the three well known tests of the theory derived from the Schwarzschild solution of the field equations. (Advance in the perihelion of planetary orbits, curvature of geodesics near massive bodies, gravitational redshifts.) An additional prediction of the Schwarzschild solutions for spherically symmetric gravitation systems is the existence of the potential limit, $2GM/c^2R < 1$, where M and R are respectively the mass and radius of the system, G the gravitational constant and c the velocity of light. The theory forbids the packing of more mass than M within the radius R, or defines an upper limit to the size of a body of given density.

With unusual properties of bodies at the Schwarzschild limit, bodies whose potentials approach the Schwarzschild limit might be expected to manifest properties somewhat different from those of gravitating systems with the much smaller potentials which obtain for most observed bodies (e.g., $2GM/c^2R$ for the sun is $\sim 10^{-6}$). It has been shown, for example, that gravitating polytropes become unstable well before the Schwarzschild limit is reached, and would be expected to collapse, expand, or disintegrate. Accordingly, a study of systems having large potentials was recommended as possibly providing additional evidence bearing on the nature of gravitational forces and on validity of the general theory of relativity.

During 1966 the values of potentials of various cosmic bodies - stars, galaxies, clusters of galaxies - were derived from old and new observational data. The search for gravitating systems with large potentials disclosed the remarkable fact that there was a common maximum value for observed potentials in the neighborhood of $2GM/c^2R \sim 10^{-4}$. This value, four orders of magnitude smaller than the Schwarzschild limit, appears to constitute some sort of stability limit; although neither the general theory of relativity, nor any other theory, predicts the existence of such a limit.

The investigation of large potentials divides itself into two phases:

1. Additional observational evidence confirming or refuting the existence of the 10^{-4} bound is being sought. Observational investigations are under way to derive the potentials of additional clusters of galaxies, and the potential of at least one second order cluster (i.e., a cluster whose elements are clusters of galaxies).

2. Theoretical hypotheses as to possible causes of a potential bound with the value 10^{-4} are being examined. Theoretical inferences of such a bound are being compared with observations.

In addition to the commonality of the 10^{-4} upper limit to potentials of stable systems observations which are consistent with the interpretation that there exists a transition from gravitational stability to instability at potentials of 10^{-4} include (1) expansion of massive radio sources which appear to have potentials exceeding 10^{-4}, (2) banded structure in the distribution of all large redshifts of radio sources and rich clusters of galaxies. Such a banded structure is what would be expected if there existed higher orders of clustering than second as demanded by a potential bound.

(3). The general expansion itself. Current estimates of the mean density of matter in the observed sample of the universe require that for a uniform distribution of matter subject to a 10^{-4} bound, instability onset at linear distances of the order of 25 megaparsecs, (one megaparsec equals three and a quarter million light years) the observed size of second order clusters.

Applications of the hypothesis of the existence of a universal potential bound of 10^{-4} to homogeneous cosmological models can be shown to lead to a positive cosmological constant, a positive curvature, and a deceleration parameter less than -1. Such a model universe would not oscillate but continue to expand for all future time.

Mémoires de la Société Royale des Sciences de Liège
also Proceedings of the 1966 Symposium

11. — DISTRIBUTIONS OF REDSHIFTS OF RADIO SOURCES AND RICH CLUSTERS

A. G. WILSON
Douglas Advanced Research Laboratory, Huntington Beach, California, U.S.A.

The distribution of optical redshifts of radio sources is non-uniform (Figure 1 upper part), showing marked clumping into bands separated by distinct gaps. With the errors of individual redshifts not much larger than the line thickness; these bands and gaps appear not to be statistical fluctuations. Although the present total available sample of redshifts is small, the probability of statistical fluctuations creating this type of non-uniformity is remote. A Poisson -chi^2 test shows probabilities of less than $1:10^6$ that the observed distribution would occur in a random sample with either uniform density or density increasing with square of distance.

There is the possibility that observational selectivity factors have generated the banded distributions. These selectivity factors are primarily those contributing to ease of observation. There is a declination factor, most of the sample being in the northern sky. There is a brightness factor, the optically bright objects being chosen to effect shorter exposure times. While the brightness factor has biased the sample toward emphasis on the giant D type radio galaxies, it is difficult to see how this factor or a declination factor could generate apparent bandings in a distribution which is in reality uniform.

If the distribution is not attributable to either statistical fluctuations or to observational selectivity factors, the next most likely hypothesis is that the radio sources are physically clustered. An investigation of the angular distribution of the sources indicates that physical clustering partially accounts for the clumping, but many sources having nearly equal redshifts are widely separated in angle. Consequently, assumption of large scale non-uniform distribution beyond the scale of recognized clusters seems necessary to account for the observed distribution.

A second type of cosmic object with large redshifts which also exhibits a similar banded redshift density distribution is the rich cluster of galaxies. In (Fig. 1 lower part) are shown the mean redshifts of all clusters of galaxies in which redshifts of one or more individual galaxies have been measured.

In the cluster sample, the Abell [5] *richness* of the cluster is correlated with the position of the clusters's redshift within a band. The richer clusters are found toward the center of a band, the sparser clusters toward the edges of a band. The variation of richness across the bands in this manner strongly reinforces an interpretation of the bands and gaps as reflecting an actual distribution of matter and not being due to random fluctuations or selectivity effects.

If the radio source and cluster redshift distributions are compared. Beyond the redshift value, log = — 1.4, three corresponding bands are identifiable. From the respective positions of the bands, it appears that there is a displacement of the redshifts of the radio sources with respect to the redshifts of the clusters.

If we compare the values of the redshifts at the lower and upper limits of the corresponding bands, we find the redshift displacement, Δz, of the radio sources with respect to the cluster redshifts is systematic following in approximation the relation

$$\Delta z = (\text{constant}) \cdot (\bar{z}_c)^{3/2}.$$

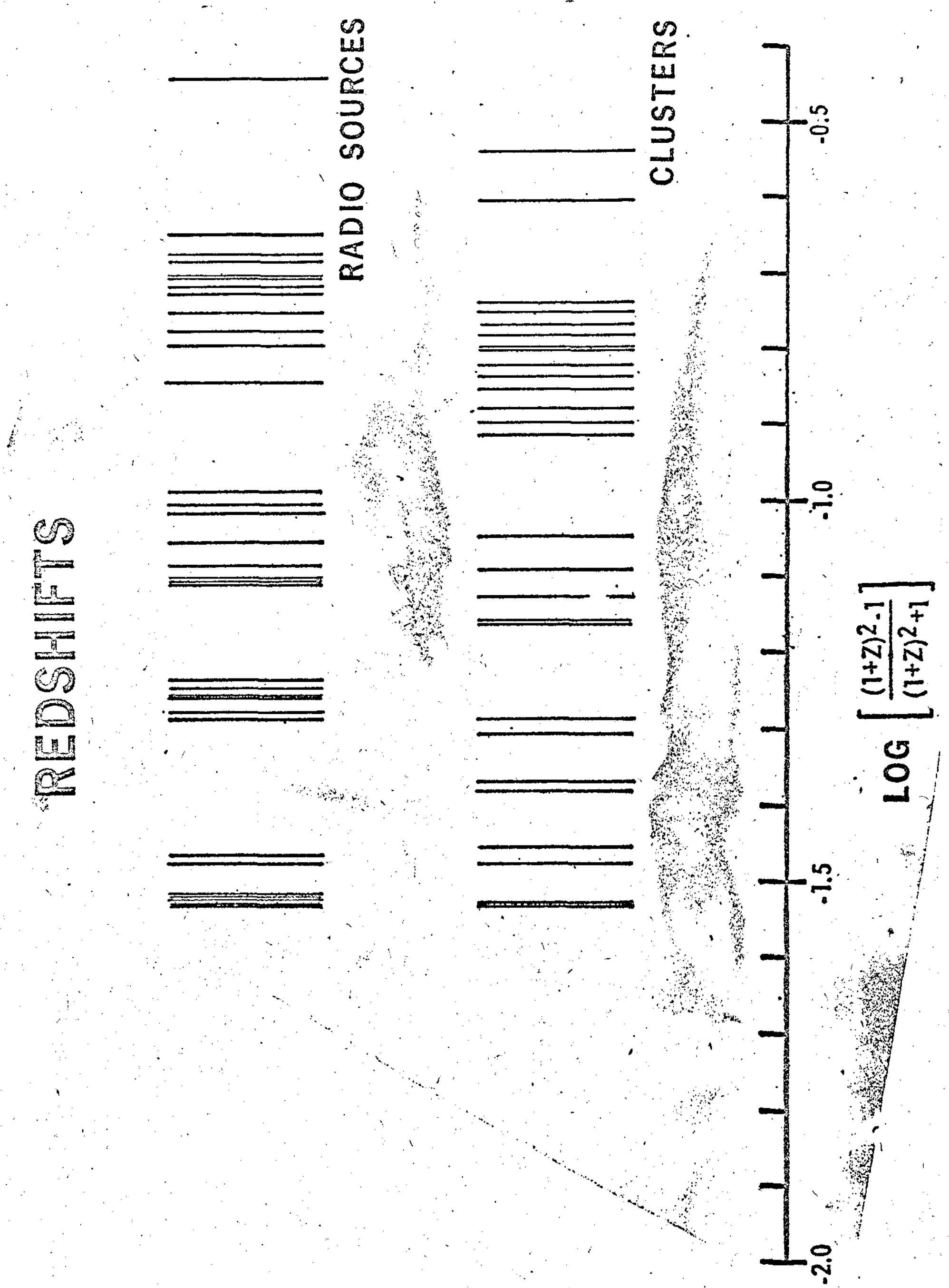

Fig. 1. — Comparison of distributions of optical redshifts greater than $z = 0.035$. Upper distribution redshifts of radio galaxies, lower distribution primarily redshifts of cD type galaxies in rich clusters [1, 2, 3, 4].

In the ranges under consideration in Figure 1, we are in essence comparing the optical redshifts of the two types of D galaxies — those which are radio sources and those which are not.

It is not understood why certain D galaxies are radio sources and others are not, nor why the radio D galaxies are found in the poorer clusters and the radio quiescent D galaxies in the richer clusters [6]. Morgan suspects that the radio D galaxies are the largest and most massive single structures known [7]. If this be so, we have a possible explanation for the differentiation between radio and non-radio D galaxies in the displacement of the radio source redshifts with respect to the non-radio redshifts : the displacement may be due to an Einstein shift. If the radio sources are more massive than the ordinary D galaxies, part of the observed redshift may be a gravitational shift.

If we assume that radio sources and clusters are cosmically distributed in the same manner, the observed bands and gaps for both radio sources and clusters must be identically distributed in *distance* requiring that the differences in *redshifts* between the radio source and cluster bands be attributed to some other cause than cosmic distance operating in accord with Hubble's Law. In other words, the cosmic or Hubble components of the observed redshifts of radio sources and clusters at the same distance must be the same, but superimposed on the cosmic redshifts is a second component of the observed redshift which is different for the radio sources and cluster D galaxies. The differences in this second component are manifested as the redshift displacement, which may be an Einstein shift. If this be the case, the emission lines in the optical spectra of radio galaxies come from sources which are located in regions of higher potential.

If the displacement is a gravitational redshift, then for a radio source and cluster having the same cosmic redshifts,

$$\Delta z = \frac{G}{c^2}\left[\frac{M_r}{R_r} - \frac{M_g}{R_g}\right]$$

where the subscript r designates a radio source and g a cluster galaxy.

If we designate nearer objects with a « second » and more distant objects with a « prime » then since $\Delta z' > \Delta z''$, we have

$$\frac{M_r'}{R_r'} - \frac{M_r''}{R_r''} > \frac{M_g'}{R_g'} - \frac{M_g''}{R_g''}$$

(the right member may be zero). Taking any change in potential of cluster D galaxies as a standard of reference with value unity, we have for all values of z beyond the local (— 1.4) boundary that

$$\frac{M_r'}{R_r'} > 1 + \frac{M_r''}{R_r''}$$

which is to say that M_r/R_r is decreasing with time. Hence, the redshift displacements between the radio sources and clusters may be interpreted as resulting from expansion and/or mass loss in the radio sources. However, only that portion of expansion in excess of any cluster Dgalaxy expansion is reflected in the displacement.

The observational conclusion that the radio sources are expanding is consistent with theoretical models, and also is in accord with explanations of the radio counts based on secular power decrease.

In addition, the superposition of the bands in the two distributions which results upon removing the Einstein shift strengthens the evidence for the non-uniform distribution of matter over distances greatly exceeding the sizes of any presently recognized clusterings, a matter which would have important cosmological implications.

Although this interpretation affords a consistent qualitative solution, quantitative conclusions must await a substantial increase in the size of the redshift samples.

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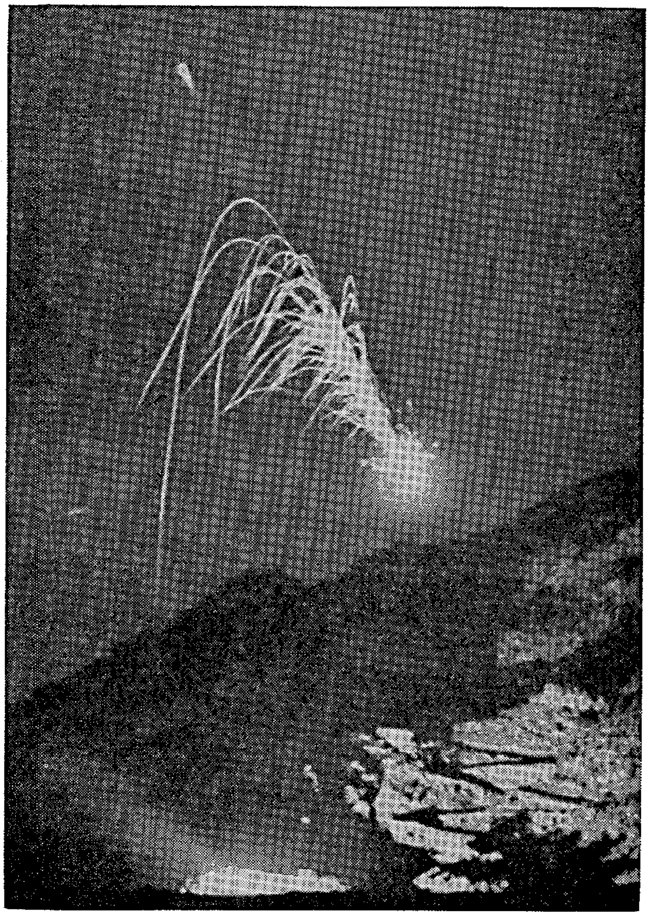

A test of artificial meteors—December 16, 1946.

THE ANNIVERSARY OF A HISTORIC FAILURE

by Albert G. Wilson

Albert G. Wilson is an associate director at the Douglas Advanced Research Laboratory in Huntington Beach, California, where he is in charge of a laboratory for environmental science. A Caltech alumnus (MS '42, PhD '47) and a staff member of the Mt. Wilson and Palomar Observatories from 1947 to 1953, he was a member of the team which took part in this "historic failure."

The pages of *Engineering and Science* magazine provide a historical record of many of the achievements and successes of Caltech researchers—alumni and staff. The dead ends and failures rarely appear in print. Fortunately for publication costs, few people want their failures recorded. However, now and then certain types of failures become historic and deserve a place in the record.

The 17th of December this year marks the 20th anniversary of such a historic failure—the first attempt to launch particles into space with escape velocity. A team of Caltech men headed by Fritz Zwicky, professor of astronomy, in cooperation with Army Ordnance, the Johns Hopkins Applied Physics Laboratory, the Harvard College Observatory, and the New Mexico School of Mines, put together a project in White Sands, New Mexico, combining the hardware components available in 1946 in a way which, theoretically, would launch a few pellets in orbit about the earth or throw them off into interplanetary space. Two marginal devices and one valid motivation made the attempt worthwhile. The devices were the V-2 rocket and the Monroe rifle grenade or "shaped charge." The motivation was to generate a shower of artificial meteors in order to calibrate the luminous efficiency of natural meteors.

The possibility of throwing something up that would not come down again fired the imagination. Although there had been 16 postwar V-2 rocket firings, this was to be the first night firing of a V-2 in the United States. In those days the launching of a V-2, with or without an instrument on board, was as much news as the launching of a Gemini today. Dr. Zwicky, who designed the experiment, placed the event in historical context: "We first throw a little something into the skies, then a little more, then a shipload of instruments—then ourselves."

A V-2 rocket was equipped with six 150-gram penolite shaped charges with 30-gram steel inserts. These were set to fire at times after launching that would eject the slugs of molten steel at heights of approximately 50, 65, and 75 kilometers. At these heights the ejection velocities of from 10 to 15 km/sec would place the slugs either in orbit or on escape trajectory. The ultimate fate of a slug would depend on its mass and velocity. Most would be meteors, but some might not be consumed.

To determine the destinies of the meteors, a battery of K4 aerial cameras equipped with rotating shutters was scattered over the White Sands Proving Range. One of these was equipped with a transparent objective grating to obtain spectra of the V-2 exhaust jet and the luminous artificial meteors launched. The sites were selected to acquire optimal triangulation data. In addition the Caltech eight-inch Schmidt camera was removed from its usual house at Palomar and set up a few miles south of the launch site to photograph the flight of the V-2 rocket and of the particles ejected from the shaped charges. Astronomers at nearby observatories with wide angle telescopes also focused in on the firing.

As this 17th postwar V-2 left the pad at 22^h 12^m 49^s mountain standard time, expectations were high. There was a feeling that history was being made. There was also the anxiety that has become as much part of every launching as the countdown. (The 16th rocket, fired a few days earlier, had tilted on lift-off and travelled 131 miles horizontally.) Lifting slowly, No. 17 filled the whole range with sound and, falling upward, held true to its course—5° tilt north. The shutters clicked and telescopes tracked—then burnout. But the rocket could still be followed by the red glow from its exhaust vanes. The time came and passed for the three pairs of charge detonations. Nothing was seen. The rocket mounted to a new record of 114 miles, then returned to earth.

Films were hastily developed in hope of seeing on the emulsion what could not be seen in the sky. But there were no trails. Tests of the charges made on previous evenings had been in every way successful. Had the charges fired, but been undetected? Subsequent investigations have not solved the mystery of just what did happen.

Just as man's first attempts at flight in the atmosphere failed, the first attempt to reach space with a chance of succeeding also failed. It is significant, however, that whereas the span between the first attempts to fly and the first successful flight is measured in centuries, the span between the first attempt to achieve orbital velocity and the successful orbiting of Sputnik was only one decade. Those who participated directly and indirectly in this experiment, though failing to launch the space age on the night of December 17, 1946, have to their credit an important contribution leading to later triumphs. Zwicky's idea was ultimately vindicated, when success crowned the *second* experimental firing of shaped charges from a rocket on October 16, 1957—twelve days after the Russians launched Sputnik.

Reprinted from Engineering and Science Magazine, December 1966
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7

TELEPHONE: ARDWICK 3333

DEPARTMENT OF ASTRONOMY,
THE UNIVERSITY,
MANCHESTER, 13.

ZK/SN

19th January, 1967.

Dr. A.G. Wilson,
Douglas Advanced Research Laboratories,
Huntington Beach,
California,
U.S.A.

Dear Dr. Wilson,

The Measure of the Moon

Proceedings of the Second International Conference on Selenodesy and Lunar Topography, held in the University of Manchester, England. May 30 - June 4, 1966.

Under the same cover I take pleasure in transmitting to you the page proofs of your contribution to the above-mentioned volume with the request that you correct it for misprints and return it to me (at the above address) at the earliest possible date.

Corrections received later than two weeks after the date of this letter, may be difficult to effect without interfering with the Publisher's schedule.

Information concerning orders of additional reprints of your contribution for yourself or your institution are being communicated to you by the Publisher, under separate cover.

Yours sincerely,

Zdenek Kopal

Zdenek Kopal.

THE USE OF THE 48-INCH SCHMIDT TELESCOPE FOR SELENODETIC OBSERVATIONS

ALBERT G. WILSON* and
Donna S. Wilson*

Photographing the Moon against star-field backgrounds is recognized as a most useful technique for measuring the difference between universal and ephemeris times, determining various higher-order motions of the Earth and Moon, and investigating certain geodesic problems. Because simultaneous photographic observations of the Moon and stars might be effectively used with new reduction techniques developed by Kopal for determining lunar librations from star positions independent of reference to the lunar limb, Kopal recently requested that we investigate the feasibility of utilizing the 48-inch Palomar Schmidt as a moon-star camera. The Schmidt is rarely in use when the Moon is above the horizon and good supplementary use could be made of the instrument if it should prove adaptable for lunar work.

The problem of photographing the Moon against its star-field background is complicated by the motion of the Moon, the large difference in brightness between Moon and stars, and the brightness of the sky in the neighborhood of the Moon. It is also necessary that the Moon and stars be photographed so that the position of the telescope when the Moon is exposed be the same as the average position for the stars in order to minimize refraction and off-axis effects.

A highly successful, but sophisticated, camera for making simultaneous photographs of the moon and a star field has been developed by Markowitz (1954, 1962). In this camera the differential motion of the Moon is corrected by continuously changing the tilt of a plane parallel glass filter. An exposure of from about 10 to 25 seconds is made of the star field with the Moon being tracked by the changing tilt filter. The filter drive is set so that the filter is parallel to the photographic plate at exactly mid-exposure. At the instant the filter is parallel, the Moon image is exposed, being recorded in its correct position with respect to the stars.

In considering the Schmidt for this problem, it was felt that since the $f/2.5$ speed allows the recording of 12th-magnitude stars (under dark sky conditions) in about 5 seconds, that the Moon-motion problem could be surmounted by driving the telescope at lunar rate and making a very short exposure. The star images would be only slightly trailed being still quite useful as fiduciary marks. The Schmidt has no declination rate control but for the short exposure, this should introduce very little blurring of the Moon's image.

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The Moon-brightness problem was to be solved by introducing a neutral filter having a dense central circle sufficiently large to mask the lunar image. Mr. George Kocher prepared a set of experimental filters by exposing 5×7 103a−O plates in contact with a template having a suitable central hole. The scale of the Schmidt is only 69″ arc/mm and marginal at best for recording lunar features. Only the sharpest of properly exposed images could be of use for libration measures.

The sky problem could be handled by photographing the star field through a suitable filter. However, since this extends the exposure time and the star images are trails, the usual gains from reducing sky brightness cannot be effected.

Time was secured on the nights of May 31 and June 1 through the courtesy of the Director of the Mt. Wilson-Palomar Observatories to make a feasibility study on full moon. The 5×7 plate holder with field flattening lens was employed. The chief limitation to the experiment was the Schmidt shutter which has an operating time of three seconds and a minimum effective exposure time of some 4 to 5 seconds. The mounting of an auxiliary shutter would constitute a major and expensive modification for which neither time nor money was available.

The results were highly unpromising. It was not possible to record the lunar surface and stars on the same plate. The intense brightness of the sky in the neighborhood of the Moon, although the transparency was good, vitaited the attempt.

The conclusions of the experiment are that the Schmidt is not useful as a Moon-camera without extensive and expensive modifications. While it may be possible to adapt the Schmidt for simultaneous moon-star field photography through diaphragms to change the f ratio, building a system of dual shutters, introducing suitable combinations of filters, and a declination rate drive, none of this seems worthwhile. Since telescopes built for one purpose are rarely adaptable to quite different purposes, I we recommend development of instruments following the successful designs of Markowitz.

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Dir. Report 1965–1966
Mt. Wilson + Palomar

unusually bad weather of the winter season permitted completion of the observations for only a single field (four plates in each color) and fragmentary observations for three other fields.

Dr. George Wallerstein of the University of Washington obtained spectra in the visual and red regions of the hydrogen-poor star HD 30353. With the help of two graduate students, T. Greene and L. Tomley, he is analyzing the spectrum to obtain abundances of H, He, C, N, O, and Ne. The problem is intrinsically difficult because the source of opacity is unknown, necessitating a procedure of successive approximations starting with electron scattering as the source of opacity and computing abundances to first order. With first-order abundances, the opacity can be improved. To obtain the distance and reddening of HD 30353, nearby B stars have been observed for comparison of the strengths of their interstellar D lines. In addition, Danziger has obtained photoelectric scans of HD 30353 and three nearby B stars to establish the effective temperature and reddening.

Wallerstein has started observing F and G stars of luminosity class Ib in the red region to search for lithium-rich supergiants. Seventeen stars show no lithium line, indicating that the F-GIb stars have probably mixed to considerable depths at some time in their past history. Deep mixing of stars of 5 and 9 $\mathfrak{M}_\odot$ is expected from the time scales of evolutionary models by Iben. A surprising by-product of the survey was the discovery of Hα emission accompanied by asymmetric absorption in ϵ Geminorum and HR 3045. Zirin found ϵ Gem to have λ10830 of He I in emission. Since the Hα line of ϵ Gem has been observed by Kraft, Preston, and Wolff to have a normal absorption line at Hα on at least one occasion, it appears that some sort of transient chromospheric activity was taking place in January 1966.

Wallerstein obtained two more spectrograms of HD 128220, a double-line spectroscopic binary of types sd 09 and about G0 III, in both the yellow and blue. An orbit is being computed in cooperation with Mrs. S. Wolff of Berkeley. The masses of both stars lie in the range of 2–5 $\mathfrak{M}_\odot$. Such a large mass for the hot subdwarfs is interesting because it appears to be considerably greater than the masses of white dwarfs and suggests that the star must lose considerable mass before evolving into a white dwarf.

Dr. Robert L. Wildey of the United States Geological Survey Center of Astrogeology, Flagstaff, Arizona, used the 100-inch telescope for three nights in August 1965, beginning just past full moon. He employed the coudé scanner with the new cold boxes, pulse amplifiers, and digitized output to attempt to detect lunar luminescence along lines of drift, at lunar orbital rate, across the equatorial belt of the moon and at selected spots in the vicinity of Tycho, Copernicus, Kepler, and Aristarchus. The observations are negative to a limit of detectability of about two to three per cent of continuum level. Greater precision may emerge on further reduction, however, enabling differences of a small fraction of a per cent to be detected if they are present. The technique used is probably the most accurate yet employed in searches for lunar luminescence.

The nebular (B) spectrograph with the 100-inch telescope has been used by Dr. A. G. Wilson of the Douglas Aircraft Company to observe the redshifts of bright galaxies in nearby clusters. The purpose of the program is to study the spatial distribution of clusters and to investigate suspected regularities in redshift distributions. The nearby clusters so far observed appear not to be randomly distributed. Mean redshifts of clusters beyond the local Virgo–Ursa Major complex and closer than $z = \delta\lambda/\lambda = 0.09$ appear to possess an unexplained regularity that is closely represented by the one-parameter expression

$$\log_{10} z = -\frac{5}{3} + \frac{n}{4}\log_{10} 2 \qquad n = -1, 0, 1, 2, \ldots, 9$$

For most of the clusters in this range, the relative error, $\delta z/z$, of this formula is less than 1%. More distant clusters appear to be nonuniformly distributed, their redshifts showing a nonstatistical banded distribution. Comparison with Schmidt's redshifts of radio sources shows the existence of a similar banded distribution for the radio sources. Wilson suggests that these distributions may be indicative of the clustering of both clusters and radio sources on a larger scale than that of any currently recognized aggregate of matter.

Robert L. Younkin of the Jet Propulsion Laboratory of the California Institute of Technology has continued work on spectrophotometric measurements of the planets. Measurements of Mars during the 1965 apparition have shown: (1) The two near-infrared reflectance spectral features of limonite (commonly assumed to be the surface material of Martian bright regions) are both absent from the integrated radiation from the disk. (2) The reflectances of a typical Martian bright area and a dark area exhibit no observable difference in the limonite spectral features. (3) In the visible spectral region the absolute energy distribution of areas of both types increases steeply to the red.

Younkin and Münch have measured the energy distribution of the rings of Saturn when the rings were fairly open. The rings were found to be colorless beyond the visible to 1.1 μ. No evidence was found for the weak ice absorption bands at 1.05 μ previously reported in the literature.

Mr. Younkin, with Dr. Hyron Spinrad of the University of California at Berkeley, has measured the vanadium-oxide abundance in several cool stars. A decreasing abundance with phase was exhibited by *o* Ceti. A relatively small abundance in χ Cygni was attributed to a lower O/C ratio. The variation of water vapor with phase in *o* Ceti was studied by Dr. Spinrad and Miss D. M. Pyper of Berkeley and R. L. Newburn, Jr., of the Jet Propulsion Laboratory. Measurements at Mount Wilson and at Kitt Peak indicated a reduction of water with approach of maximum light and possible real variations of water abundance from one cycle to the next.

INSTRUMENTATION

Electronics Laboratory

Under the supervision of Dennison, the major effort of the Astro-Electronics Laboratory at Caltech during the past year has been centered on a new data-acquisition system for the 200-inch Hale telescope. The purpose of this system is to collect basic data from photoelectric photometers along with all relevant observing parameters, and to record this information for subsequent computer reduction.

Because of the general success of pulse counting, probably almost all future photoelectric measurements will be made with this technique. In the case of photomultiplier tubes, each photoelectron released from the photocathode generates a pulse that can be amplified and counted. The output of photoconductive devices, such as those used for infrared measurements, is a direct current that can be amplified and converted by a voltage-to-frequency converter into a series of pulses. The pulse rate at the output of the converter is proportional to the photoconductor current.

The new 200-inch data system is designed to count pulses from either photomultipliers or voltage-to-frequency converters in two reversing counters. In general, one photomultiplier is exposed to the sky, the other to sky plus object. A mechanical interchange of the two light paths at a subaudio frequency is carried out with synchronous commutation of the two reversing counters. The net counts are a measure of the difference in light flux at the two photometer apertures.

(Wilson)

The paper, "Homogeneous Cosmological +3
Models with Bounded Potential," applies +3
a consistency argument within general +9
relativity theory to select allowable ho- +5
mogeneous relativistic cosmological mod- -2
els. Specifically, the existence of the +9
"Schwarzschild Limit," implied by gen- +4
eral relativity, requires that non-empty +4
homogeneous relativistic models be closed -3
and expand monotonically and accelera- +5
tingly for all future time.

<u>Homogeneous Cosmological Models with</u> +4
<u>Bounded Potential</u>, A. G. Wilson and +12
D. G. B. Edelen, Douglas Paper 4488, +7
DARL Research Communication No. 29. +4
Under the assumptions that the gravita- +5
tional potential $2Gm/c^2r$ is everywhere +6
less than unity, and that the physically +8
realizable pressure in the universe is +10
everywhere no less than zero and no +14
greater than photon gas pressure, it is +7
shown that homogeneous models based on +1
the Walker-Robertson line element have +4
zero density if the curvature parameter k 0
is 0 or -1. Allowable closed universes +6
(k = +1) have a positive cosmological con- -1
stant, λ, and must in all future time ex- +3
pand monotonically without limit. As- +8
ymptotic values for distant future times +5
are presented in the usual notations: +14
$H = c\sqrt{\lambda/3}$, $q = -1$, $p = 0$, and $\rho = 0$. The -1
cosmological constant is bounded below +6
by $3H_0^2/c^2$, where H_0 is the present value 0
of the Hubble parameter.

HOMOGENEOUS COSMOLOGICAL MODELS WITH

BOUNDED POTENTIAL

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ABSTRACT

Under the assumptions that the gravitational potential, $2Gm/c^2r$, is everywhere less than unity, and that the physically realizable pressure in the universe is everywhere no less than zero and no greater than photon gas pressure, it is shown that homogeneous models based on the Walker-Robertson line element have zero density if the curvature parameter k is 0 or -1. Allowable closed universes (k = +1) have a positive cosmological constant, λ, and must in all future time expand monotonically without limit. Asymptotic values for distant future times are in usual notations: $H = c\sqrt{(\lambda/3)}$, $q = -1$, $p = 0$, and $\rho = 0$. The cosmological constant is bounded below by $3H_o^2/c^2$ where H_o is the present value of the Hubble parameter.

I. POTENTIAL BOUNDS

The existence of an upper bound for the value of gravitational potential which can develop anywhere in the universe is suggested by both theory and observation. If the dimensionless potential, Φ, is taken to be $2Gm/c^2r$ where G is the gravitational coupling constant; c, the velocity of light; m the total mass contained within <u>any</u> radius r (usually, but not necessarily, measured from the center of some cosmic body), then the existence of a universal bound, U, requires that

$$\Phi \leq U \qquad (1.1)$$

everywhere for all r.

From the field equations of general relativity several theoretical values for U have been derived. The largest bound is the Schwarzschild limit for which U has the value unity. This bound is derived by equating the Schwarzschild exterior solution of the general relativistic field equations to the interior solution which results when the interior is assumed to consist of an incompressible perfect fluid with constant proper density (Tolman 1934). The Schwarzschild limit constitutes an "ultimate" bound on the potential in the sense that above the value of unity the metric no longer preserves normal space time characteristics. Eddington (1923) showed that the pressure in the Schwarzschild solution becomes infinite at the value $\Phi = 8/9$. As a consequence, no physical system consisting of a perfect fluid with uniform proper density could ever actually attain the Schwarzschild limit. More recently several investigators have established theoretical upper bounds for Φ under more general and realistic assumptions. Buchdahl (1959) has established 8/9 as an upper bound for the potential in a manner which is independent of the relation between pressure and energy density. Bondi (1964) has also generalized these results. He has derived a value of $4[3\sqrt{2}-4] = 0.970$ as a rigorous upper bound for the potential independent of <u>any</u> assumptions about the equation of state subject only to the restriction that the density be nowhere negative. Bondi has also found under certain assumptions governing adiabatic stability that Φ must be less than 0.62. Chandrasekhar (1964) has considered the dynamic stability of polytropic gas spheres under relativistic and post Newtonian conditions and finds that for any finite value of the specific heat ratio, γ, that dynamical instability always occurs before the potential reaches 8/9. The Newtonian condition for dynamical stability is that $\gamma > 4/3$. Chandrasekhar finds that for γ slightly larger than 4/3, the relativistic condition for radial stability is $\Phi < (\gamma - 4/3)/K$, where K is a constant of the order of unity (dependent on the polytropic index). Similar results have been derived by Fowler (1966).

In addition to these relativistic and post-Newtonian bounds suggested by theory, there is also observational evidence suggesting the existence of a potential bound for non-degenerate cosmic bodies (Wilson 1966). The maximum values of the mass/radius ratio which occur in the samples of stars, galaxies, and clusters of galaxies which have been measured are found to be closely the same for each species of cosmic body. The common value is approximately 10^{23} g/cm. The ratio is derived by three separate methods: from orbits of eclipsing binaries in the case of stars; from rotational dynamics of galaxies, and from the virial theorem applied to clusters. Table 2 summarizes the results

TABLE 1

RELATIVISTIC POTENTIAL BOUNDS

Bound	Symbol	Constraints	Upper Limit $2Gm/c^2r$
Schwarzschild	U_S	$\rho \equiv$ constant	1
Eddington	U_E	$\rho \equiv$ const., p finite	0.888
Bondi I	U_B	ρ not increasing from center, $p \leq \rho c^2/3$	0.638
Bondi II	U_A	adiabatic stability $p \leq \rho c^2/3$	0.620

TABLE 2

MAXIMUM OBSERVED GRAVITATIONAL POTENTIALS

System	Object	$\log_{10}(m/r)$ g/cm
Star	V444 Cyg A	23.27
Galaxy	M87	23.6
Cluster	Coma	23.5

for those bodies found to have the largest m/r ratios (Allen 1963). The value of the m/r ratio for a second order cluster has not been observed directly. However, based on the mean cluster mass, the observed number of clusters in second order clusters, and their independently estimated radii (Abell 1961, de Vaucouleurs 1960), the same value of approximately 10^{23} g/cm is found. The cosmic bodies for which this bound appears to obtain are those which are non-degenerate and stable. Whether it also holds for quasars, radio galaxies and white dwarf stars, is at present uncertain. Taking the bounding value for m/r to be no smaller than $10^{23.6}$ g/cm (value for M87), the observed upper bound U_o, of Φ at the present epoch is $10^{-4.3}$.

The value of the observed bound, U_o, is not equal to any of the several theoretical values of U derived from general relativity or post-Newtonian approximations. Although the nature of U_o, and even its reality, are uncertain, a nearly identical limit governing systems that are aggregates of particles that may be either atoms, stars, or galaxies suggests the existence of a bound which is independent of the equation of state or of any assumptions concerning pressure-energy density relationships. Despite the discrepancy in values, there exists in effect an observed potential upper bound which corresponds phenomenologically to the existence of potential bounds,as predicted by theory. This phenomenological correspondence may be taken as the rationale for postulating the existence of a universal potential upper bound and investigating its cosmological inferences. The discrepancy between observed and theoretical values for the upper bound dictates that no single value for the upper bound be assumed, but that relation (1.1) be adopted as a postulate without specifying the value of U. Whenever the values of the various bounds possess diverse inferences, the implications of each bound must be discussed separately.

In postulating the bounding relationship, (1.1) we shall wish to require that the upper bound, U, be global, a less restrictive condition than universal. By a global potential upper bound we shall mean that if a sphere of radius r is circumscribed about any point, P, as center, then for every choice of P and for r sufficiently large, the total mass, m, contained within the sphere will be such that

$\Phi = 2Gm/c^2r \leq U$. By a <u>universal potential bound</u>, on the other hand, we shall mean that $\Phi \leq U$ for <u>all</u> P and <u>all</u> r. Clearly, every universal bound is global, but it is quite possible for a global bound to be valid and at the same time for there to exist local regions within r for which $\Phi > U$. For example, the potentials of certain white dwarf stars may exceed U_o, but this does not violate the inequality $\Phi < U_o$ in the large. It is probable that U_s or U_E are universal bounds in the above sense while U_o may be merely global. Both global and universal potential bounds, like other universal constants, will here be assumed to be time independent. However, if other basic physical constants vary with time, it is likely that the potential bounds also vary with time.

An important class of relativistic cosmological models has been constructed under the assumption that the matter in the universe may be approximated <u>in the large</u> by a uniform perfect fluid (Robertson 1933) whose density is a function of time only. These models do not reflect observed density fluctuations nor do they allow incorporation of the cosmological information implicit in the existence of stars, galaxies, clusters, and other bodies. Nonetheless, these models provide a zero order physical approximation and useful mathematical simplifications. They have constituted the main stream of theoretical and observational cosmology for the past four decades and many cosmological questions are formulated relative to their structure with observables expressed in terms of the basic parameters which characterize them. For these reasons we shall wish to consider the inferences of potential bounds for homogeneous cosmological models. However, before proceeding we must ascertain whether it is consistent to impose simultaneously the conditions of uniform density and the existence of a global potential bound.

The mean density, $\langle \rho(P,r) \rangle$ of matter contained within a sphere of radius r centered at P will be equal to $m/(\Gamma r^3)$ where m is the mass inside the spherical surface defined by P and r and Γ is a factor which depends on the curvature of space. The assumption of uniform density is equivalent to the statement that the limit of $\langle \rho(P,r) \rangle$ as r becomes very large is the same for all P. It follows that Φ on the spherical surface of radius r is equal to $2G\langle \rho \rangle \Gamma r^2/c^2$. From (1.1) the conditions of uniform density and the existence of a global potential bound are seen to be compatible provided

$$(1.2) \qquad \langle \rho \rangle < c^2 U/(2G\Gamma r^2)$$

for all $r \leq R$, the "radius of the universe." The inequality (1.2) indicates that there are no physical inconsistencies in simultaneously postulating uniform density and a potential bound. However, there are limitations placed on R by the value of $\langle \rho \rangle$. While it is thus physically consistent and meaningful to impose the two conditions simultaneously _in the large_, more locally a global potential bound implies the existence of some sort of _hierarchal_ structure in the distribution of matter. Cosmological models with hierarchal structures subject to a potential bound will be discussed in a subsequent paper; the remaining sections of the present paper will be restricted to the discussion of homogeneous isotropic models.

II. HOMOGENEOUS MODELS

a) Basic Equations

Our discussion of homogeneous isotropic models will follow the development of Robertson (1933). Using the standard metric,

$$(2.1) \quad ds^2 = c^2 dt^2 - R(t)^2 du^2$$

with $du^2 = (dr^2 + r^2 \, d\theta^2 + r^2 \sin^2\theta \, d\phi^2)/(1 + kr^2/4)^2$ being taken as the line element of three dimensional space sections which are of constant positive, zero, or negative curvature according as $k = +1$, o, or -1; the forms assumed by the field equations in the notation of Robertson are:

$$(2.2) \quad \kappa\rho c^2 = -\lambda + 3(k + \dot{R}^2/c^2)/R^2 \quad \text{and}$$

$$(2.3) \quad \kappa p = \lambda - 2\ddot{R}/(c^2 R) - (k + \dot{R}^2/c^2)/R^2.$$

Through the substitutions $\epsilon(\chi) = r/(1 + kr^2/4)$ and $d\chi = dr/(1 + kr^2/4)$, the spatial line element may be written in the form,

$$(2.4)\quad du^2 = d\chi^2 + \epsilon(\chi)^2\,(d\theta^2 + \sin^2\theta\, d\phi^2),$$

with $0 \le \theta \le \pi$, $0 \le \phi < 2\pi$, and

$$\epsilon(\chi) = \begin{cases} \sin(\chi) \text{ for } k = +1,\ 0 \le \chi \le \pi, & (2.4a) \\ \chi \quad\text{ for } k = 0,\ 0 \le \chi \le \infty, & (2.4b) \\ \sinh(\chi) \text{ for } k = -1,\ 0 \le \chi \le \infty. & (2.4c) \end{cases}$$

If m(X,t) denotes the mass within the "coordinate sphere", $0 < \chi < X$, at time t, then, since the mean mass density, $\langle\rho(t)\rangle$, is a function of time only,

$$m(X,t) = \langle\rho(t)\rangle\, R(t)^3 \int_0^X \epsilon(\chi)^2 d\chi \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi.$$

Using the values of $\epsilon(\chi)$ from (2.4) we have

$$m(x,t) = 4\pi\langle\rho(t)\rangle R(t)^3 \begin{cases} [2X - \sin(2X)]/4, \quad k = +1 & (2.5a) \\ X^3/3, \quad k = 0 & (2.5b) \\ [\sinh(2X) - 2X]/4, \quad k = -1. & (2.5c) \end{cases}$$

Since χ is a geodesic coordinate, the radius $r(x,t)$ of the coordinate sphere, $0 \le \chi < X$, at time t will be

$$r(x,t) = \int_0^r ds = R(t)\int_0^X d\chi = R(t)X$$

We may thus write for the potentials on the surfaces $\chi = X$,

$$\frac{2Gm(X,t)}{c^2 r(X,t)} = \frac{8\pi G\langle\rho(t)\rangle R(t)^2}{c^2} \begin{cases} [1 - \sin(2X)/(2X)]/2, & k = +1 \quad (2.6a) \\ X^2/3, & k = 0 \quad (2.6b) \\ [\sinh(2X)/(2X)-1]/2, & k = -1 \quad (2.6c) \end{cases}$$

where $0 \le X \le \pi$ for $k = +1$; $0 \le X \le \infty$ for $k = 0$; and $0 \le X \le \infty$ for $k = -1$.

b) Open Universes

Under the assumption of a global bounded potential, by equation (1.1), the left member of equations (2.6a, b,c) will be $\le U$ for all X in the allowable ranges, therefore, the right members are subject to the same bound. In the cases $k = 0$, and $k = -1$, the allowable range for X is infinite. Since $R(t_o) \neq 0$ at the present epoch (the subscript "o" will be used in the remaining portion of the paper to designate the present epoch), it follows that the inequality (1.1) can be satisfied for all X if and only if $\langle \rho(t_o) \rangle \equiv 0$ at the present epoch. This leads to the conclusion that the only universes with zero or negative constant curvatures which are isotropic and homogeneous and have bounded potential possess zero mean density. This must be true not only at the present, but for all times for which $R(t) \neq 0$, and for every finite U. Furthermore, this conclusion holds regardless of the dynamical processes involved, since it was obtained without use of the field equations.

Two possibilities thus exist for open homogeneous universes with bounded potentials. They may be empty of all matter - which contradicts observation - or they may be hierarchically structured. Charlier (1922) has shown that a hierarchal universe with infinite orders of clustering has a vanishing mean density.

c) Closed Universes

A similar argument cannot be made in the case $k = +1$, since the function $S(X) = [1 - \sin(2X)/(2X)]$ is bounded for all X. Accordingly, in order to discuss the implications of bounded potentials on closed universes, we must use the properties of the field equations and an inequality governing physically allowable pressures. The finite volume of a closed universe may be either $2\pi^2 R(t)^3$ or $\pi^2 R(t)^3$ according as to whether X has the range $0 \leq X \leq \pi$ or $0 \leq X \leq \pi/2$. The first case is usually termed a spherical space, the second, an elliptical space. At the values $X = \pi$ and $X = \pi/2$, the so called spherical and elliptic horizons, the potential $2Gm(X,t)/[c^2 r(X,t)]$, takes on the same value, $\kappa c^2 \langle\rho(t)\rangle R(t)^2/2$. We shall designate twice this horizon value of the potential, namely, $\kappa c^2 \langle\rho\rangle R^2$ by $\Phi^*(t)$. Substituting in (2.6a) and using (1.1), the potential bound assumes the form,

$$\Phi^*(t)\; S(X)/2 \leq U$$

In spherical space S(X) assumes the maximum value of

1.217 at X = 2.245 radians. Hence,

$$(2.7a) \quad \Phi^*(t) \leq 1.644U$$

In elliptical space S(X) assumes the maximum value of unity at X = $\pi/2$ giving,

$$(2.7b) \quad \Phi^*(t) \leq 2U$$

Equations (2.7a and b) are taken to hold at the present epoch $t = t_o$.

In order to make use of these bounds on Φ^*, we write the field equations (2.2) and (2.3) in terms of Φ^* and its time derivative. Setting k = +1, in (2.2) we obtain

$$(2.8) \quad \Phi^* - 3 = 3R'^2/c^2 - \lambda R^2$$

Differentiating (2.8) and substituting in (2.3) yields

$$(2.9) \quad \Phi^* + \Phi'^*/H = -3\kappa p R^2 \quad \text{where } H(t) = R'/R.$$

From (2.3), (2.8), and (2.9), we may derive,

$$(2.10) \quad -\dot{\Phi}^*/(2H) = \lambda R^2 + 3\dot{R}^2 q/c^2$$

where q is the "deceleration parameter", $- R\ddot{R}/\dot{R}^2$.

For all of the bounds, U_S, U_E, etc., by either equation (2.7a) or (2.7b), the left member of (2.8) is seen to be strictly negative in some neighborhood of the present epoch. It follows that for an isotropic homogeneous model subject to a potential bound, in the case $k = +1$ the cosmological constant, λ, must be <u>greater</u> <u>than</u> <u>zero</u>.

The existence of the Schwarzschild Limit, $\Phi < 1$, (or any smaller potential bound) is inconsistent with all homogeneous closed models with vanishing or negative cosmological constant as, for example, closed Friedman models. For Friedman models with zero pressure, the curvature parameter, k, will be +1, 0, or -1, according as the deceleration parameter, q_o, is greater than, equal to, or less than one-half. Hence, the selection of a value of q_o on the basis of theoretical curves derived from Friedman models is inconsistent if the observations indicate a value of $q_o > 1/2$. On the basis of comparison with theoretical curves derived from a Friedman model, Sandage (1966) reports observations leading to a value of $q_o = 1.65 \pm 0.3$. If this value is assumed, one or more of the following must be concluded: 1) the assumption $\lambda = 0$ is invalid, 2) the potential of the universe exceeds the Schwarzschild Limit, or 3) the general relativistic field equations under homogeneity assumptions do not consistently account for the observations. When evolutionary effects in the galaxies have been provisionally accounted for, Sandage revises his value of q_o to 0.5 ± 0.3 which avoids the inconsistency with the Schwarzschild limit.

However, two other difficulties are encountered in interpreting observational data on the basis of $\lambda = 0$ models. The first of these is the problem of time scale. The presently adopted value of the Hubble parameter, 75 km/sec/mpc, together with a value of q_o of 1.65, corresponds to a time since beginning of expansion of 6.5×10^9 years (for $q_o = 0.5$, the age since expansion = 8.7×10^9 years). On the other hand, a time greater than 20×10^9 years is required to account for observed stellar evolutionary effects (Sandage 1961).

The second problem is the discrepancy between the observed value of the mean density and the observed value of the deceleration parameter as related through the Friedman equation, $q_o = 4\pi G\langle\rho\rangle/3H_o^2)$. Oort (1958) derives a value for $\langle\rho_o\rangle$ of 3.1×10^{-31} gms/cm^3. This density is consistent with an open universe and for $H_o = 75$ km/sec/mpc corresponds to $q_o = 0.02$. Sandage's observed $q_o = 1.65$, on the other hand, is consistent with a closed universe and for the same H_o corresponds to $\langle\rho_o\rangle = 3.5 \times 10^{-29}$ gms/cm^3. The critical density separating open from closed models, corresponding to $q_o = 0.5$, is 1.06×10^{-29} gms/cm^3. The invisible matter necessary to bring Oort's density to a high enough value to close the universe and be consistent with Sandage's observed q_o's is almost 100 times as great as visible matter. Missing matter of this amount is difficult to account for. Since both the time-scale and the density-deceleration inconsistencies may be removed by abandoning the $\lambda = 0$ assumption, we take the potential bound-curvature inconsistency as further evidence in opposition to the assumption of a zero-valued cosmological constant.

The maximum physical pressure possible under homogeneous isotropic assumptions is that for a photon gas, $\rho c^2/3$ (Sandage 1961), giving the physically realizable pressure range, $0 \le p \le \rho c^2/3$. From (2.9) and the definition of Φ^*, the pressure inequalities lead to the potential inequalities,

$$(2.11) \quad 0 \le \Phi^*/2 \le -\Phi^{*\prime}/(2H) \le \Phi^*$$

Substituting the right hand inequality in (2.10) and adding (2.8) gives

$$(2.12) \quad (1 + q)\, R^2/c^2 \le 2\Phi^*/3 - 1.$$

Using the inequality (2.11), it can be shown from (2.8) and (2.10) that a sufficient condition for q_o to be positive is for $\Phi^* > 3$. However, since for all U's, $\Phi^* \le 2$, the possibility of a positive q_o cannot be established without additional information. On the other hand by (2.12), $q_o < 0$ (actually < -1) whenever $\Phi^* < 3/2$. Hence by (2.7a), q_o will be negative for spherical space whenever $U < 0.912$. The Eddington limit, the Buchdahl limit, the Bondi U_A and U_B limits, and the Observational limit are all less than 0.912. By (2.7b), q_o will be negative for elliptic space whenever $U < 0.75$. This condition is met by U_A, U_B, and U_O. In addition, since H is positive at the present epoch, it follows from (2.11) that $\Phi_o^{*\prime} < 0$ at present.

We conclude that, except possibly for hierarchal universes of infinite order after the model of Charlier, the only non-empty, homogeneous, isotropic cosmological models satisfying a global bounded potential equal to U_A, U_B, or U_O are those with $k = +1$, $\lambda > 0$, $q_o < -1$, and $\Phi' < 0$. This conclusion also holds in spherical - but not elliptic - space for the bounds equal to 8/9 or smaller.

III. THE FUTURE STATE OF HOMOGENEOUS UNIVERSES

The set of possible homogeneous models with $k = +1$ and $\lambda > 0$ includes models which expand monotonically from $R = 0$ (Lemaitre's model); expand from a non-zero critical radius, R_c (Lemaitre-Eddington model); remain static at $R \equiv R_c$ (Einstein model); expand asymptotically to $R = R_c$; contract from infinity at $t = -\infty$ to a minimum radius, then expand monotonically; oscillate between zero and a finite radius (Bondi 1952).

In order to isolate the model or models which are consistent with the inferences of a bounded potential we may either 1) require, as with other evolutionary models, that the total mass of the universe is invariant (we have already eliminated the continuous creation model or steady state universe for which $k = 0$ and $q \equiv -1$, by identifying $k = +1$; or 2) invoke a more general argument based on the pressure limit set by a photon gas.

a) Constant Mass

In the first approach we proceed by writing equation (2.2) in terms of the mass of the universe, M, by taking $\kappa c^2 \langle\rho\rangle R^2 = a\kappa c^2 M/R$, where a is a small constant. We have

$$(3.1) \quad a\kappa c^2 M/R = -\lambda R^2 + 3 + 3R'^2/c^2$$

where M will be independent of the time. Differentiating (3.1), we obtain

$$(3.2) \quad R'' = \lambda c^2 R/3 - a\kappa c^4 M/6R^2$$

From the inequality $q_o < 0$, we have that $R''_o > 0$ at the present epoch. For any interval of time in which R''

remains positive, R' must remain positive and R must continue to increase. Equation (3.2) with $\lambda > 0$ assures that an increasing R can only make R'' more positive. We conclude then that R'' and R' will remain positive for all future time and that R will continue to increase indefinitely. Under the assumption of invariant mass a universe subject to future oscillations is accordingly ruled out.

b) Bounded Pressure

More generally, without any assumptions regarding the constancy of mass, we may investigate the possible evolutionary properties of homogeneous models with bounded potentials by using the conclusions of the previous section as initial conditions and investigate the behavior of equations (2.8) and (2.9) subject to the assumed bounds on the potential and the pressure. Since $\Phi_o^{*\prime} < 0$ and $\Phi_o^* < 2$, for U_s or any smaller bound, there will exist some time interval T containing the present, during which the left member of (2.8), $\Phi^* - 3$, will remain strictly negative. We rewrite (2.8) in the form

(3.3) $(\Phi^* - 3)/\lambda = 3R'^2/(\lambda c^2) - R^2 = -g(t)^2 < 0,$

which is valid for t on the interval T. The function g(t) defined by equation (3.3) is real valued on the interval T, so we may make the change of variables,

(3.4) $R(t) = g(t) \cosh w(t).$

Substituting (3.4) in (3.3), we obtain

(3.5) $[(g'/g) \cosh w + w' \sinh w]^2 = (c^2\lambda/3) \sinh^2 w$

Hence (3.4) will be a solution of (3.3) provided the function w(t) satisfies the relation

(3.6) $w' = \pm c \sqrt{(\lambda/3)} - g' \cosh w/g \sinh w$

Equation (3.6) may be used to eliminate w' from the derivative of (3.4) giving,

(3.7) $R' = \pm c \sqrt{(\lambda/3)}\, g \sinh w$

Since R' and g(t) are positive at the present epoch, we may have either

$$R' = c\sqrt{(\lambda/3)}\ g \sinh w, \qquad \text{with } w(t_o) > 0$$

or

$$R' = -c\sqrt{(\lambda/3)}\ g \sinh w, \qquad \text{with } w(t_o) < 0$$

The evolutionary behavior of R' during the interval T is thus seen to be controlled by the functions w(t) and g(t). To ascertain the behavior of R' and R, we must determine whether g' and w' are positive or negative, i.e., whether g(t) and w(t) are increasing or decreasing in the neighborhood T.

From (3.3)

$$g^2 = (3 - \Phi^*)/\lambda$$

differentiating this expression, we get,

$$(3.8) \quad g' = g\,\Phi^{*\prime}/(2\Phi^* - 6).$$

Since $\Phi_o^{*\prime} < 0$, $(\Phi_o^* - 3) < 0$, and $g_o > 0$, g(t) must be an <u>increasing</u> function of t in the neighborhood of the present epoch.

In order to investigate w(t), we substitute (3.8) and the relation $H = \pm c \sqrt{(\lambda/3)} \sinh w/\cosh w$, which is obtained by dividing (3.4) by (3.7), in (3.6) yielding

$$w'(t) = \pm c\sqrt{(\lambda/3)}\,\{1 + \Phi'^{*}/[2H(3-\Phi^{*})]\}$$

By means of (2.9) this may be written in terms of the pressure, p,

$$(3.9) \quad w'(t) = \pm c \sqrt{(\lambda/3)}\,\{[3(2-\Phi^{*}) - 3\kappa pR^{2}]/(6-2\Phi^{*})\}$$

The evolutionary track of the universe will be determined by the pressure, the potential, and the rate of change of the potential in the manner that these physical quantities determine g(t) and w(t) through relations (3.8) and (3.9).

As pointed out in § II, all physical states of the universe must lie between the state of vanishing pressure and the state of the photon gas. These limits to the pressure allow us to define bounds for the evolutionary tracks which are permitted to w'(t). The pressure p, may be written as $\eta\langle\rho\rangle c^{2}/3$ in terms of a parameter η defined on the interval, $0 \le \eta \le 1$. When

$\eta = 0$, we have the condition of vanishing pressure; when $\eta = 1$, we have the limiting radiation pressure. It follows from the definition of Φ^* that $3\kappa p R^2 = \eta\Phi^*$. Replacing the expression involving the pressure by $\eta\Phi^*$ in (3.9), the permitted values of $w'(t)$ must lie in the interval,

$$(3.10) \quad c\sqrt{(3\lambda)}\,\frac{(3-2\Phi^*)}{(3-\Phi^*)} \leq \pm\, w' \leq c\sqrt{(3\lambda)}\,\frac{(2-\Phi^*)}{2(3-\Phi^*)} \;,$$

the left member corresponding to $\eta = 1$, the right member to $\eta = 0$. Both the left and right hand expressions in Φ^* will be positive for $\Phi^* < 3/2$. This is the same condition as that governing the validity of the bounds given in the last paragraph of § II. Hence, if these same global bounds are valid for all t, $\pm\, w'$ will always be bounded above and below by two positive numbers. It follows that if the + sign is selected, $w' > 0$; while if the - sign is selected, $w' < 0$. Hence, if $w'_o > 0$, by the sign convention in (3.7) governing R', $w_o > 0$ and R' is increasing. If $w'_o < 0$, then $w_o < 0$ and $[-\sinh w(t)]$ is increasing, leading again to an increasing R'. We thus have that in the neighborhood T of the present epoch, $R''_o > 0$, and $q_o < 0$.

The present expansion will continue so long as R'' remains positive. Consequently a constant or increasing R'' will assure that the universe will expand indefinitely. From (2.9) and the definition of η, the potential gradient may be written, $\Phi^{*\prime} = -(1 + \eta)\Phi^* H$. Substituting this quantity in (2.10) we obtain,

$$(3.11) \quad R'' = c^2\lambda R/3 - (1 + \eta)c^2\Phi^*/6R$$

Since R_o'' is positive, at the present epoch the first term is positive and is larger in absolute value than the negative second term. With increasing R the first term and the denominator of the second term are increasing. Further Φ^* and η are decreasing, so the second term is decreasing. It follows that R'' will remain positive for all future time. We conclude, with the assumption of the validity of the potential bounds and for all states of the universe between the extremes of vanishing pressure and that of photon gas, that the future history of the universe is described by a monotonically increasing function, R(t).

c) Asymptotic Values for Large t

In view of this conclusion, it is of interest to examine the asymptotic situation of large future times. From the relation $H = c\sqrt{(\lambda/3)}\sinh w(t)/\cosh w(t)$, it follows that the Hubble parameter H(t) is a monotone increasing function which approaches $c\sqrt{(\lambda/3)}$ asymptotically for all states of the universe. It thus follows that $3\ H_o^2/c^2$ gives a lower bound for the cosmological constant and that the asymptotic value of H(t) is an invariant over the set of all possible states of the universe at the present epoch. For H_o = 75 km/sec/mpc., the lower bound of the cosmological constant is $2 \times 10^{-56}/cm^2$; for $H_o = 100$, it is $3.5 \times 10^{-56}/cm^2$. With ρ and p tending to zero, as t increases without limit, it follows from (2.3) that $\ddot{R}/R \rightarrow \lambda c^2/3$. Hence, the asymptotic value of the deceleration parameter, q is -1. These results hold for U_s or any smaller bound. In the event that Φ_o^* is less than 3/2, then $q_o < -1$ and the asymptotic value is approached from below. This condition is satisfied by U_A, U_B, and U_O.

d) Model Selection

In the argument based on the assumption of constant mass, equation (3.2) excludes oscillating universes. Under the more general assumptions of the pressure bounds, $0 \le p \le \rho c^2/3$, oscillating universes are also excluded on the basis of equation (3.11). The Einstein universe listed because $\lambda > 0$, is excluded because it is static. With regard to the future, it remains to determine whether R(t), though increasing monotonically, increases without limit or increases to some finite asymptotic value. By (3.11) allowable future states are those for which R'' will be positive for all future time. This assures that R' continue to increase monotonically and without limit, which in turn implies that R increases without limit. Since R' increases without limit, there must exist a future time for which $R' > c$. This "paradox" does not constitute an inconsistency with the special theory of relativity since the quantity R is a geometric, not a physical, entity.

We conclude that under the assumptions of a potential upper bound less than or equal to unity and pressure in the range $0 \le p \le \rho c^2/3$, that the ultimate future state of the universe is uniquely determined for the set of all allowable states of the present universe. Specifically, R'' will be positive for all future time, R and R' will increase monotonically and without limit, and H will increase monotonically to $c\sqrt{(\lambda/3)}$. The asymptotic values of q, p, and ρ are -1, 0, and 0, respectively.

Whereas the future is uniquely determined under the assumptions of bounded potential and pressure the question remains,which of the permitted homogeneous models with positive cosmological constant represents the past history of the universe: contraction from $R = \infty$ to a minimum radius then expansion; expansion from $R = 0$ radius; or expansion from a non-zero radius. While the use of homogeneous relativistic models to represent the present and future state of the universe may afford a valid approximation to the physical situation, the validity of extrapolation to the past when other forces than gravitation played larger roles is open to question. However, there is a period of past time during which the representation given by equations (2.2) and (2.3) are still valid. Extrapolation backwards through this period gives the contextual conditions needed for constructing models of an earlier evolutionary era.

In the case of constant mass, since $R' \leq c$, we have immediately from (3.1) that

$$(3.12) \quad a\kappa c^2 M/R \le 6 - \lambda R^2$$

As R decreases the left member increases without limit, but the right member remains finite. Hence, to preserve the inequality for all t, there must exist a minimum value of $R > 0$ which bounds the radius R below. This rules out a universe exploding from the singular $R = 0$ condition. Either the universe followed the Lemaitre-Eddington pattern, being initially an Einstein static universe for an indefinite time until a chance perturbation initiated the expansion, or there was contraction through a minimum R, "bouncing off the Schwarzschild singularity," followed by monotone expansion.

The detailed behavior of homogeneous models subject to bounded potentials for times earlier than the present will be treated in a subsequent paper.

IV. SUMMARY

It has been shown that in order for open homogeneous cosmological models ($k = 0$, or -1) to be consistent with the Schwarzschild Limit, or any finite potential limit, that the mean density, $\langle\rho\rangle$, of the universe must vanish. A zero density universe which contains matter is physically realizable if it is structured in an hierarchal manner with an infinite number of orders of clustering as has been shown by Charlier. Except for this possibility, under the assumptions of homogeneity and bounded potential we are restricted to closed universes. For all potential bounds less than or equal to the Schwarzschild Limit, closed homogeneous universes must possess a positive cosmological constant. All Friedman models, such as those discussed by Sandage (1961) are inconsistent with a bounded potential and, as pointed out by Sandage, if $\lambda > 0$, the deceleration parameter, q_o, loses its discriminatory utility.

Under the additional assumption that the physically realizable conditions of pressure are bounded for all epochs by a zero pressure below and photon gas pressure above, it has been shown that the only homogeneous relativistic models consistent with a bounded potential expand monotonically without limit from a non-zero radius. The parameters, R', and H, also increase monotonically while ρ, p, and Φ decrease monotonically. The asymptotic values of H and q, are $c\sqrt{(\lambda/3)}$, and -1, respectively. The pressure and density tend toward zero. Further, if H_o is the present value of the Hubble parameter, the cosmological constant is bounded below by $3H_o^2/c^2$. All of these conclusions are valid if the global bound is the Schwarzschild Limit or any smaller potential bound.

It is evident from the conclusions of this paper that the observational establishment of the existence of a global potential bound would provide a test allowing discrimination between various homogeneous cosmological models. The three classical Hubble-Tolman observational tests - the redshift-magnitude relation, the redshift-count relations, and the redshift-diameter relation - are beset with severe observational difficulties which have to date precluded identification of the homogeneous model that best represents the observed sample of the universe. Further, the Hubble-Tolman tests, even if successful in ascertaining q_o, do not determine the value of the cosmological constant. Whether λ vanishes or not requires additional information such as that given by a bounded potential.

The present results pose new observational and theoretical problems. Observational determination that the order of clustering terminates would establish that $k = +1$. Observational confirmation that the U_o bound or some other bound is global would establish the conclusions of this paper. Theoretical problems include the past histories of homogeneous models with $\lambda > 0$, $q_o < 0$, $k = +1$, and $\rho > 0$. More important, however, is the investigation of non-homogeneous models capable of including and utilizing the information contained in the observed structure in the universe - the stars, galaxies, and clusters.

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A Hierarchal Cosmological Model. ALBERT WILSON, *Douglas Advanced Research Laboratories.*— The observation of equal maximum values of gravitational potential for stars, galaxies, and clusters of galaxies (Wilson, A. G., *Astron. J.* **71**, 402, 1966) suggests the existence of a universal potential bound governing gravitational stability. The assumption that systems whose potentials lie in the zone between the observed maximum value and the Schwarzschild limit ($10^{-4} < 2GM/c^2R < 1$), are unstable, whatever their densities or total energies, prohibits the stable existence of uniformly distributed matter of indefinite extent. Large masses in order to form stable systems must be structured hierarchically.

The existence of banded structures in the distributions of the redshifts of rich clusters of galaxies and radio sources (Wilson, A. G., *Proceedings of the 14th International Astrophysical Symposium, Liege, 1966*, to be published) indicates the existence of one or more possible additional members of a hierarchal structure which would be expected as a consequence of the assumed universal stability bound. Estimates of the potentials of these indicated super systems place them within the instability zone, consistent with, and possibly causally related to, the observed general expansion.

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green (λ5100–5600) and the low surface brightnesses of nearly all planetaries, conventional techniques have yielded only fragmentary information. The advantages of the Lallemand image converter in this spectral region are impressive. We have observed the spectral region λ4600–5900, overlapping part of the conventional "blue" region so as to tie in the newly observed line intensities to the older system. By proper choice of electronographic emulsion and development conditions, a linear relationship between density and intensity can be obtained. Observations of suitable comparison stars then enable one to establish relative intensities over the observable range. Both high and low excitation planetaries are included. Low excitation objects such as IC 418 show few lines in the 5010–5900 region, e.g., [N I] λ5876; moderate excitation objects show [Cl III] and high excitation objects also show He II λ5411, [Ca V], [Fe III], [Fe VI], [Fe VII] and C IV. The Orion nebula shows a rich visual-region spectrum. Some nebulae show pronounced excitation differences at different points. In objects such as NGC 6826 or NGC 7009, the background continuum limits the brightness of the observable lines.

Composition of the Hydrogen-Poor Star HD 30353. George Wallerstein, Thomas Greene, and Leslie Tomley, *University of Washington.*—A coarse analysis of the hydrogen-poor star HD 30353 has yielded the following parameters: $T_{eff} = 10\,000$°K; $\log P_e = +0.3$. By comparison of observations of Hγ with theoretical profiles from hydrogen-poor model atmospheres by E. Böhm-Vitense we find $N_H/N_{He} = 10^{-4}$, Other important abundance ratios are $N_C/N_N = 10^{-3}$, $N_O/N_N = 2\times10^{-2}$. The abundance ratios are probably accurate within a factor of 3. Starting with neon, the heavier elements have about normal abundances per gram of material.

The results may be interpreted in terms of the loss of the entire outer envelope of the star revealing material that has had its hydrogen converted to helium primarily by the carbon cycle. The high nitrogen abundance strongly favors carbon cycling but the very low abundances of both carbon and oxygen preclude a simple interpretation of carbon equilibrium at a single temperature. We prefer to hypothesize that the carbon cycle has been operative during the period of mass loss resulting in a gradually falling central temperature. At a temperature greater than 20 million degrees much of the oxygen could have been converted to carbon and at a later stage with a temperature less than 15 million degrees the carbon could have been converted to nitrogen.

Microwave Radiation Temperature of Space at 1.5 cm Wavelength. Wm. J. Welch, S. Keachie, D. D. Thorton, and G. Wrixon, *University of California, Berkeley.*—The intensity of diffuse 1.5 cm radiation incident upon the earth was measured during July and August of 1966 at Mount Bancroft in the White Mountains of eastern California. The site was chosen because at that altitude, 13 000 ft. the confusing emission at 1.5 cm from atmospheric water vapor is very much smaller than at sea level, A small horn antenna and conventional microwave radiometer were used for the observations. The radiometer was calibrated with the aid of blackbody terminations at liquid helium, liquid nitrogen and ambient temperatures. The emission from the atmosphere, ranging between 3° and 6°K at the zenith, was obtained from the observed variation of antenna temperature with zenith angle.

The mean isotropic background radiation temperature (after subtraction of the atmospheric emission) was 2.0°, +0.8°, −0.8°K. The indicated errors are two standard deviations. Observations at other wavelengths, as summarized by Howell and Shakeshaft (*Nature* **210**, 1318, 1966), are consistent with a blackbody spectrum of approximately 3°K temperature for this radiation. The present result is somewhat lower and suggests a departure from a simple blackbody spectrum for the radiation near 1 cm wavelength. On the other hand, the experiments are difficult to do with precision, as is indicated by the large errors quoted by all the observers, and a blackbody temperature of about 2.5°K would be consistent with all the observations including the present one.

Molecular Mantles on Interstellar Graphite Grains. David A. Williams, *Goddard Space Flight Center, Greenbelt, Maryland.*—The conditions affecting the growth of mantles on graphite grains are investigated at various places in the atmospheres of carbon stars. It is shown that graphite grains formed close to the carbon star and blown out by radiation pressure do not accrete hydrocarbon mantles. Since mantles do not form on graphite under quiescent interstellar conditions it must be assumed that interstellar graphite grains from this source do not have mantles. However, it is shown that grains associated with the interstellar gas contracting in the process of star formation will become coated with molecules: These grains may be significant in reflection nebulae.

A Hierarchal Cosmological Model. Albert Wilson, *Douglas Advanced Research Laboratories.*—The observation of equal maximum values of gravitational potential for stars, galaxies, and clusters of galaxies (Wilson, A. G., *Astron. J.* 71, 402, 1966) suggests the existence of a universal potential bound governing gravitational stability. The assumption that systems whose potentials lie in the zone between the observed maximum value and the Schwarzschild limit ($10^{-4}<2GM/c^2R<1$), are unstable, whatever their densities or total energies, prohibits the stable existence of uniformly distributed matter of indefinite extent. Large masses in order to form stable systems must be structured hierarchically.

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***UBV* Extinction Predictions Based on Numerical Integrations of Star and Blackbody Radiation Functions.** ROBERT E. WILSON, *University of South Florida.*—In deriving the law of atmospheric extinction as a function of air mass, one comes to the following step:

$$\Delta m = -2.5 \log \int I_\lambda R_\lambda e^{-k_\lambda X} d\lambda, \qquad (1)$$

where Δm is the extinction in magnitudes, I_λ is the monochromatic outside atmosphere intensity, R_λ is the response of the instrument, k_λ is the monochromatic extinction coefficient, and X is the true air mass. In order to find a linear dependence of Δm on X one must remove the exponential from under the integral. Thus k_λ must be independent of wavelength if the extinction is to be a strictly linear function of air mass in broad-band photometry. The author has numerically integrated Eq. (1) on the USF IBM 1410 computer using the published U, B, V response functions for R_λ and an $A+B\lambda^{-4}$ law for k_λ. I_λ was given by the Planck function at 2000° increments from 3000° to 25 000°K, and by observed star radiation curves published by Bahner (*Astrophys. J.* 138, 1314, 1963). The error made by fitting a straight line to the resulting curves over the X range 1.2 to 3.6 was then computed for different spectral types. Observed zero air-mass magnitudes are always fainter than true magnitudes. Summary of results: (1) Near sea level, the (untransformed) error in V is negligible, that in B is about $0^m.01$ and that in U is about $0^m.04$. (2) Results for stars differ little from those for blackbodies. (3) The effect depends somewhat on spectral type. As a byproduct of such integrations, one obtains the color index dependence of the U, B, and V extinction coefficients. In regard to both accuracy and economy of observing time, it is suggested that such a procedure may be more satisfactory for finding this dependence than the standard method of observing stars having a suitable range of color indices.

Venus: on an Inverse Variation with Phase in the 3.4-mm Emission During the 1965/1966 Apparition. W. J. WILSON, *U. S. Air Force*, E. E. EPSTEIN, *Aerospace Corporation*, J. P. OLIVER, *Aerospace Corporation*, and *University of California, Los Angeles*, R. A. SCHORN, *Jet Propulsion Laboratory*, AND S. L. SOTER, *Aerospace Corporation* and *Cornell University.*—Observations of Venus were made for 250 h on 98 days from August 1965 through August 1966 at a wavelength of 3.4 mm (88 GHz) to (1) extend the microwave spectrum to shorter wavelengths, (2) determine the nature of the phase effect, if any, and (3) search for temporal variations in the emission. We used a dual-beam observing procedure with the 15-ft (4.57 m) Cassegrain antenna of the Aerospace Corporation Space Radio Systems Facility. The 3.4-mm brightness temperature of Venus was found to have a small *inverse* phase effect which is represented by the expression: $T_B=297(\pm 2)-12(\pm 2)\cos[i\pm 6°(\pm 5°)]$°K. The errors are statistical standard errors only; system calibration errors are estimated to be ±30°K. No statistically significant temporal variations were found; an upper limit to day-to-day variations is approximately ±10%.

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A Revised Orbit and Masses for the System 31 Cygni. K. O. WRIGHT AND R. E. HUFFMAN, *Dominion Astrophysical Observatory.*—The system 31 Cygni consists of a K4Ib primary and a B4V secondary star. The K-type atmosphere can be studied near eclipse. A new orbit, based on 145 high-dispersion (3–4 Å/mm) spectrograms obtained since 1951 at the Dominion Astrophysical Observatory,

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A Hierarchal Cosmological Model. ALBERT WILSON, *Douglas Advanced Research Laboratories.*—The observation of equal maximum values of gravitational potential for stars, galaxies, and clusters of galaxies (Wilson, A. G., *Astron. J.* 71, 402, 1966) suggests the existence of a universal potential bound governing gravitational stability. The assumption that systems whose potentials lie in the zone between the observed maximum value and the Schwarzschild limit ($10^{-4} < 2GM/c^2R < 1$), are unstable, whatever their densities or total energies, prohibits the stable existence of uniformly distributed matter of indefinite extent. Large masses in order to form stable systems must be structured hierarchically.

The existence of banded structures in the distributions of the redshifts of rich clusters of galaxies and radio sources (Wilson, A. G., *Proceedings of the 14th International Astrophysical Symposium, Liege, 1966*, to be published) indicates the existence of one or more possible additional members of a hierarchal structure which would be expected as a consequence of the assumed universal stability bound. Estimates of the potentials of these indicated super systems place them within the instability zone, consistent with, and possibly causally related to, the observed general expansion.

HOMOGENEOUS COSMOLOGICAL MODELS WITH

BOUNDED POTENTIAL

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ABSTRACT

Under the assumption that there exists a finite global upper bound to the gravitational potential, $2Gm/c^2r$, it is shown that simply connected homogeneous models based on the Walker-Robertson line element must have zero mean density if the curvature parameter k is 0 or -1. For homogeneous models with k = +1, all global potential bounds less than or equal to the Schwarzschild Limit imply a positive cosmological constant. If the potential is globally less than 0.912 (spherical space) or 0.75 (elliptical space), and if the physically realizable pressures are at all future epochs bounded below by zero and above by photon gas pressure, $\rho c^2/3$, k = +1 models will expand monotonically without limit - for all future time. Asymptotic values for large times are in the usual notations: $H = c\sqrt{(\lambda/3)}$, $q = -1$, $p = 0$, and $\rho = 0$. Under the same conditions, the present value of the deceleration parameter is less than minus one, so that the asymptotic value is approached from below. The cosmological constant is bounded below by $3H_o^2/c^2$ where H_o is the present value of the Hubble parameter.

I. POTENTIAL BOUNDS

The existence of an upper bound to the local value of the gravitational potential which may obtain anywhere in the universe is suggested by both theory and observation. Following Schwarzschild, several authors (Eddington, 1923, Tolman, 1934, Buchdahl, 1959, Chandrasekhar, 1964, Fowler, 1966, Bondi, 1967) have shown theoretically that the potential $\Phi = 2Gm/c^2r$ (where G is the gravitational coupling constant, c the velocity of light, and m the total mass contained within radius r), of a static spherically symmetric system immersed in a region of zero mass density is bounded. These bounds result from solutions of the relativistic or post-Newtonian field equations under various assumptions regarding the equation of state. Examples of some of these theoretical potential limits are given in Table 1. Schwarzschild found the limiting value of unity for the potential assuming that the interior of the spherical system consisted of an incompressible perfect with constant proper density. Eddington derived the limiting value of 8/9 under the additional assumption that the pressure be everywhere finite. Bondi has shown that 0.970 is a rigorous upper bound for the potential independent of any assumptions concerning the equation of state and subject only to the restriction that the density nowhere be negative. Other Bondi limits are given in the Table. INSERT TABLE I

TABLE 1

RELATIVISTIC POTENTIAL BOUNDS

Bound	Symbol	Constraints: Spherical Symmetry plus	Upper Limit to $2Gm/c^2r$
Schwarzschild	U_S	$\rho \equiv$ constant	1
Eddington	U_E	$\rho \equiv$ const., p finite	0.888
Bondi I	U_B	ρ not increasing from center, $p \leq \rho c^2/3$	0.638*
Bondi II	U_A	adiabatic stability $p \leq \rho c^2/3$	0.620*

* Instead of the proper radius, Bondi uses $\sqrt{(A/4\pi)}$ where A is the proper area.

In addition to these relativistic and post-Newtonian bounds suggested by theory, there is also observational evidence suggesting the existence of a potential bound for stable nondegenerate cosmic bodies (Wilson 1966). The maximum values of the mass/radius ratio which occur in those samples of stars, galaxies, and clusters of galaxies that have been measured are found to be nearly the same for each species of cosmic body. This bounding value is approximately $10^{23.6}$g/cm, or $\Phi \leq 10^{-4.3}$. Table 2 gives the largest values of observed potentials (Allen 1963). The value (U_o) of this observed bound and its significance are uncertain. The same value of approximately 10^{23}g/cm may be derived for second order clusters on the basis of their radii and cluster contents (de Vaucouleurs 1960, Abell 1961). Despite the fact that U_o is markedly less than the theoretical limits, as might be expected in an expanding system, a nearly equal upper potential limit of systems that are aggregates of particles which are atoms, stars, or galaxies suggests a bound independent of the equation of state and in phenomenological correspondence with the predictions of theory.

INSERT TABLE 2

The above theoretical and observational results suggest the following <u>hypothesis</u>: there exists a constant <u>global</u> potential upper bound, U, such that for any sphere of proper radius r circumscribed about any point

TABLE 2

MAXIMUM OBSERVED GRAVITATIONAL POTENTIALS

System	Object	$\log_{10}(m/r)$ g/cm
Star	V444 Cyg A	23.27
Galaxy	M87	23.6
Cluster	Coma	23.5

P as center, the total mass contained within the sphere will satisfy

$$\Phi = 2Gm/c^2 r \leq U, \tag{1}$$

provided r is sufficiently large. This hypothesis, if true, would have interesting cosmological implications. The intent of this paper is to investigate the effects of this hypothesis on a class of relativistic cosmological models constructed under the assumption that the matter in the universe may be approximated <u>in the large</u> by a uniform perfect fluid (Robertson 1933) whose density and pressure are functions of time only. In order for condition (1) to be consistent with the assumption of uniform density in the large as postulated for homogeneous models, it is necessary that the mean density in every sphere of radius r and center P also be bounded. This condition of bounded density is consistent with both observation and the properties assumed for homogeneous isotropic models.

II. HOMOGENEOUS MODELS

a) Basic Equations

The bounded potential condition (1) will be applied to those homogeneous isotropic models with the usual assumption of simply connected covering spaces. Following Robertson (1933), we take coordinates such that the standard metric is,

$$ds^2 = c^2 dt^2 - R(t)^2 du^2 \tag{2}$$

where

$$du^2 = d\chi^2 + \epsilon(\chi)^2 (d\theta^2 + \sin^2\theta \; d\phi^2) \tag{3}$$

is the line element on a three-dimensional space of constant curvature, $k = (1, 0, -1)$. In (3)

$$0 \le \theta \le \pi, \quad 0 \le \phi < 2\pi$$

and

$$\epsilon(\chi) = \begin{cases} \sin(\chi), & 0 \le \chi \le \pi \\ \chi, & 0 \le \chi \le \infty \\ \sinh(\chi), & 0 \le \chi \le \infty \end{cases}$$

accordingly as the curvature is positive, zero, or negative. The forms assumed by the field equations in Robertson's notation are:

$$\kappa \rho c^2 = -\lambda + 3(k + R'^2/c^2)/R^2 \text{ and} \tag{4}$$

$$\kappa p = \lambda - 2R''/(c^2 R) - (k + R'^2/c^2)/R^2. \tag{5}$$

If m(X,t) denotes the mass within the "coordinate sphere," $0 < \chi < X$ at time t; then, since the mean mass density $\langle p(t) \rangle$ is a function of time only,

$$m(X,t) = \langle \rho(t) \rangle R(t)^3 \int_0^X \epsilon(\chi)\, d\chi \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi$$

Since χ is a geodesic coordinate, the proper (geodesic) radius r(X,t) of the coordinate sphere $0 \leq \chi < X$ at time t will be

$$r(x,t) = R(t) \int_0^X d\chi = R(t)X$$

For positive, zero, and negative curvatures, the potentials on the surfaces $\chi = X$ are taken to be

$$\frac{2GM(X,t)}{c^2 r(X,t)} = \frac{8\pi G\langle\rho(t)\rangle R(t)^2}{c^2}\begin{cases}[1 - \sin(2X)/(2X)]/2, \\ X^2/3, \\ [\sinh(2X)/(2X) - 1]/2,\end{cases} \tag{6}$$

where $0 \le X \le \pi$; $0 \le X \le \infty$; and $0 \le X \le \infty$ respectively.

b) Zero and Negative Curvature

By the global bounded potential condition, (1) the right members of equation (6) will be $\le U$ for all X in the allowable ranges.

In the cases of zero and negative curvature ($k = 0$, and $k = -1$) the allowable range for X is infinite. Since $R(t_o) = R_o \neq 0$ at the present epoch (the subscript "o" will be used throughout to designate the value at the present epoch), it follows that the inequality (1) can be satisfied for all X if and only if $\langle \rho_o \rangle \equiv 0$. It is also evident that $\langle \rho \rangle = 0$ for all t for which $R(t) \neq 0$. This leads to the conclusion that for any finite potential bound U, the only universes with zero or negative curvatures which are isotropic and homogeneous have zero mean density. Furthermore, this conclusion holds regardless of the dynamical processes involved, since it was obtained without use of the field equations.

It must be noted that this conclusion may not be valid if the spaces are not simply connected. There exist 18 different topological space forms for a three-dimensional space with zero curvature and infinitely many in the case of negative curvature. Some of these space forms are known to be closed in which case the arguments of this section do not hold. (Heckmann and Schücking 1962).

Two possibilities thus exist for open simply connected homogeneous universes with bounded potentials. They may be empty of all matter - which contradicts observation - or they may be hierarchically structured. Charlier (1922) has shown that a hierarchal universe with infinite orders of clustering has a vanishing mean density.

c) Positive Curvature

A similar argument cannot be made in the case $k = +1$, since the function $S(X) = [1 = \sin(2X)/(2X)]$ occurring in (6) is bounded for all X. Accordingly, in order to discuss the implications of bounded potentials for this case, we must use the properties of the field equations. For $k = +1$ we consider only the two customary topological cases: <u>spherical</u> space for which $0 \le X \le \pi$ and <u>elliptical</u> space for which $0 \le X \le \pi/2$. At the values $X = \pi$ and $X = \pi/2$, the so called spherical and elliptic <u>horizons,</u>

the potential $2Gm(X,t)/[c^2 r(X,t)]$, takes on the same value, namely, $\kappa c^2 \langle \rho(t) \rangle R(t)^2/2$. In the following it will be convenient to introduce the quantity

$$\Phi^*(t) = \kappa c^2 \langle \rho \rangle R^2, \tag{7}$$

equal to twice the horizon value of the potential. By (6), (7), and the definition of $S(X)$, the inequality (1) becomes,

$$\Phi^*(t)\ S(X)/2 \leq U.$$

In spherical space $S(X)$ assumes the maximum value of 1.217 at $X = 2.245$ radians, and hence

$$\Phi^*(t) \leq 1.644U. \tag{8a}$$

In elliptical space $S(X)$ assumes the maximum value of unity at the "end point" $X = \pi/2$ and hence

$$\Phi^*(t) \leq 2U. \tag{8b}$$

In order to make use of these bounds on Φ^*, we write the field equations (4) and (5) with $k = +1$ in terms of Φ^* and its time derivative:

$$\Phi^* - 3 = 3R'^2/c^2 - \lambda R^2, \quad (9)$$

$$\Phi^* + \Phi'^*/H = 3\kappa p R^2 \quad (10)$$

where $H(t) = R'/R$. From (5), (9), and (10), we may derive,

$$-\Phi'^*/(2H) = \lambda R^2 + 3R'^2 q/c^2 \quad (11)$$

where q is the "deceleration parameter", $-RR''/R'^2$.

For all potential bounds less than or equal to the Schwarzschild Limit, ($U = 1$), in both spherical and elliptical cases the left member of (9) is strictly negative in some neighborhood of the present epoch. Hence, the cosmological constant, λ, must be greater than zero. The Schwarzschild Limit taken as a global potential bound is accordingly inconsistent with homogeneous $k = +1$ models with vanishing or negative cosmological constant such as closed Friedmann models.

If we introduce the additional assumption that physically realizable pressures lie in the range $0 \leq p \leq \rho c^2/3$, the upper bound being photon gas pressure (Sandage 1961), by (10) the potential Φ^* must satisfy the inequalities,

$$0 \le \Phi^*/2 \le -\Phi'^*/(2H) \le \Phi^* \tag{12}$$

Substituting the right hand inequality in (11) and adding (9) gives

$$(1 + q)\, R'^2/c^2 \le 2\Phi^*/3 - 1. \tag{13}$$

This inequality requires that $q < -1$ whenever $\Phi^* < 3/2$. Hence by (8a), q will be less than -1 for spherical space whenever $U < 0.912$. The Eddington limit, U_E, and smaller limits including the observed U_o limit, all satisfy this requirement. Similarly by (8b), q will be less than -1 for elliptic space whenever $U < 0.75$. In addition, since $H_o > 0$, it follows from (12) that $\Phi'^*_o < 0$.

It may thus be concluded that, except possibly for hierarchical universes of infinite order, the only non-empty, homogeneous, isotropic cosmological models with simply connected topologies and a global bounded potential less than or equal to 0.75 are those with $k = +1$, $\lambda > 0$, $q_o < -1$, and $\Phi'_o < 0$.

III. THE FUTURE STATE OF HOMOGENEOUS UNIVERSES

a) Conditions in the Neighborhood of the Present

We may investigate allowable evolutionary paths of homogeneous models with bounded potentials by using the conclusions of the previous section as initial conditions and investigate the behavior or equations (9) and (10) subject to the assumed bounds on the potential and the pressure. Since $\Phi'^*_o < 0$ and $\Phi^*_o < 2$, for U_s (or any smaller bound) there will exist some time interval T containing the present, during which the left member of (9) will remain strictly negative. Accordingly the inequality,

$$-g(t)^2 \equiv (\Phi^* - 3)/\lambda = 3R'^2/(\lambda c^2) < 0 \qquad (14)$$

holds for t on the interval T. The function g(t) defined by equation (14) is real valued on the interval T, permitting the change of variables,

$$R(t) = g(t)\cosh[w(t)]. \qquad (15)$$

Whereby (14) becomes

$$[(g'/g)\cosh w + w' \sinh w]^2 = (c^2\lambda/3) \sinh^2 w \quad (16)$$

Hence (15) will be a solution of (14) provided the function w(t) satisfies the relation

$$w' = \pm c \sqrt{(\lambda/3)} - g' \coth(w)/g. \qquad (17)$$

Equation (17) may be used to eliminate w' from the derivative of (15) giving,

$$R' = \pm c \sqrt{(\lambda/3)}\, g \sinh w \qquad (18)$$

Since R'_o and g_o are positive, we may have either

$$R' = c \sqrt{(\lambda/3)}\, g \sinh w, \qquad \text{with } w_o > 0$$

or

$$R' = -c \sqrt{(\lambda/3)}\, g \sinh w, \qquad \text{with } w_o < 0$$

The evolutionary behavior of R' during the interval T is thus controlled by the functions w(t) and g(t). To ascertain the behavior of R' and R, we must determine whether g(t) and w(t) are increasing or decreasing in the neighborhood T. Differentiating (14), we get,

$$g' = g\Phi'^*/(2\Phi^* - 6). \qquad (19)$$

Since $\Phi'^*_o < 0$, $(\Phi^*_o - 3) < 0$, and $g_o > 0$, $g(t)$ must be an increasing function of t in the neighborhood T.

In order to investigate w(t), we substitute (19) and the relation $H = \pm c\sqrt{(\lambda/3)}\tanh w$, (obtained from (15) and (18)), in (17):

$$w'(t) = \pm c\sqrt{(\lambda/3)}\{1 + \Phi'^*/[2H(3-\Phi^*)]\}.$$

For the physically permissable range discussed in § II, the pressure, which may be written as $\eta\langle\rho\rangle c^2/3$ in terms of a dimensionless parameter η defined on the interval $0 \le \eta \le 1$, affords $3\kappa pR^2 = \eta\Phi^*$. Using this relation and (10), it may be shown that the permitted values of w'(t) must lie in the interval,

$$c\sqrt{(3\lambda)}\,\frac{(3-2\Phi^*)}{(3-\Phi^*)} \le \pm w' \le c\sqrt{(3\lambda)}\,\frac{(2-\Phi^*)}{2(3-\Phi^*)}, \quad (20)$$

where the left member corresponds to $\eta = 1$ and the right member to $\eta = 0$. Both the left and right hand expressions in Φ^* will be positive for $\Phi^* < 3/2$. This is the same condition as that governing the validity of the bounds given in the last paragraph of § II. Hence, if these same global bounds are valid for all t in T, $\pm w'$ will always be bounded above and below by two positive numbers. It follows from the sign convention used for R' in equation (18) and from $g > 0$, $g' > 0$ in T that R' is increasing in T and that the values of R and R' are independent of the choice of sign, $-w'$ or $+w'$. Hence throughout T, R'' is strictly positive and q negative.

b) Asymptotic Values for Large t

From (10) and the definition of η, we have that $\Phi'^* = -(1 + \eta)\Phi^* H$. Substituting this quantity in (11) we obtain,

$$R'' = c^2 \lambda R/3 - (1 + \eta) c^2 \Phi^*/6R. \tag{21}$$

Since R'' is positive in T, the first term of the right member must be larger than the second throughout T. Further, with R increasing and Φ^* decreasing, R'' will remain positive for all future time, and T is unbounded above. It follows that for all $t > t_o$, $R'(t)$ and $R(t)$ will be exponentially increasing functions. The Hubble parameter, $H(t) = c\sqrt{(\lambda/3)} \tanh [w(t)]$, is monotonically increasing and will approach the value $c\sqrt{(\lambda/3)}$ asymptotically for all allowable states of the model. This fact establishes $3H_o^2/c^2$ as a lower bound for the cosmological constant. ($H_o = 100$ km/sec/mpc $\sim \lambda > 3.5 \times 10^{-56}/\text{cm}^2$.) The pressure and density decreases to zero as t increases without limit. It follows from (5) that

$R''/R \rightarrow \lambda c/3$ and $q \rightarrow -1$. If $\Phi_o^* < 3/2$, then $q_o < -1$ and the asymptotic value is approached from below.

Possible homogeneous models with $k = +1$ and $\lambda > 0$ are: a) a model which expands monotonically without limit from a singular value (Lemaitre); b) expands from a non-zero critical radius (Lemaitre-Eddington); c) a static model (Einstein); d) expands asymptotically to a finite radius; e) contracts from infinity to a minimum finite radius, then expands monotonically without limit; f) oscillates between zero and a finite radius (Bondi 1952).

The behavior of R and R' for large t rules out all models except a), b), and e). These three cases are indistinguishable on the basis of their future paths. Thus while the future is uniquely determined under the above assumptions of bounded potential and pressure the questions remains, which of the three permitted homogeneous models with positive cosmological constant represents the past history of the universe.

While the use of homogeneous relativistic models to represent the present and future state of the universe may afford a valid approximation to the physical situation, the validity of indefinite extrapolation to the past when other forces than gravitation played major roles is open to question. Additional information such as that contained in the natural aggregates - the elements, stars, galaxies, clusters - must be used to discriminate between past histories.

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NEW METHODS OF THOUGHT AND PROCEDURE

Edited by

F. Zwicky and A. G. Wilson

Contributions to the Symposium on

METHODOLOGIES

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The format followed at the conference and in this book is that the first paper of each section presents a general review of the subject area and subsequent papers within the section provide examples of specific applications of the methodology.

PROLOGUE

There have been numerous attempts throughout the recorded history of man to review, classify and systematize the basic methods of thought and of procedure which are being used to deal both with the practical and the theoretical problems of life. To mention a very few of these there are the ORGANON of Aristotle's, the NOVUM ORGANUM by Francis Bacon and the Discours de la Méthode by René Descartes. These works were, of course, based on knowledge available at the time, which was both limited and in many cases false. Since the treatise of Descartes was written, science, technology and life in general have become so complex that renewed meditation on the essential aspects of fundamental constructive thought and procedure is in order. The necessity for such meditation has obviously been recognized in many countries and in many quarters and has in some instances led to a successful reevaluation of old principles and procedures as well as to the development of new thoughts, while again in other cases a lack of perspective resulted in more confusion. To achieve technically and humanly satisfactory results three prerequisites must be fulfilled, namely first unbias, that is absolute detachment from bias and prevaluations, second, sufficient knowledge about the true nature of the world and third, freedom of action. These three conditions have seldom been fulfilled in the past, today we have hopes of achieving them.

Perhaps the most far reaching and successful effort of all times which did satisfy all of the mentioned prerequisites is the idea which was promoted by Henri Pestalozzi (1746-1827) and first practiced by him -- that knowledge must be made available to every child and adult. Pestalozzi is thus the initiator of general education, some of whose ideas were so profound and far reaching that even today they have not yet been sufficiently taken advantage of inasmuch as we have not yet achieved the training of the whole of man which he visualized.

To go back to World War I, those who lived through it, well remember the general gloom all over the world and also the ardent desire by many of the best minds to understand what had gone wrong and to develop universal vistas and to devise methods of thought and procedure which would enable man to deal more effectively than before with the ever multiplying complexities of life. As a result the world was completely remodeled through the planned actions of communism, of the Versailles Treaty, of the League of Nations and the advents of Fascism and Naziism. The nations which remained free likewise started on many planned developments of their various potentials and they in particular recognized the need of organizing and integrating their scientific and technological capabilities. Institutions like the brain trust of the New Deal thus came into being and were subsequently diversified and expanded manyfold during the Second World War in order to insure the defeat of the dictators.

Since World War II enormous efforts have been made to develop general methods of thought and procedure which would allow us to deal efficiently with all of the complex problems of the world and which would eventually enable us to reach that most desired goal, a unified world based on mutual respect and esteem of all men and of all nations. The United Nations, the agreements about Antarctica, about nuclear testing and most recently the Outer Space Treaty resulted from these efforts. In addition, literally hundreds of groups all over the world were established for the purpose of applying their technical and human knowledge toward the construction of a sound and stabilized world.

By way of illustration I shall mention a few groups of which I have some personal knowledge since it has been my good fortune to have been associated with them. For instance experts from all fields were brought together by the ingenious founder of the Pestalozzi Foundation of America, Mr. H. C. Honegger, to establish war orphan villages on all continents and to deal effectively with the problems of destitute children all over the world. In way of large scale constructive actions for adequate housing the efforts of Professor Constantinos Doxiades in Athens are outstanding, partly because of the yearly Delos Conference which he has organized and in which experts in absolutely every field of human endeavor participate. The establishment of the Cité de Généralisation du Canisy near Deauville, which is being promoted by the French, is intended for occupation by experts and men of universal outlook who will deal with large scale problems, one after another. The Conférence des Sommets (Cultural Top Conference) in Brussels in 1961 should also be mentioned. At the invitation of the King and the Belgian Government, outstanding representatives of all sciences, technologies and the arts were invited to attempt an integration of all essentials of present-day knowledge. The organizer of this conference, Francois Le Lionnais, president of the French association of scientific writers, had previously edited a book, LA MÉTHODE DANS LES SCIENCES MODERNES, which may be regarded as a sequel to Descartes' "Discours de la Méthode" and which contains articles by some forty authors.

Finally, we organized the Society for Morphological Research, one of whose purposes it is to bring all new methods and procedures to the attention of a larger public. In this endeavor two major projects have been started, namely

1. A series of some two dozen comprehensive volumes on new methods of thought and procedure in the sciences, engineering, medicine, law, and the arts and so on.

2. To arrange conferences periodically at which the experts of the different methods and procedures will be brought together for discussions. Along this line a first proposal was made several years ago to Dr. A. H. Warner, then Director of the Office for Industrial

Associates at the California Institute of Technology. After Dr. Warner retired, the project had to be postponed but is now being realized in this symposium through the cooperation of Richard P. Schuster, the present Director of the Office for Industrial Associates.

The purpose of the present symposium was to awaken a universal self-awareness of methodology as a discipline. Dr. Simon Ramo in his inaugural address expressed the belief that such a trend in thinking is already discernible and will develop more or less automatically. However this may be, those among us who are active in the invention and application of new methods of thought and procedure want to make sure that all knowledge gained is effectively integrated and widely disseminated. As Professor Henry Borsook of the Biology Department of C.I.T. stated in his introduction of the session on morphological research there has been nothing like this since the latter part of the 5th century B.C. in Greece. At that time there was a great outburst and activity in the subjects of logic, the nature of knowledge, its transmission, exercise of power and the use of it. While the Greek atomists were intensely interested in the facts of nature, the sophists taught techniques, how to be successful politicians, lawyers, generals, ignoring, however, moral considerations. Justice for them was nothing more than the interest of the stronger. Thus knowledge without wisdom produced some monstrous consequences. On the other hand the wisdom without empirical knowledge of Plato's Academy could be nothing but ineffectual.

Today, after a period of more than 2000 years of accumulation of disconnected thoughts and procedures we are attempting to integrate them and to make them available to every man, woman and child for the purpose of training the whole of man.

F. Zwicky

EPILOGUE

METHODOLOGY--A DISCIPLINE

A primary purpose of this conference has been to consider whether the various methodologies employed in solving problems when taken together constitute in themselves a useful scientific and technological discipline. The descriptions of the several approaches to problems that have been presented here--Operations Research, Systems Engineering, Morphological Analysis, etc.--have made visible some common principles which have been independently developed for structuring, analyzing, and solving complex problems of many types. Though using different names and terminologies, the identities and overlaps contained in these approaches, taken with the fact of their independent discovery in many diverse contexts, strongly suggest the developability of a useful discipline that we may call "methodology." Although the presentations during this conference have only partially defined the subject area of methodology, they have demonstrated that it would now be meaningful to take steps toward systematic definition and organization of the concepts so far developed and establish a formal discipline.

Specific problem areas from hospitals to codes to jet engines have been treated at this conference. However, in all the variety of problems discussed, almost nothing has been said concerning how to select which problems to solve. It seems most important that any discipline of methodologies for problem solving be concerned not only with the definition and solution of specific problems but also with the totality of that growing complex consisting of the set of problems competing for our attention. The discipline of methodology should investigate criteria by which to assign priorities, the appropriate levels of resources--funds and talent--to be thrown against a problem, the nature of the interrelatedness of problems, the consequences of solutions to problems and the anticipation of derivative problems.

Neglecting an overview of the interrelated complex of problems has given rise to some serious unbalances in our culture. Dr. Ramo, in his introduction, pointed out a few of these unbalances. In 90 minutes we can travel around the earth in Gemini while in 90 minutes in our cities we sometimes can travel only a few blocks. We can provide pure breathable air 100 miles above the earth for our astronauts, but not within a hundred surface miles of our major cities. We have developed remote sensing equipment that can tell us everything going on inside a space capsule, but have not equipped the physician with comparable equipment for monitoring what is going on inside his patient. There is no need to enumerate our disparate and desperate social unbalances. We might now add that a conference on methodologies for solving problems without consideration of how to choose which problems to solve in itself constitutes an unbalance.

In addition to unbalances, there are other shortcomings inherent in our present approach to the growth and application of scientific and technological knowledge. For example, early this year, the world's largest oil tanker of 120,000 tons was wrecked off the east coast of England, releasing thousands of tons of crude oil which floated ashore and polluted hundreds of miles of shore line. This developed into a tragedy that assumed national proportions in England. It is estimated that extensive portions of beach will be polluted for decades, perhaps even permanently; and since the feedback on the ecology of major environmental alterations of this sort are sometimes delayed, the full extent of the damage created by the pollution probably will not be evident for some years. As expected, there was widespread comment on this disaster. However, criticism did not focus on the navigational situation which was the immediate cause of the wreck, nor on the structural feasibility of large tankers (they are quite feasible--there is a tanker of 300,000 tons currently under construction and one of 500,000 tons on the drawing boards), rather comment focussed on the defects in a technology that could blindly and blandly create the set up for this sort of disaster. This isolated example made some of the blind spots of technology visible to many for the first time. One of our own cabinet officers commented, "The environmental backlash we confront today cannot be eliminated just by applying more of the same science and technology that put us in our present predicament."

There is growing feeling in some quarters that the time has come to ring the bell on applying technology without responsibility to the environment or to the future; on synthesizing complexity without regard for social and human consequences; on continuously injecting change into society without direction or evaluation. We must now face the great responsibilities of what we choose or do not choose to do with our technological capabilities. We have reached the precarious level of technological development in which we have the power significantly to alter our environment without having either the power totally to control the means by which we effect the alterations, or an understanding adequate to predict the properties of the environmental states we bring about. Not only must the proposed discipline of methodology be able to derive knowledge concerning the limits to the controllability and predictability of specific applications of technology but also be able to derive the summary consequences resulting from the piecewise solutions of the various portions of the total problem complex.

Some of the methodologies reviewed at this conference pointed to the importance of the elimination of prejudice as basic to the problem solving process. Prejudices are often habits of thought that we unconsciously carry to new situations in which they are no longer applicable. An example of such a habit of thought that affects our application of technology is the making of decisions primarily on the basis of feasibility. One of the severe deficiencies in the present use of technology is the failure to note that at some level of the state of the art the answers to the two questions: how big can we build a tanker, and how big should we build a tanker, begin to diverge. For decades technology has been primarily concerned with finding ways to do things hitherto impossible. The emphasis has been on pushing back the limitations of nature and ignorance in order to make more products and activities feasibile and broaden our spectrum of choice. In an increasing number of technological areas we have recently moved from the regime of finding a way to the regime of choosing the best way. The task is no longer to remove natural limitations but to set up limitations of our own, to define the constraints and restraints which are prerequisite to sensible choice. In a regime of limited capability, choice is usually properly made for the limit of feasibility--build a plow that will cut as many furrows simultaneously as possible. However, the habit of thinking developed in this regime tends to carry over into the second regime; the difficult problems of choice being ignored and option being made simply for the limit of feasibility. For example, in typical past wars the level of tolerance to destruction and ability to recover was higher than the level of any enemy's capabilities to destroy. However, in the past two decades, this inequality has been reversed. It is now possible to destroy beyond any nation's tolerance to absorb. We have entered the regime of choice. There is the necessity for limited and restrained actions, but some spokesmen still adhere to first regime thinking.

Although this phenomena of regime change seems tautological to many, and is well understood by many business and government leaders, the oil on the beaches bears witness that one of our urgent problems is to spread more broadly the awareness of the regime change and replace feasibility thinking with some of the new methodological tools that are now available for making difficult decisions.

We had best rapidly acquire the techniques essential for decisions in a choice regime. The new developments in biology, for example, are leading us to a capability level where we may shortly be able to determine the sex of our offspring, extend our life spans indefinitely, and even create new varieties of organisms. Clearly the responsibilities of choice imposed by such developments are likely to be as demanding as any ever faced by man. The temptation to be guided purely by feasibility, say in producing selective viruses, could put an end to the human experiment.

In a choice regime, it becomes necessary to formulate every problem, not only in terms of the internal capability parameters, but also in terms of the contextual parameters, considering environmental effects and interrelationships and possible synergistic developments. Our failure to do this reveals another prejudice--the prejudice to settle for the reductionist factors and ignore the holistic ones. This is a pattern of thought which derives partially from the past successes of reductionism, especially in physics, and partially from the unwarranted association of holistic effects with supernaturalism.

Besides facing up to these and other prejudices such as fadism, the proposed discipline of methodology must derive techniques for treating the increasing complexity of our problems and systems, complexity leading to such occurrences as regional power blackouts or postal service breakdowns. Oftimes feedback signals from complex systems cannot be interpreted promptly. The signals may be delayed or lost in other effects. Pollution is an example of a problem area whose feedback signals have been unheeded until the environmental backlash has reached proportions whose correction will require major technological and social surgery. Development of techniques for prompt interpretation of feedback signals are an urgent problem area of the discipline of methodology.

Other new problem situations are on the horizon. The trend toward longer development times and shorter life times for new systems with the impossibility of paying off development costs before obsolescence may place us in the same situation as an organism whose life span drops below its gestation period.

There are many other aspects of the subject of how to select, define, and solve problems which will concern the methodologist. If the future comes to be dominated by unknown and uncontrolled parameters arising from the interaction of the random application of technology to specific problems in agriculture, medicine, manufacture, space, defense, etc., then planning becomes illusory and the course that our civilization will take is that of a car without a driver. It will be useless to construct one of our usual "good guy--bad guy" explanations for the situation. There is no villain, only complexity, and it is not too early to bring out best research talents to grips with it.

Albert Wilson

Read C.I.T.
May 24, 1967
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Chapter 2

MORPHOLOGY AND MODULARITY

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The morphological approach is not only a methodology for solving problems, it is an attitude toward problems. It is an attitude that demands that no problem be considered in isolation of all relevant contexts. It is an attitude that would try to take off our customary blinders before looking at the problem. It tries to obtain an unfiltered view by comparing views through as many different filters as possible. It looks for all possible solutions by also looking at many of the impossible ones. It attempts fresh views of the problem by looking at similar problems. In short, the morphological approach uses whatever methodologies are available to arrive at the most complete and unbiased representation of the structure of the problem and its solutions as is possible.

This attitude will be recognized as basic not only to the morphological method but to some of the other methodologies described in this symposium. In order that ground previously covered not be repeated, this paper will restrict discussion to examples of two important methodological procedures not hitherto considered. The first of these is an exercise toward the development of useful non-mathematical modeling. The second is an example of

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hypothesis generation through morphological analogy.

I. NON-QUANTIFIED MODELS

One of the extensions of problem solving capability needed for many important problems today is the development of methodologies for handling problems not easily quantified.

It frequently happens that many of the parameters that we know have relevance to a problem are not readily amenable to measurement or quantification. There is a tendency to concentrate on those parameters for which numerical values are obtainable and to neglect those parameters which are not measurable even though their relative weight in the problem may be high. To offset this tendency a methodology is required by which we are somehow able to incorporate the effects of those parameters which cannot be quantifiably represented. Examples are esthetic, ethical and moral values, psychological factors, and unquantifiable requirements of the future.

When standard numerical methodologies fail, or when non-quantifiable factors must be taken into account, a relevance type morphological procedure proposed by Alexander and Manheim may be applicable. The idea is based on the predication that any form or structure may be thought of as resulting from the interaction of a set of abstract forces or tendencies. These are general, not merely physical, forces. They may be quantifiable or unquantifiable, with no restriction on their variety. The totality of these forces generates a solution that reflects the contribution of each. The problem is to find a representation of the forces that allows them to be combined. Said in another way the problem is posed in an abstract space in which the representative elements are the generalized forces. The aggregate of such elements defines a form. If the aggregate is complete and in balance, the form becomes a stable object or solution.

A concrete example of this approach attempted by Alexander and Manheim may be found in the MIT report entitled, "The Use of Diagrams in Highway Route Location." Alexander and Manheim's problem was to locate the route for a freeway covering a 20 mile stretch in Massachusetts starting at Springfield and ending somewhere near Northampton. They first morphologically derived all of the individual abstract forces whose interaction would determine the path which the freeway should take. Shown on Table 1 of the freeway design parameters, is the goal or objective of the study, which was a freeway to meet major current traffic desires. In this case the aggregate solution was restricted to be a new freeway, rather than a morphological examination of all possible solutions to meet current traffic requirements. This new freeway had to be considered in the context of its interaction with existing freeway systems and in support of the competition with other transportation systems. Future transportation systems as visualized also had to be given representation. However, the largest number of constituent forces fall into two classes; those which determine the internal structure and behavior of the freeway, and those reflecting the interaction of the freeway with the environment. Table 2 of freeway design parameters shows the decomposition of the internal and environmental parameters into their different values. Under internal parameters, are first the construction parameters including earthwork costs, bridge costs, pavement and subgrade costs, and construction interference. Secondly, there are economic factors: land costs, public financial losses, user costs, obsolescence; and thirdly, operational factors: travel time, local accessability, safety, maintenance, and self-induced congestion.

The environmental parameters may be divided into physical, economic and esthetic. The physical environment includes questions of drainage patterns and catchment areas, effects of weather, air pollution. The economic environmental factors include the effect of the freeway on regional and local land development, public and private losses, such as the obliteration of historical, commercial, or other structures due to the routing of the freeway. Finally, esthetic con-

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siderations such as eyesores and noise must be considered.

Certainly not all of these parameters are easily measured, nor is it even possible to assign numerical values to some of them. Alexander and Manheim developed a method by which each factor would be reflected in the overall selection of the freeway route. They employed a modification of the method Zwicky has termed composite analytical photography. Each of the modular forces listed on the charts by itself favors a particular location for the highway. For example, consider earth work costs. The requirement to minimize earth work favors the location of the freeway in areas where the land is relatively flat. A transparent map is made in which the flat portions are rendered dark and the hilly portions light, the degree of hilliness and flatness can be represented by a corresponding density or opacity on the map.

Thus the tendency or force to locate the freeway in accordance with the minimization of earth work costs is to put the path in regions of maximum density on the map. Similarly for each of the other forces. If a separate transparent map of each of the forces which contributes to the location of the freeway is made so that the dark area favors location and the light area rejects location; if the forces are then combined through the process of composite photography, the resulting density on the photograph made from superimposing all the individual photographs would give the location that all the forces in combination tend to favor. The darkest strip would mark the best route.

By using this method, those parameters or forces which cannot be quantified can be weighted either through the density used on their representative maps or through the way in which the maps are superimposed. A subset of three or four parameters given equal weight and densities can be combined to produce a composite density which might then be reduced in order to adjust the joint weight of the set before combining with the maps of other parameters or sets. The structuring of the combinations thus provides the ability to weight the various factors.

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II. MORPHOLOGICAL ANALOGY

a) Artifical and Natural Systems

An interesting emergent property of recent times is that the source of new concepts and basic scientific knowledge is not only the natural order but also the structures and organizations created by man. The collection of concepts which are called "cybernetics" were derived jointly from the study of animal nervous systems and man-made control systems. The idea of information came from the study of special communication networks but merged with the concept of entropy. Today the important basic concepts underlying structure and organization are being brought to light by the designer of complex systems as well as by the observer of the natural order. In the sense of discovery vis-a-vis application the historic distinction between science and technology is thus tending to disappear.

As a result of the abstract parallelisms between natural and artifical systems we are able to create objects for study which provide us with the equivalent of new views of the natural order with temporal and spatial resolving powers hitherto unavailable. For example, the freeway provides us with a new type of fluid, called traffic, whose properties can be made useful to us in developing more comprehensive theories of fluid dynamics, extending to new realms the laws of fluids as observed in nature. The growing sample of such structures and organizations which have been made available for study as a result of our own creations provides still another positive feedback contributing to the accelerated development of science and technology. In effect we are creating another powerful epistemological methodology simply through constructing and studying systems that occupy some of the gaps in the natural order. (Even the reasons for the natural gaps may be learned in time if our creations prove unstable.)

b) Hierarchical Modular Structures

One possible source of information on systems of

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complex structures is in the analysis of how complexity and bigness are treated in the natural order. We observe throughout nature that the large and complex is constructed in a hierarchical modular manner from the small and simple.' Direct confrontation of the large and small is avoided, a hierarchical linkage is always interposed. Bigness is avoided in the sense that the ratio between the size of any structure and the modules out of which it is built is functionally bounded. If there are demands for a structure to continue to grow in size or complexity, then a new level in the hierarchy and a new module are introduced so that aggregate to module ratios may remain bounded.

Formally, by a hierarchical modular structure we shall mean an aggregate or organization of modules that are in turn hierarchical modular structures. Such a structure may be closed in the sense that there is ultimately a lowest level whose modules are not decomposable. Examples of hierarchical modular structures are ubiquitous: in the macrocosmos, there is the grouping of stars into galaxies, galaxies into clusters, etc.; in the microcosmos, the grouping of atoms into molecules, molecules into crystals, etc.; in the mesocosmos, there are the organizations and structures of man, armies, hospitals and hierarchical coding models.

What can we learn through comparing the properties of these hierarchical modular structures, artificial and natural, that will be useful in deriving a syntax to structure and increasingly complex systems of today's world, or that will be useful in understanding the limitations of our own organizations and structures? As an example of the method of morphological comparison we propose to look at two hierarchical systems--one social, one physical.

Martin Ernst's paper in this volume on city planning from the operations research point of view discusses the modular parameters basic to urban structure and evolution. The paper elaborates on one model, affording techniques through which planners and city officials could control the

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direction of changes in an urban complex. Ernst's approach might be called a reductionist approach, decomposing the city into components and sub-components, and looking at the "portfolio of possibilities." This is an important part of the analysis of any complex problem. However, the morphologist wants to add something. There may exist some parameters which place limits on the portfolio of possibilities but which are not evident in the reductionist approach. I would like to look at the city in this alternate manner. For this purpose the important properties of hierarchical modular structures to abstract are the bounds or limits to which the modules and the aggregates may be subject.

There are indications that our cities may be approaching some kind of critical limits. What kind of limits might these be, and how may we avoid difficulties without having to test to destruction to see where the failure occurs? To do this, let us compile sufficient modular forces to close the form we call a city, and see what the limitations on that form might be.

First, human beings as modules are subject to aggregating forces as are other so-called social creatures. These forces tend to draw people into physically compact aggregates. Historically, humans aggregated into towns and walled cities for trade and physical security. Today natural gregariousness is still very much a force bringing men together for physical, economic, and emotional security and growth.

Next, there are density limits governing how closely people may satisfactorily live together. These limits depend on the amount of freedom of movement and privacy we require. The higher densities in prisons and concentration camps are possible because of the restriction of movement and loss of privacy. Without knowing the value of the density limit, we can definitely assert that such a limit exists. (If you want an absolute limit, you may take the value of one person per 1.83 sq. ft., provided by Surajah Dowlah's experiment in close packing of humans

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in Calcutta in 1756.) However, we must bear in mind that in the modern city for purposes of density limits, the real inhabitants are motor vehicles, not people. The maximum density is determined by the minimum space needed for maneuvering, parking, and servicing automobiles.

A second limit exists in city life. This is the limit on the time required to be in movement to transact the city's business, or the bound on the maximum fraction of the day that the average commuter can tolerate spending in commuting. Doxiadis' studies show in cities of the past, the maximum distance from their centers was ten minutes by walking. We have certainly moved a long way from this value toward the commuting time limit. Three hours, or one-eighth of the day is not uncommon although the average is still considerably less than one hour per day. Both the city and the human modules which come together to make it are governed by the characteristic time period of 24 hours. This is an "absolute" value that is not at our disposal appreciably to modify. It is more basic than the day-night cycle imposed by the earth's rotation since this period is also set by the biological clock in each inhabitant. Even though adjustments in basic commuting problems can be made by some people, such as going to work on Monday, living near their work, and returning home on Friday for the week end, such practices can not alter the basic 24-hour period set by the needs of the city and its population. With present work and sleep requirements commuting time must be no greater than 1/3 of 24 hours.

These limits may readily be combined symbolically to define a closed entity. Let $\hat{\sigma}$ be the density bound and $\hat{\tau}$ the commuting time bound. (The latter may be expressed in terms of the natural period of the city $T = 24^h$ by $\hat{\tau} \leq \zeta T$ where $\zeta < 1$.) For a simplified model of a two dimensional city, $N = a\bar{\sigma}R^2$ where R is the maximum length path through the city and a is a shape factor. A limiting velocity which depends on the state of the art will be designated by c. The realizable commuting velocity will be less than c.

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Since $R \leq c\bar{\tau} \leq c\hat{\tau}$ and $\bar{\sigma} < \hat{\sigma}$, where barred quantities are mean values, we have $N = a\bar{\sigma}R^2 < ac^2\bar{\sigma}\bar{\tau}^2 < ac^2\hat{\sigma}\,\hat{\tau}^2 \leq ac^2\hat{\sigma}\zeta^2T^2$. In a three dimensional model we may introduce the mean height, $\bar{h}$, of the city and use three dimensional densities, $\bar{\rho}$ and $\hat{\rho}$, giving.

$$N < a'c^2\bar{h}\hat{\rho}\,\hat{\tau}^2$$

If we designate the absolute limit $\hat{\rho}\,\hat{\tau}^2$ by $1/H$, then

$$\frac{HN}{c^2\bar{h}} < a' \qquad (1)$$

These particular limits combined with an aggregating force may indeed have some significance with regard to cities, for it is interesting that a similar relation obtains in cosmic aggregates.

In 1907 before the development of modern cosmological theories and before the establishment of the existence of white nebulae as external galaxies, the Swedish mathematician C. V. L. Charlier showed that in a universe containing an infinite number of stars the sum of gravitational forces acting at every point would still be finite provided the universe were structured in a hierarchical modular manner. Quite independently of possible relevance to cosmology, Charlier's inequalities showed in general that a hierarchical modular structure could be used to bound density and inverse square type forces.

Under assumptions of uniform density and spherical symmetry, Schwarzschild showed that the field equations of general relativity predicted the existence of a bound on the gravitational potential

$$\frac{GM}{c^2R} \leq \frac{1}{2} \qquad (2)$$

where M is the mass and R the radius of the gravitating sphere. Under the assumption of uniform density this limit demands the existence of hierarchical modular structure. If the equation is written in the form

$$\bar{\rho}R^2 \leq B$$

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where $\overline{\rho}$ is the density and B is a fixed bound (we assume that G and c are constants), we see that for a given density--as for example, mean stellar density--the maximum possible radius of a star is determined. Such an inequality not only defines a limit to stellar size but forbids close packing of stars in space. Stars can be organized together into a larger aggregate only if a lower value of $\overline{\rho}$ obtains. If $\overline{\rho}$ assumes the mean value of galactic density the argument may be repeated. The maximum size of a galaxy is determined by the same bound but with a lower value of $\overline{\rho}$. The repeated application of a potential bound, like in the Schwarzschild inequality, can account for the levels in the hierarchical modular structure observed in the universe. However, the inequality does not explain the particular set of $\overline{\rho}$'s which are observed in the universe nor does it indicate at what level the hierarchical modular structure may terminate. Potential bounds like the Schwarzschild level may also be interpreted as bounding the maximum velocity a module may possess in a coordinate system at rest with respect to the aggregate. With this last interpretation, we see from Figure 1 that cosmic bodies are either "density limited" or "velocity limited." The "slope 3" line represents the limiting density of matter in a non-degenerate form. Solid cosmic bodies lie on or to the right of this line. (On the logarithmic scales used in the diagram, the planetary bodies appear to have essentially the same densities.) The "slope 1" line represents the observed location of the velocity limited bodies, i.e., the star, galaxy, cluster, and derived super cluster having the largest potentials or escape velocities. (This is an observed potential bound and differs in numerical value from the theoretical Schwarzschild bound. The objects falling on the observed bound, like those on the density bound, are non-degenerate.) The inequalities (1) and (2) may be put in the respective forms, 1.8

$$\overline{\rho}\,\tau^2 < B^* \quad \text{and} \quad \overline{\rho}\,R^2 < B$$

These inequalities have the same ingredients and we might expect them to have the same significance even though the values of the coupling constants are quite different.

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On the basis of these similarities we might propose a theorem of the form:

Given

1. The existence of an aggregating force tending to bring modules into a condition of maximum compactness, (gravity in the case of cosmic bodies.)
2. The existence of a maximum limiting density, (the limit set by non-degenerate matter in the cosmic example.)
3. The existence of a potential bound or its equivalent, (such as the Schwarzschild Limit, in the gravitational case.)

then hierarchical modular structures provide a way for accommodating indefinite size while satisfying these intrinsic limitations. Specifically we are led to inequalities of the $\overline{\rho} R^2 < B$ or $\rho \tau^2 < B^*$ type. If we assume we may apply such a theorem to a city, then from $\overline{\sigma} R^2 \leq a c^2 \hat{\sigma} \hat{\tau}^2$, we see that for a given density, the size depends on a bound set by the effective velocity of travel and the maximum acceptable commuting time. The bound may be satisfied as N increases by increasing c, or alternatively the solution may be found in hierarchical structure.

If a polynucleated city develops on hierarchical lines, it will be stable so long as each nucleus and the complex of all the nuclei (with an overall lower density) satisfy the inequality, $\sigma R^2 < B$. However, the nuclei will not close pack, which means that if subsequent urban development fills in the areas between the nuclei bringing the mean density up to the level obtaining within a nucleus, the complex will surpass the limit. This sort of "filling in" process is occurring in the "megapolis" areas of the Eastern United States and Southern California. If these derived inequalities are valid, we will not escape with impunity the destruction of our open spaces or the low density background between present cities.

Since no physical restrictions governing the distribution of density in the city exist as in the cosmic case, there are other possible solutions. It can be shown that

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the bound may be satisfied by selecting a density distribution $\sigma(r) \sim r^{-(\gamma+1)}$ where $\gamma > 1$. In this case, the city may grow and still satisfy the bound if it is built in a ring shape. Several suggestions of this sort have been made including a city which is nothing but a series of linear structures several stories high with freeways on top.

Additional limit theorems on the structure of cities may be derived. However, these require more sophisticated models and exceed the parallelisms in the hierarchical modular analogy given. Since our purpose here is not to develop a general theory of urban structure, but to illustrate the method of morphological parallelism, this one analogy will suffice. The method of morphological analogy does not per se generate valid theories. It produces hypotheses and ideas on which models may be constructed. These must then be tested by the usual canons of scientific verification.

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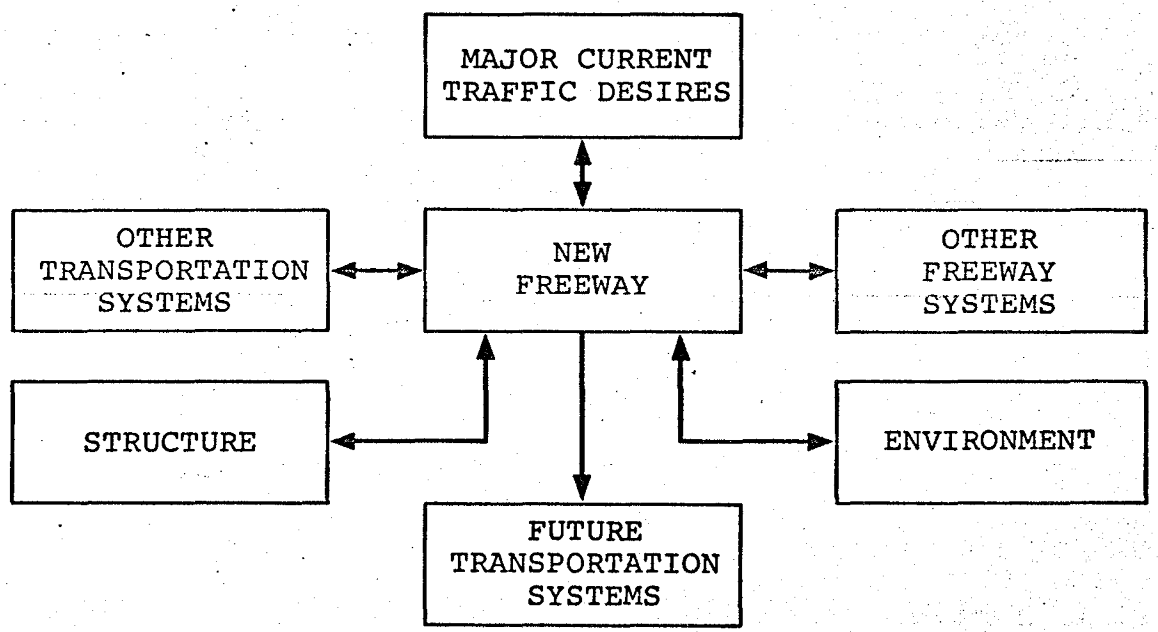
FREEWAY DESIGN PARAMETERS
I



FREEWAY DESIGN PARAMETERS
II

Internal

Constructional
- Earthwork costs
- Bridge costs
- Pavement and subgrade costs
- Construction interference

Economic
- Land costs
- Public financial losses
- User costs
- Obsolescence

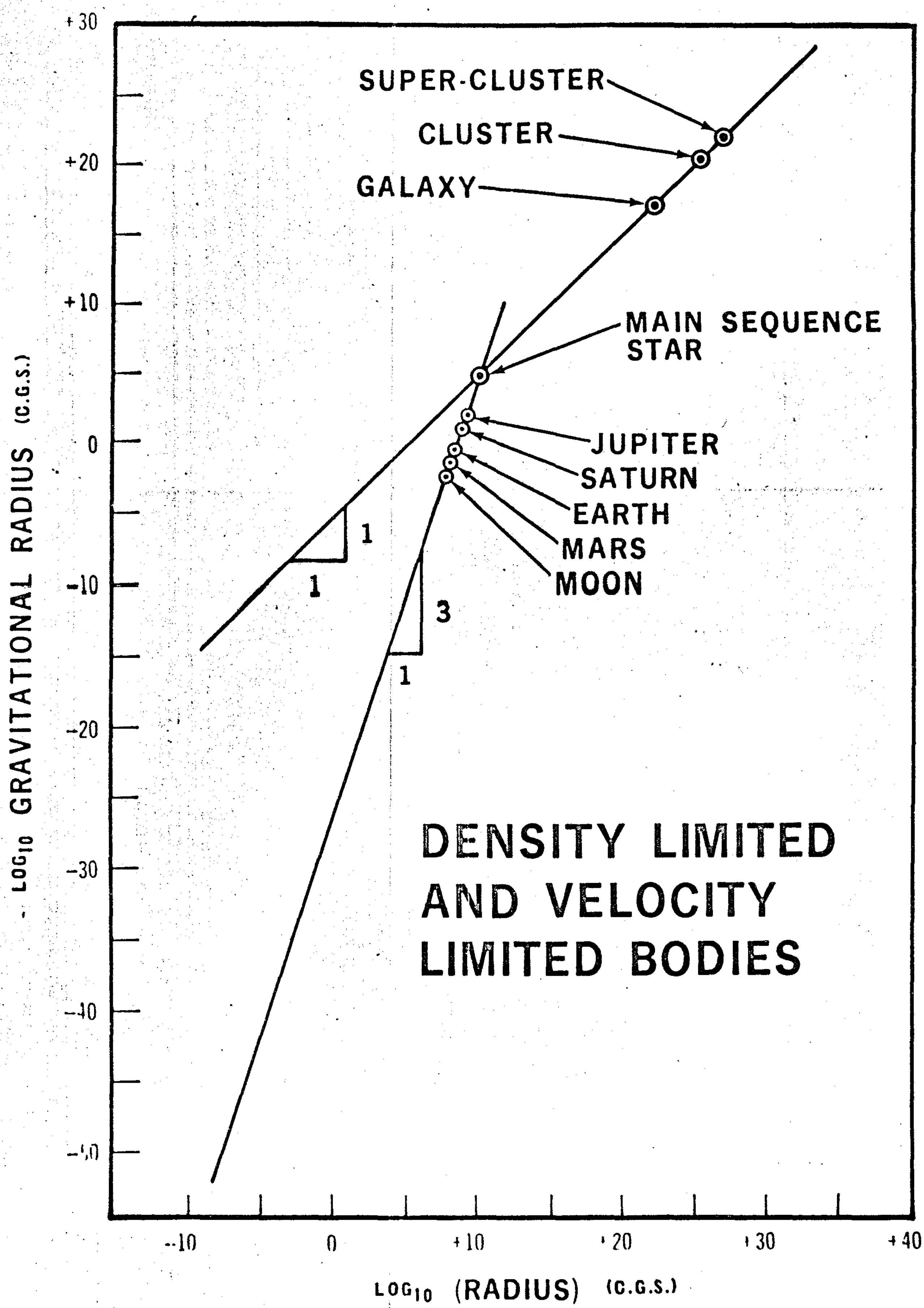
Operational
- Travel time
- Local accessibility and integrity
- Safety
- Maintenance and services
- Self induced congestion

Environmental

Physical
- Catchment areas
- Drainage patterns
- Weather effects
- Air pollution

Economic
- Regional land development
- Local land development
- Non-recompensable public and private loss

Esthetic
- Eyesores
- Noise


SUPER-CLUSTER
CLUSTER
GALAXY
MAIN SEQUENCE STAR
JUPITER
SATURN
EARTH
MARS
MOON
1
1
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DENSITY LIMITED AND VELOCITY LIMITED BODIES
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NEW METHODS OF THOUGHT AND PROCEDURE

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MORPHOLOGY AND MODULARITY

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This attitude will be recognized as basic not only to the morphological method but to some of the other methodologies described in this symposium. In order that ground previously covered not be repeated, this paper will restrict discussion to examples of two important methodological procedures not hitherto considered. The first of these is an exercise toward the development of useful non-mathematical modeling. The second is an example of

hypothesis generation through morphological analogy.

I. NON-QUANTIFIED MODELS

One of the extensions of problem solving capability needed for many important problems today is the development of methodologies for handling problems not easily quantified.

It frequently happens that many of the parameters that we know have relevance to a problem are not readily amenable to measurement or quantification. There is a tendency to concentrate on those parameters for which numerical values are obtainable and to neglect those parameters which are not measurable even though their relative weight in the problem may be high. To offset this tendency a methodology is required by which we are somehow able to incorporate the effects of those parameters which cannot be quantifiably represented. Examples are esthetic, ethical and moral values, psychological factors, and unquantifiable requirements of the future.

When standard numerical methodologies fail, or when non-quantifiable factors must be taken into account, a relevance type morphological procedure proposed by Alexander and Manheim may be applicable. The idea is based on the predication that any form or structure may be thought of as resulting from the interaction of a set of abstract forces or tendencies. These are general, not merely physical, forces. They may be quantifiable or unquantifiable, with no restriction on their variety. The totality of these forces generates a solution that reflects the contribution of each. The problem is to find a representation of the forces that allows them to be combined. Said in another way the problem is posed in an abstract space in which the representative elements are the generalized forces. The aggregate of such elements defines a form. If the aggregate is complete and in balance, the form becomes a stable object or solution.

A concrete example of this approach attempted by Alexander and Manheim may be found in the MIT report entitled, "The Use of Diagrams in Highway Route Location." Alexander and Manheim's problem was to locate the route for a freeway covering a 20 mile stretch in Massachusetts starting at Springfield and ending somewhere near Northhampton. They first morphologically derived all of the individual abstract forces whose interaction would determine the path which the freeway should take. Shown on Table 1 of the freeway design parameters, is the goal or objective of the study, which was a freeway to meet major current traffic desires. In this case the aggregate solution was restricted to be a new freeway, rather than a morphological examination of all possible solutions to meet current traffic requirements. This new freeway had to be considered in the context of its interaction with existing freeway systems and in support of the competition with other transportation systems. Future transportation systems as visualized also had to be given representation. However, the largest number of constituent forces fall into two classes; those which determine the internal structure and behavior of the freeway, and those reflecting the interaction of the freeway with the environment. Table 2 of freeway design parameters shows the decomposition of the internal and environmental parameters into their different values. Under internal parameters, are first the construction parameters including earthwork costs, bridge costs, pavement and subgrade costs, and construction interference. Secondly, there are economic factors: land costs, public financial losses, user costs, obsolescence; and thirdly, operational factors: travel time, local accessability, safety, maintenance, and self-induced congestion.

The environmental parameters may be divided into physical, economic and esthetic. The physical environment includes questions of drainage patterns and catchment areas, effects of weather, air pollution. The economic environmental factors include the effect of the freeway on regional and local land development, public and private losses, such as the obliteration of historical, commercial, or other structures due to the routing of the freeway. Finally, esthetic con-

siderations such as eyesores and noise must be considered.

Certainly not all of these parameters are easily measured, nor is it even possible to assign numerical values to some of them. Alexander and Manheim developed a method by which each factor would be reflected in the overall selection of the freeway route. They employed a modification of the method Zwicky has termed composite analytical photography. Each of the modular forces listed on the charts by itself favors a particular location for the highway. For example, consider earth work costs. The requirement to minimize earth work favors the location of the freeway in areas where the land is relatively flat. A transparent map is made in which the flat portions are rendered dark and the hilly portions light, the degree of hilliness and flatness can be represented by a corresponding density or opacity on the map.

Thus the tendency or force to locate the freeway in accordance with the minimization of earth work costs is to put the path in regions of maximum density on the map. Similarly for each of the other forces. If a separate transparent map of each of the forces which contributes to the location of the freeway is made so that the dark area favors location and the light area rejects location; if the forces are then combined through the process of composite photography, the resulting density on the photograph made from superimposing all the individual photographs would give the location that all the forces in combination tend to favor. The darkest strip would mark the best route.

By using this method, those parameters or forces which cannot be quantified can be weighted either through the density used on their representative maps or through the way in which the maps are superimposed. A subset of three or four parameters given equal weight and densities can be combined to produce a composite density which might then be reduced in order to adjust the joint weight of the set before combining with the maps of other parameters or sets. The structuring of the combinations thus provides the ability to weight the various factors.

II. MORPHOLOGICAL ANALOGY

a) Artifical and Natural Systems

An interesting emergent property of recent times is that the source of new concepts and basic scientific knowledge is not only the natural order but also the structures and organizations created by man. The collection of concepts which are called "cybernetics" were derived jointly from the study of animal nervous systems and man-made control systems. The idea of information came from the study of special communication networks but merged with the concept of entropy. Today the important basic concepts underlying structure and organization are being brought to light by the designer of complex systems as well as by the observer of the natural order. In the sense of discovery vis-a-vis application the historic distinction between science and technology is thus tending to disappear.

As a result of the abstract parallelisms between natural and artifical systems we are able to create objects for study which provide us with the equivalent of new views of the natural order with temporal and spatial resolving powers hitherto unavailable. For example, the freeway provides us with a new type of fluid, called traffic, whose properties can be made useful to us in developing more comprehensive theories of fluid dynamics, extending to new realms the laws of fluids as observed in nature. The growing sample of such structures and organizations which have been made available for study as a result of our own creations provides still another positive feedback contributing to the accelerated development of science and technology. In effect we are creating another powerful epistemological methodology simply through constructing and studying systems that occupy some of the gaps in the natural order. (Even the reasons for the natural gaps may be learned in time if our creations prove unstable.)

b) Hierarchical Modular Structures

One possible source of information on systems of

complex structures is in the analysis of how complexity and bigness are treated in the natural order. We observe throughout nature that the large and complex is constructed in a hierarchical modular manner from the small and simple. Direct confrontation of the large and small is avoided, a hierarchical linkage is always interposed. Bigness is avoided in the sense that the ratio between the size of any structure and the modules out of which it is built is functionally bounded. If there are demands for a structure to continue to grow in size or complexity, then a new level in the hierarchy and a new module are introduced so that aggregate to module ratios may remain bounded.

Formally, by a hierarchical modular structure we shall mean an aggregate or organization of modules that are in turn hierarchical modular structures. Such a structure may be closed in the sense that there is ultimately a lowest level whose modules are not decomposable. Examples of hierarchical modular structures are ubiquitous: in the macrocosmos, there is the grouping of stars into galaxies, galaxies into clusters, etc.; in the microcosmos, the grouping of atoms into molecules, molecules into crystals, etc.; in the mesocosmos, there are the organizations and structures of man, armies, hospitals and hierarchical coding models.

What can we learn through comparing the properties of these hierarchical modular structures, artificial and natural, that will be useful in deriving a syntax to structure and increasingly complex systems of today's world, or that will be useful in understanding the limitations of our own organizations and structures? As an example of the method of morphological comparison we propose to look at two hierarchical systems--one social, one physical.

Martin Ernst's paper in this volume on city planning from the operations research point of view discusses the modular parameters basic to urban structure and evolution. The paper elaborates on one model, affording techniques through which planners and city officials could control the direction of changes in an urban complex. Ernst's approach might be called a reductionist approach, decomposing the city into components and sub-components, and looking at the "portfolio of possibilities." This is an important part of the analysis of any complex problem. However, the morphologist wants to add something. There may exist some parameters which place limits on the portfolio of possibilities but which are not evident in the reductionist approach. I would like to look at the city in this alternate manner. For this purpose the important properties of hierarchical modular structures to abstract are the bounds or limits to which the modules and the aggregates may be subject.

There are indications that our cities may be approaching some kind of critical limits. What kind of limits might these be, and how may we avoid difficulties without having to test to destruction to see where the failure occurs? To do this, let us compile sufficient modular forces to close the form we call a city, and see what the limitations on that form might be.

First, human beings as modules are subject to aggregating forces as are other so-called social creatures. These forces tend to draw people into physically compact aggregates. Historically, humans aggregated into towns and walled cities for trade and physical security. Today natural gregariousness is still very much a force bringing men together for physical, economic, and emotional security and growth.

Next, there are density limits governing how closely people may satisfactorily live together. These limits depend on the amount of freedom of movement and privacy we require. The higher densities in prisons and concentration camps are possible because of the restriction of movement and loss of privacy. Without knowing the value of the density limit, we can definitely assert that such a limit exists. (If you want an absolute limit, you may take the value of one person per 1.83 sq. ft., provided by Surajah Dowlah's experiment in close packing of humans

in Calcutta in 1756.) However, we must bear in mind that in the modern city for purposes of density limits, the real inhabitants are motor vehicles, not people. The maximum density is determined by the minimum space needed for maneuvering, parking, and servicing automobiles.

A second limit exists in city life. This is the limit on the time required to be in movement to transact the city's business, or the bound on the maximum fraction of the day that the average commuter can tolerate spending in commuting. Doxiadis' studies show in cities of the past, the maximum distance from their centers was ten minutes by walking. We have certainly moved a long way from this value toward the commuting time limit. Three hours, or one-eighth of the day is not uncommon although the average is still considerably less than one hour per day. Both the city and the human modules which come together to make it are governed by the characteristic time period of 24 hours. This is an "absolute" value that is not at our disposal appreciably to modify. It is more basic than the day-night cycle imposed by the earth's rotation since this period is also set by the biological clock in each inhabitant. Even though adjustments in basic commuting problems can be made by some people, such as going to work on Monday, living near their work, and returning home on Friday for the week end, such practices can not alter the basic 24-hour period set by the needs of the city and its population. With present work and sleep requirements commuting time must be no greater than 1/3 of 24 hours.

These limits may readily be combined symbolically to define a closed entity. Let $\hat{\sigma}$ be the density bound and $\hat{\tau}$ the commuting time bound. (The latter may be expressed in terms of the natural period of the city $T = 24^h$ by $\hat{\tau} \leq \zeta T$ where $\zeta < 1$.) For a simplified model of a two dimensional city, $N = a\bar{\sigma}R^2$ where $\underline{R}$ is the maximum length path through the city and $\underline{a}$ is a shape factor. A limiting velocity which depends on the state of the art will be designated by $\underline{c}$. The realizable commuting velocity will be less than c.

Since $R \leq c\bar{\tau} \leq c\hat{\tau}$ and $\bar{\sigma} < \hat{\sigma}$, where barred quantities are mean values, we have $N = a\bar{\sigma}R^2 < ac^2\bar{\sigma}\bar{\tau}^2 < ac^2\hat{\sigma}\,\hat{\tau}^2 \leq ac^2\hat{\sigma}\zeta^2T^2$. In a three dimensional model we may introduce the mean height, $\bar{h}$, of the city and use three dimensional densities, $\bar{\rho}$ and $\hat{\rho}$, giving

$$N < a'c^2\bar{h}\hat{\rho}\hat{\tau}^2$$

If we designate the absolute limit $\hat{\rho}\hat{\tau}^2$ by $1/H$, then

$$\frac{HN}{c^2\bar{h}} < a' \qquad (1)$$

These particular limits combined with an aggregating force may indeed have some significance with regard to cities, for it is interesting that a similar relation obtains in cosmic aggregates.

In 1907 before the development of modern cosmological theories and before the establishment of the existence of white nebulae as external galaxies, the Swedish mathematician C. V. L. Charlier showed that in a universe containing an infinite number of stars the sum of gravitational forces acting at every point would still be finite provided the universe were structured in a hierarchical modular manner. Quite independently of possible relevance to cosmology, Charlier's inequalities showed in general that a hierarchical modular structure could be used to bound density and inverse square type forces.

Under assumptions of uniform density and spherical symmetry, Schwarzschild showed that the field equations of general relativity predicted the existence of a bound on the gravitational potential

$$\frac{GM}{c^2R} \leq \frac{1}{2} \qquad (2)$$

where M is the mass and R the radius of the gravitating sphere. Under the assumption of uniform density this limit demands the existence of hierarchical modular structure. If the equation is written in the form

$$\bar{\rho}R^2 \leq B$$

where $\bar{\rho}$ is the density and B is a fixed bound (we assume that G and c are constants), we see that for a given density--as for example, mean stellar density--the maximum possible radius of a star is determined. Such an inequality not only defines a limit to stellar size but forbids close packing of stars in space. Stars can be organized together into a larger aggregate only if a lower value of $\bar{\rho}$ obtains. If $\bar{\rho}$ assumes the mean value of galactic density the argument may be repeated. The maximum size of a galaxy is determined by the same bound but with a lower value of $\bar{\rho}$. The repeated application of a potential bound, like in the Schwarzschild inequality, can account for the levels in the hierarchical modular structure observed in the universe. However, the inequality does not explain the particular set of $\bar{\rho}$'s which are observed in the universe nor does it indicate at what level the hierarchical modular structure may terminate. Potential bounds like the Schwarzschild limit may also be interpreted as bounding the maximum velocity a module may possess in a coordinate system at rest with respect to the aggregate. With this last interpretation, we see from Figure 1 that cosmic bodies are either "density limited" or "velocity limited." The "slope 3" line represents the limiting density of matter in a non-degenerate form. Solid cosmic bodies lie on or to the right of this line. (On the logarithmic scales used in the diagram, the planetary bodies appear to have essentially the same densities.) The "slope 1" line represents the observed location of the velocity limited bodies, i.e., the star, galaxy, cluster, and derived super cluster having the largest potentials or escape velocities. (This is an observed potential bound and differs in numerical value from the theoretical Schwarzschild bound. The objects falling on the observed bound, like those on the density bound, are non-degenerate.) The inequalities (1) and (2) may be put in the respective forms,

$$\bar{\rho}\tau^2 < B^* \quad \text{and} \quad \bar{\rho}R^2 < B$$

These inequalities have the same ingredients and we might expect them to have the same significance even though the values of the coupling constants are quite different.

On the basis of these similarities we might propose a theorem of the form:

Given

1. The existence of an aggregating force tending to bring modules into a condition of maximum compactness, (gravity in the case of cosmic bodies.)
2. The existence of a maximum limiting density, (the limit set by non-degenerate matter in the cosmic example.)
3. The existence of a potential bound or its equivalent, (such as the Schwarzschild Limit, in the gravitational case.)

then hierarchical modular structures provide a way for accommodating indefinite size while satisfying these intrinsic limitations. Specifically we are led to inequalities of the $\bar{\rho} R^2 < B$ or $\rho \tau^2 < B^*$ type. If we assume we may apply such a theorem to a city, then from $\bar{\sigma} R^2 \leq ac^2 \hat{\sigma} \hat{\tau}^2$, we see that for a given density, the size depends on a bound set by the effective velocity of travel and the maximum acceptable commuting time. The bound may be satisfied as N increases by increasing c, or alternatively the solution may be found in hierarchical structure.

If a polynucleated city develops on hierarchical lines, it will be stable so long as each nucleus and the complex of all the nuclei (with an overall lower density) satisfy the inequality, $\sigma R^2 < B$. However, the nuclei will not close pack, which means that if subsequent urban development fills in the areas between the nuclei bringing the mean density up to the level obtaining within a nucleus, the complex will surpass the limit. This sort of "filling in" process is occurring in the "megapolis" areas of the Eastern United States and Southern California. If these derived inequalities are valid, we will not escape with impunity the destruction of our open spaces or the low density background between present cities.

Since no physical restrictions governing the distribution of density in the city exist as in the cosmic case, there are other possible solutions. It can be shown that

the bound may be satisfied by selecting a density distribution $\sigma(r) \sim r^{-(\gamma+1)}$ where $\gamma > 1$. In this case, the city may grow and still satisfy the bound if it is built in a ring shape. Several suggestions of this sort have been made including a city which is nothing but a series of linear structures several stories high with freeways on top.

Additional limit theorems on the structure of cities may be derived. However, these require more sophisticated models and exceed the parallelisms in the hierarchical modular analogy given. Since our purpose here is not to develop a general theory of urban structure, but to illustrate the method of morphological parallelism, this one analogy will suffice. The method of morphological analogy does not per se generate valid theories. It produces hypotheses and ideas on which models may be constructed. These must then be tested by the usual canons of scientific verification.

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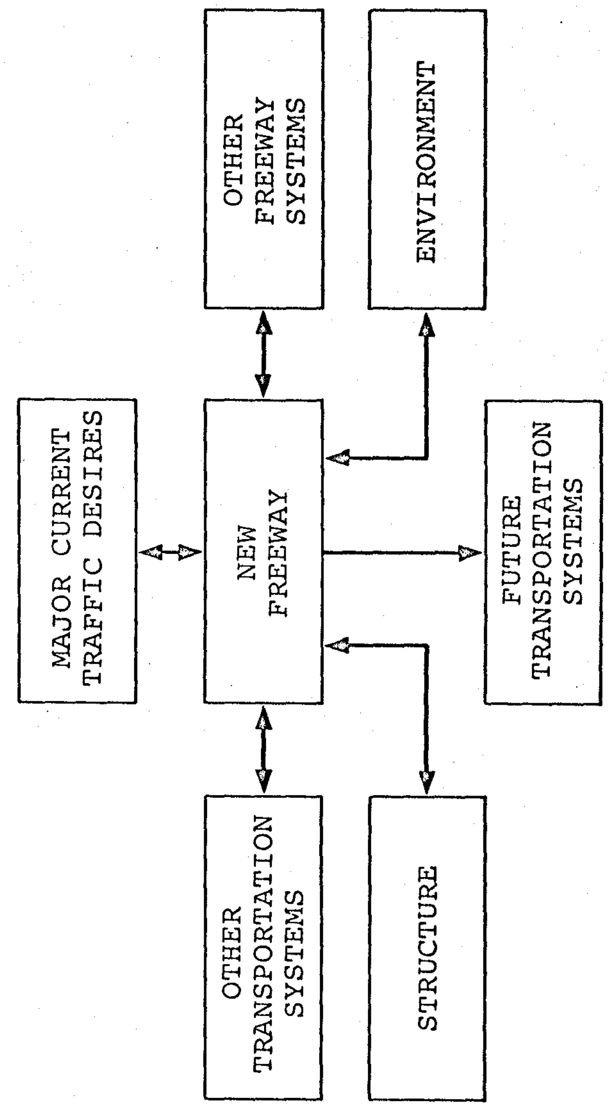
FREEWAY DESIGN PARAMETERS
I



FREEWAY DESIGN PARAMETERS
II

Internal

Constructional
- Earthwork costs
- Bridge costs
- Pavement and subgrade costs
- Construction interference

Economic
- Land costs
- Public financial losses
- User costs
- Obsolescence

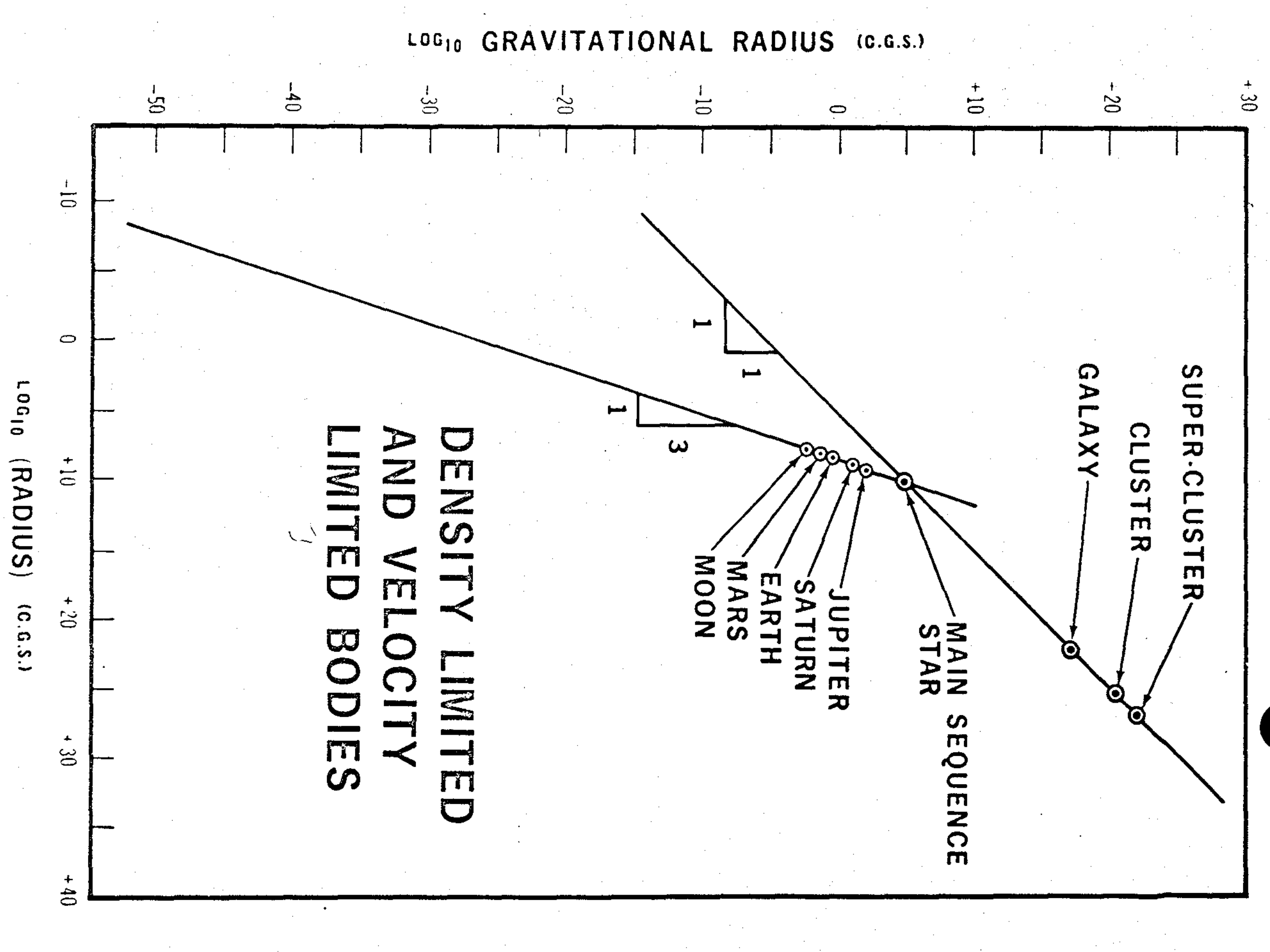
Operational
- Travel time
- Local accessibility and integrity
- Safety
- Maintenance and services
- Self induced congestion

Environmental

Physical
- Catchment areas
- Drainage patterns
- Weather effects
- Air pollution

Economic
- Regional land development
- Local land development
- Non-recompensable public and private loss

Esthetic
- Eyesores
- Noise



EPILOGUE

METHODOLOGY--A DISCIPLINE

A primary purpose of this conference has been to consider whether the various methodologies employed in solving problems when taken together constitute in themselves a useful scientific and technological discipline. The descriptions of the several approaches to problems that have been presented here--Operations Research, Systems Engineering, Morphological Analysis, etc.--have made visible some common principles which have been independently developed for structuring, analyzing, and solving complex problems of many types. Though using different names and terminologies, the identities and overlaps contained in these approaches, taken with the fact of their independent discovery in many diverse contexts, strongly suggest the developability of a useful discipline that we may call "methodology." Although the presentations during this conference have only partially defined the subject area of methodology, they have demonstrated that it would now be meaningful to take steps toward systematic definition and organization of the concepts so far developed and establish a formal discipline.

Specific problem areas from hospitals to codes to jet engines have been treated at this conference. However, in all the variety of problems discussed, almost nothing ha been said concerning how to select which problems to solve It seems most important that any discipline of methodologie for problem solving be concerned not only with the definition and solution of specific problems but also with the totality of that growing complex consisting of the set of problems competing for our attention. The discipline of methodology should investigate criteria by which to assign priorities, the appropriate levels of resources--funds and talent--to be thrown against a problem, the nature of the interrelatedness of problems, the consequences of solutions to problems and the anticipation of derivative problems.

Neglecting an overview of the interrelated complex of problems has given rise to some serious unbalances in our culture. Dr. Ramo, in his introduction, pointed out a few of these unbalances. In 90 minutes we can travel around the earth in Gemini while in 90 minutes in our cities we sometimes can travel only a few blocks. We can provide pure breathable air 100 miles above the earth for our astronauts, but not within a hundred surface miles of our major cities. We have developed remote sensing equipment that can tell us everything going on inside a space capsule, but have not equipped the physician with comparable equipment for monitoring what is going on inside his patient. There is no need to enumerate our disparate and desperate social unbalances. We might now add that a conference on methodologies for solving problems without consideration of how to choose which problems to solve in itself constitutes an unbalance.

In addition to unbalances, there are other shortcomings inherent in our present approach to the growth and application of scientific and technological knowledge. For example, early this year, the world's largest oil tanker of 120,000 tons was wrecked off the east coast of England, releasing thousands of tons of crude oil which floated ashore and polluted hundreds of miles of shore line. This developed into a tragedy that assumed national proportions in England. It is estimated that extensive portions of beach will be polluted for decades, perhaps even permanently; and since the feedback on the ecology of major environmental alterations of this sort are some-

times delayed, the full extent of the damage created by the pollution probably will not be evident for some years. As expected, there was widespread comment on this disaster. However, criticism did not focus on the navigational situation which was the immediate cause of the wreck, nor on the structural feasibility of large tankers (they are quite feasible--there is a tanker of 300,000 tons currently under construction and one of 500,000 tons on the drawing boards rather comment focussed on the defects in a technology tha could blindly and blandly create the set up for this sort of disaster. This isolated example made some of the blind spots of technology visible to many for the first time. One of our own cabinet officers commented, "The environmenta backlash we confront today cannot be eliminated just by applying more of the same science and technology that put us in our present predicament."

There is growing feeling in some quarters that the time has come to ring the bell on applying technology without responsibility to the environment or to the future; on synthesizing complexity without regard for social and huma consequences; on continuously injecting change into society without direction or evaluation. We must now face the grea responsibilities of what we choose or do not choose to do with our technological capabilities. We have reached the precarious level of technological development in which we have the power significantly to alter our environment witho having either the power totally to control the means by whic we effect the alterations, or an understanding adequate to predict the properties of the environmental states we bring about. Not only must the proposed discipline of methodolog be able to derive knowledge concerning the limits to the controllability and predictability of specific applications of technology but also be able to derive the summary consequences resulting from the piecewise solutions of the vario portions of the total problem complex.

Some of the methodologies reviewed at this conferenc pointed to the importance of the elimination of prejudice as basic to the problem solving process. Prejudices are ofter habits of thought that we unconsciously carry to new situations in which they are no longer applicable. An example of such a habit of thought that affects our application of technology is the making of decisions primarily on the basis of feasibility. One of the severe deficiencies in the present use of technology is the failure to note that at some level of the state of the art the answers to the two questions: how big can we build a tanker, and how big should we build a tanker, begin to diverge. For decades technology has been primarily concerned with finding ways to do things hitherto impossible. The emphasis has been on pushing back the limitations of nature and ignorance in order to make more products and activities feasibile and broaden our spectrum of choice. In an increasing number of technological areas we have recently moved from the regime of finding a way to the regime of choosing the best way. The task is no longer to remove natural limitations but to set up limitations of our own, to define the constraints and restraints which are prerequisite to sensible choice. In a regime of limited capability, choice is usually properly made for the limit of feasibility--build a plow that will cut as many furrows simultaneously as possible. However, the habit of thinking developed in this regime tends to carry over into the second regime; the difficult problems of choice being ignored and option being made simply for the limit of feasibility. For example, in typical past wars the level of tolerance to destruction and ability to recover was higher than the level of any enemy's capabilities to destroy. However, in the past two decades, this inequality has been reversed. It is now possible to destroy beyond any nation's tolerance to absorb. We have entered the regime of choice. There is the necessity for limited and restrained actions, but some spokesmen still adhere to first regime thinking.

Although this phenomena of regime change seems tautological to many, and is well understood by many business and government leaders, the oil on the beaches bears witness that one of our urgent problems is to spread more broadly the awareness of the regime change and replace feasibility thinking with some of the new methodological tools that are now available for making difficult decisions.

We had best rapidly acquire the techniques essential for decisions in a choice regime. The new developments in biology, for example, are leading us to a capability level where we may shortly be able to determine the sex of our offspring, extend our life spans indefinitely, and even create new varieties of organisms. Clearly the responsibilities of choice imposed by such developments are likely to be as demanding as any ever faced by man. The temptation to be guided purely by feasibility, say in producing selective viruses, could put an end to the human experimen

In a choice regime, it becomes necessary to formulat every problem, not only in terms of the internal capability parameters, but also in terms of the contextual parameters considering environmental effects and interrelationships and possible synergistic developments. Our failure to do this reveals another prejudice--the prejudice to settle for the reductionist factors and ignore the holistic ones. This is a pattern of thought which derives partially from the past successes of reductionism, especially in physics, and partially from the unwarranted association of holistic effects with supernaturalism.

Besides facing up to these and other prejudices such as fadism, the proposed discipline of methodology must derive techniques for treating the increasing complexity of our problems and systems, complexity leading to such occurrences as regional power blackouts or postal service breakdowns. Oftimes feedback signals from complex syster cannot be interpreted promptly. The signals may be delaye or lost in other effects. Pollution is an example of a problem area whose feedback signals have been unheeded until the environmental backlash has reached proportions whose correction will require major technological and social surgery. Development of techniques for prompt interpretation of feedback signals is an urgent problem area of the discipline of methodology.

Other new problem situations are on the horizon. Th trend toward longer development times and shorter life time for new systems with the impossibility of paying off development costs before obsolescence may place us in the same situation as an organism whose life span drops below its gestation period.

There are many other aspects of the subject of how to select, define, and solve problems which will concern the methodologist. If the future comes to be dominated by unknown and uncontrolled parameters arising from the interaction of the random application of technology to specific problems in agriculture, medicine, manufacture, space, defense, etc., then planning becomes illusory and the course that our civilization will take is that of a car without a driver. It will be useless to construct one of our usual "good guy--bad guy" explanations for the situation. There is no villain, only complexity, and it is not too early to bring out best research talents to grips with it.

Albert Wilson

type A (**56**). The examples of 3C 305 and NGC 1275 show that, as should be expected, the identifications of strong radio sources as spheroidal galaxies on the basis of their appearance on 48-inch Schmidt plates are not entirely reliable. Some galaxies of types E and D with peculiarities may turn out to be spirals or irregulars on more detailed observations.

F. Sato (**57**) has investigated the physical conditions of the radio galaxies NGC 1068, NGC 1275 and Cyg A in terms of the ionization mechanism of hydrogen atoms. Three possible mechanisms were examined, collisions of thermal electrons, thermal radiation from hot stars and ultra-violet synchrotron radiation. The conclusion is that every mechanism meets difficulties. If collisions of thermal electrons play a role in ionization of hydrogen atoms, the kinetic energy of electrons are not high enough to maintain the ionization for the life time of emission line phenomena.

It would seem reasonable to suppose that ultraviolet radiation from hot stars is responsible for the ionization, but this possibility is excluded because of absence of Bowen resonance-fluorescence lines of O III in the case of NGC 1068. If the ionization is produced by synchrotron radiation, relativistic electrons must be provided either by acceleration or by continuous production. Both the acceleration and the production of the relativistic electrons meet difficulties from a point of view of energy balances.

G. de Vaucouleurs has investigated peculiar galaxies showing signs of instability (**58**).

B. E. Westerlund and L. F. Smith (**59**) report identifications and investigations of southern radio sources. An investigation of the Parkes radio source 0521–36 by B. E. Westerlund and N. R. Stokes (**60**) shows that it is a compact (N) galaxy of high optical luminosity and strong radio emission.

D. E. Osterbrock and R. A. R. Parker (**117**) measured the intensities in the spectrum of the nucleus of the Seyfert galaxy NGC 1068, and showed that in the ionized gas $T > 8000°$ and that substantial amounts of neutral gas are mixed into the ionized regions. Frequent collisions between high-velocity clouds probably produce a large part of the observed ionization.

GALAXY COUNTS. CLUSTERS OF GALAXIES

An exhaustive search for dwarf galaxies in the Fornax cluster of galaxies was carried out by P. Hodge, M. Pyper and J. Webb (**61**). Fifty dwarf objects were chosen as probable members of the cluster, and photometry revealed that some are probably sculptor-type dwarfs and others are dwarf irregular galaxies. The spatial distributions of giant and of dwarf galaxies in the cluster are sufficiently different to suggest the presence of equipartition of galaxies of different masses.

The distributions of the stars in all six dwarf galaxies of the Local Group have been used to compare observed and predicted limiting radii (**62**). For the three nearest galaxies, the observed limits agree with those computed, but for the three more distant ones, the observed limits are too small. This was interpreted to be in agreement with the fact that relaxation times computed for these objects are extremely large, 10^{12} to 10^{15} years. The orbital parameters of the galaxies were estimated, and it was found that for the Fornax galaxy, the radial velocity of which is known, parameters for the orbit could be fairly completely determined.

T. Kiang (**63**) has examined the problem of clustering of Abell's rich clusters of galaxies by adapting Neymann and Scott's model of uniform clusters to these objects.

The parameters in the model were determined from the variation of the index of clumpiness K^2 with the cell size z. Random realizations of the model so specified were then effected to produce (*i*) synthetic surface distribution and (*ii*) the variation with separation h of the coefficient of quasi-correlation, $Q(h;z)$ for 2 values of z, and these were then compared with their observed counterparts. Discrepancies of the same sort as found by Neyman and Scott in their examination of the clustering of galaxies show up, namely (*i*) the synthetic surface distribution did not reproduce all the 'clumpiness' of the observed surface distribution and (*ii*) the two synthetic $Q(k)$ curves were close together whereas the observed curves for the same two cell-sizes were far apart. The present result on the clustering of Abell's objects, combined with Neyman and Scott's result on the clustering of galaxies and the phenomenon of sub-clustering (**64**) suggest that galaxies are clustered on all scales. This picture is in no way inconsistent with the apparent absence of clustering among radio sources when the great numerical disparity between radio sources and galaxies is remembered. Further, Kiang believes that while there is clustering on all scales, it does not take the form of physically distinct clusters. Kiang also points out that this picture of indefinite clustering naturally results from the mechanism of continual creation proposed by W. H. McCrea (**65**).

H. Neckel (**66**) finds that the brightness distribution within the galaxies plays a serious role in the determination of the optical thickness of the galactic absorption layer from the latitude variation of galaxy counts and from the mean surface brightness of the galaxies. A value of about $0^m\!.9$ is found for the photographic optical thickness of the absorption layer. General agreement is achieved between the hitherto different results of (1) Hubble's counts of galaxies, (2) the counts by C. D. Shane, C. A. Wirtanen and U. Steinlin (**67**), and (3) the observations of the mean surface brightness. The value of $0^m\!.45$ for the absorption at the galactic pole and the color excess $B - V = 0^m\!.05$ or $0^m\!.06$, which remains unchanged, lead to a value of about eight for the ratio between total photographic absorption and color excess.

C. D. Shane and G. E. Kron are determining the limiting photographic magnitudes of galaxy counts made with the 20-inch (51 cm) astrograph at the Lick Observatory. While not all of the final corrections have been applied, the average galaxy at the limit of identification is quite close to magnitude 19·0. Shane and Kron have also measured the color of several hundred galaxies in different galactic latitudes. Tentative discussion of only a fraction of the measures reveals a small or even negligible relation between latitude and color.

A new analysis of Hubble's galaxy counts has been carried out by G. de Vaucouleurs and G. Malik (**68**) who found serious bias in earlier analyses; in particular the half thickness of the galactic absorbing layer is $A_B = 0{\cdot}5$ magnitude (not 0·25 magnitude) in agreement with Shane's analysis of the Lick counts. An improved expression for galactic extinction as a function of latitude and longitude was obtained.

G. de Vaucouleurs (**69**) has derived distance indicators based on magnitudes and diameters have been established for galaxies in small groups or clusters and distances of 54 nearby groups; these are in good agreement with distances from van den Bergh's luminosity classification.

G. C. Omer, T. L. Page, and A. G. Wilson (**70**) have intercompared three independent counts of the Coma cluster of galaxies. The three different surveys appear to be consistent with each other. The conclusion is reached that the cluster contains about 800 members to photovisual magnitude about 18·8 and has a radius of about 100′. A possible spatial density distribution is derived from the combined data.

A. G. Wilson studied the distribution of rich clusters of galaxies. Regularities in their distances (**71**) and angular separations (**72**) suggest the existence of larger scale structures than any presently recognized. Radio galaxies also appear to share cluster distributions (**73**) in the sense of indicating the existence of large scaled structures. The redshift distributions may be interpreted as indicative of a continuation of a Charlier type hierarchy up to organizations of the order of 500 megaparsecs.

F. Zwicky and his collaborators (**74-78**) have made extensive studies of the distribution of clusters of galaxies which have shown: (*a*) The largest open, medium compact and compact clusters of galaxies at all distances have the same indicative dimensions, (*b*) There is no evidence for any clustering of clusters of galaxies, all irregularities being accounted for by accidental fluctuations as well as the interference of interstellar and intergalactic obscuration and (*c*) The average size of the cluster cells occupied by rich clusters of galaxies, as determined from the distribution of 7000 clusters of galaxies is about 50 million pc, in close agreement with the value determined by Zwicky in 1938 from the distribution of the hundred nearest clusters then known.

Several investigations of individual clusters of galaxies have been carried out by members of Zwicky's group. R. Okroy (**79**) has investigated the Virgo cluster and confirmed a segregation effect: the number of galaxies per square degree increases towards the center for the more luminous galaxies, but decreases for the fainter galaxies. A similar result was obtained by T. Kwast (**80**) for the Hydra I cluster. K. Rudnicki and M. Baranowska (**81**) find the segregation effect in the clusters Zwicky 156–5 and 156–14, but not (**82**) in the cluster Zwicky 97–8 (= 127–2).

COSMOLOGY AND RELATED SUBJECTS

Relevant observational information on the physical conditions in intergalactic space now begins to become available. One of the most important newly discovered phenomena is cosmic background radiation whose significance has been discussed by R. H. Dicke, P. J. E. Peebles, P. G. Roll, D. T. Wilkinson (**83**). This radiation becomes observable at wavelengths below 20 cm. At longer wavelengths galactic and extragalactic emissions dominate the spectrum. The observed background temperatures are:

2·8 ($\pm$0·6) °K at 20·7 cm by T. F. Howell and J. R. Shakeshaft (**84**)
3·5 ($\pm$1·0) °K at 7·4 cm by A. A. Penzias and R. W. Wilson (**85**)
3·2 ($\pm$0·5) °K at 3·2 cm by P. G. Roll and D. T. Wilkinson (**86**)
3·22 ($\pm$0·15) °K at 0·26 cm by G. B. Field and J. L. Hitchcock (**87**),

determined from the excitation of interstellar cyanogen. These results suggest that the background radiation has a blackbody spectrum with a temperature close to 3°K. The discordant value

1·7 ($\pm$0·4) °K at 1·5 cm by W. J. Welch (**88**)

has quite recently been obtained. Additional observations obviously are needed to establish the nature of the background radiation beyond doubt.

G. J. Whitrow and B. D. Yallop (**89**) have discussed the problem of background radiation in homogeneous isotropic models in which account is taken of possible intergalactic absorption effects due to the finite extent of the galaxies and to intergalactic matter. The possible effect of relaxing the assumption that on the average the galaxies radiate uniformly in time is also considered in a preliminary way.

In objects with a redshift of 2, Lyman-α of hydrogen and the shortward region of the spectrum become accessible to ground based optical observations. If the redshift is cosmological, neutral hydrogen between the source and us will become observable, either as an absorption continuum extending shortward from the redshifted line to its zero-velocity wavelength if the intergalactic medium is uniform (J. C. Gunn and B. A. Peterson, **90**; P. A. G. Scheuer, **91**) or as discrete absorption features if the intergalactic hydrogen is concentrated in clusters of galaxies (J. N. Bahcall and E. E. Salpeter, **92, 93**). Gunn and Peterson find from the lack of strong absorption shortward of Lyman-α that intergalactic hydrogen accounts for less than 10^{-6} or 10^{-7} of the cosmological density to be expected if the acceleration parameter $Q_0 = 1/2$. Thus, most of the hydrogen must be ionized if the discordance between the cosmological density and the density of matter in galaxies is to be explained as due to the presence of intergalactic hydrogen. G. B. Field, P. M. Solomon and E. J. Wampler (**94**) show that H_2

molecules also cannot contain a major fraction of the mass. The intergalactic gas then must be highly ionized and at temperature close to the limit of about 10^6 °K that has to be set according to G. B. Field and R. Henry (**95**) if the bremsstrahlung from the intergalactic medium is not to exceed the total flux in the X-ray region. It should be pointed out that a very small value of q_0 cannot be ruled out at present, but it is not suggested by any other evidence.

D. W. Sciama and M. J. Rees (**96, 97, 98, 99**) have made studies of the intergalactic medium in the light of recent observations of the X-ray background and the spectra of quasi-stellar objects with redshifts ~ 2. In the steady state model of the universe an intergalactic hydrogen gas of density $\sim 10^{-29}$ g cm^{-3} is required to have a kinetic temperature $\sim 10^6$ °K, which could plausibly be achieved by cosmic ray heating. If oxygen is present with its normal cosmical abundance, it could be detected in the spectra of sources with a redshift 2·2. However, the steady state model itself is not consistent with recent data on the redshifts of quasi-stellar objects, if these redshifts are cosmological in origin.

Whether the redshifts of quasi-stellar sources and objects is cosmological and whether they are correspondingly large distances is subject to doubt in view of the suggestion by J. Terrell (**100, 101**) that quasi-stellar objects are local, rapidly moving objects. The two different points of view have been discussed by F. Hoyle and G. R. Burbidge (**102**). At this time there is no conclusive argument in favor of one of the competing interpretations.

A. Sandage (**103**) has given an interim report on his recent work on the redshift-apparent magnitude relation for the brightest members of clusters of galaxies and for radiogalaxies. The results are mainly based on photoelectric photometry. The K term needed to correct the photometric data for the effects of redshift has been re-determined. The plot of redshift against the corrected V magnitude for the 1st ranked galaxy in clusters shows a linear relation with surprisingly small dispersion which emphasizes that the absolute magnitude of the brightest galaxy of a cluster is an exceedingly stable statistical parameter. The observed deceleration parameter will be very close to $q_0 = 1$. A correction must be applied, however, to the absolute magnitudes to account for the evolution of the stellar content for the individual galaxies during the light travel time. This correction gives $q_0 = +0{\cdot}2$ based on an evolutionary theory obtained from old star clusters. The uncorrected observations are consistent with a closed model, while the corrections lead to an open model of the universe.

The redshift-apparent magnitude relation for the radio galaxies fits the relation for the brightest galaxies in clusters closely, although with somewhat larger scatter. The conclusion is that the radio galaxies do not differ significantly in luminosity from the first ranked cluster galaxies and that a galaxy must be supergiant in luminosity and presumably in mass to be a strong radio source.

G. de Vaucouleurs (**103a**) finds that local departures from linearity and isotropy in the redshift-apparent magnitude relation are confirmed by an analysis of improved and enlarged magnitude and velocity data. It follows that the value of the Hubble constant derived from nearby groups or the Virgo cluster is biased and probably too low.

H. Alfvén (**104**) has further studied the properties and evolution of a meta-galaxy containing equal amounts of matter and antimatter. He analyzed the properties of an 'ambiplasma' (containing ionized matter and antimatter) and showed that a magnetized ambiplasma (under the conditions assumed to exist in the universe now) should emit synchrotron radiation but no detectable gamma-radiation. He also suggested mechanisms for separating matter from antimatter. The possible formation of proto-galaxies from an 'ambiplasma' and their subsequent evolution into galaxies was treated by H. Alfvén and A. Elvius (**105**) and reported at the IAU Symposium 29. The annihilation of protons-antiprotons as a possible source of energy for certain astronomical objects was again discussed. A. G. Ekspong, N. K. Yamdagni and

B. Bonnevier (**106**) showed that the radio spectra to be expected from extragalactic objects deriving their energy from the annihilation of nucleon-antinucleons would be similar to radio spectra actually observed for quasi-stellar radio sources and strong radio galaxies. B. Bonnevier (**107**) studied some problems related to the early development of the metagalaxy in the theory of H. Alfvén and O. Klein. His solutions of equations previously derived by Alfvén and Klein indicated that an original contraction of the metagalaxy might be converted to an expansion in agreement with observed redshifts.

R. J. Dickens and S. R. C. Malin (**108**) find that observations of quasi-stellar sources and small-diameter galaxies show no differential aberration with respect to nearby stars. The result contradicts the Ritz theory of light propagation.

P. E. Kuustanheimo (**109, 110, 111**) has developed a new kind of linear gravitational theory in which the gravitational redshift may be non-conservative. This would explain a possible limb effect in the sun, and the route dependence of the redshift could be measured in the laboratory if the redshift experiments of Pound and Rebka based on the Mössbauer effect are repeated using photons not travelling vertically.

D. Sugimoto (**112**) explains phenomena in quasi-stellar sources by the nuclear instability of a star of about 200 $M_\odot$ that are formed by the rapid fragmentation of gas of $10^8 M_\odot$ in the nuclei of galaxies.

Zwicky (**113**) has discussed the discovery and some principal characteristics of previously unknown families of compact galaxies. He also has discussed a number of circumstances which will cause an entire compact galaxy to implode or, in some cases at least to liberate and to radiate energy at a much more rapid rate than an ordinary galaxy. He has observed differential redshifts in the spectra of some compact galaxies (**114**) that suggest an explanation in terms of Einstein's gravitational effect and the possibility that large redshifts are only in small part due to the cosmological effect.

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APPENDIX I. OPTICAL DATA ON QUASI-STELLAR SOURCES

(Prepared by A. Sandage)

Introduction

There has been enormous activity in study of quasi-stellar radio sources since the 1964 Hamburg meeting. It is now generally accepted that the name 'quasars' be applied to these objects and we shall adhere to this nomenclature in this report. Work on quasars can be conveniently divided into eight sections, touching the subjects of identification, redshifts, absorption lines, photometry, optical variations, interlopers ('radio quiet' quasars, sometimes designated QSG for quasi-stellar galaxies), the controversy of cosmological versus local distances, and theoretical studies of the structure of the sources.

The present summary is a partial literature review of papers appearing between 1963 and 1966. No attempt for completeness was made. It is further confined primarily to optical observations with only a few remarks on interpretation.

Identification

The first quasars were optically identified in 1960. This was achieved when radio positions of high precision became available through the work at the Owens Valley Radio Observatory on a list of high-flux-density sources of small angular diameter, as measured at Jodrell Bank. Direct photographs of several sources in this list were obtained with the 200-inch telescope. Matthews and Sandage (**1**) found that 3C 48, 3C 196, and 3C 286 appeared to coincide with starlike objects at the radio positions (which were accurate to better than $\pm$ 10″ arc in both coordinates). These three objects all had peculiar U, B, V colors compared to normal stars. This peculiarity of bright ultraviolet has been one of the main observational methods of optical confirmation of suggested identifications.

The fourth source to be identified was 3C 273 by Schmidt (**2**), who used a precise radio position due to workers at the Owens Valley Radio Astronomy Observatory and to Hazard, Mackey, and Shimmins (**2a**) at Parkes, who obtained the position by a lunar occultation. The 13th magnitude object was bright enough for Schmidt to obtain highly widened spectra of good quality from which he discovered the general property of redshifts (**2**).

The next three sources were found by Ryle and Sandage (**3**) using a two-color photographic method invented by Haro, where an ultraviolet and a blue image of each star in a given field is obtained on the same plate. These plates are searched for extreme ultraviolet objects. Sources 3C 9, 3C 216, and 3C 245 were discovered in this way. Photoelectric photometry confirmed the peculiar colors.

Schmidt and Matthews (**4**) identified 3C 47 and 3C 147 and confirmed from spectrograms that these were quasars.

By the middle of 1964 it was realized that quasars must be numerous. Estimates indicated that about 30% of the 3*C Revised Cambridge Catalogue* were objects of this type. Concentrated efforts of identification were then made at Owens Valley by Matthews and Wyndham, at Mt Wilson and Palomar by Sandage and Véron, at Cambridge by Longair (**5**) and later by Wills and Parker (**6**), and by Bolton and his co-workers at Parkes (**13**, **13a**, **14**).

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HOMOGENEOUS COSMOLOGICAL MODELS WITH BOUNDED POTENTIAL

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ABSTRACT

Under the assumption that there exists a finite global upper bound to the gravitational potential, $2Gm/c^2r$, it is shown that simply connected homogeneous models based on the Walker-Robertson line element must have zero mean density if the curvature parameter k is 0 or -1. For homogeneous models with $k = +1$, all global potential bounds less than or equal to the Schwarzschild Limit imply a positive cosmological constant. If the potential is globally less than 0.912 (spherical space) or 0.75 (elliptical space), and if the physically realizable pressures are at all future epochs bounded below by zero and above by photon gas pressure, $\rho c^2/3$, $k = +1$ models will expand monotonically without limit—for all future time. Asymptotic values for large times are in the usual notations: $H = c\sqrt{(\lambda/3)}$, $q = -1$, $p = 0$, and $\rho = 0$. Under the same conditions, the present value of the deceleration parameter is less than -1, so that the asymptotic value is approached from below. The cosmological constant is bounded below by $3H_0^2/c^2$, where H_0 is the present value of the Hubble parameter.

I. POTENTIAL BOUNDS

The existence of an upper bound to the local value of the gravitational potential that may obtain anywhere in the Universe is suggested by both theory and observation. Following Schwarzschild, several authors (Eddington 1923; Tolman 1934; Buchdahl 1959; Chandrasekhar 1964; Bondi 1964; Fowler 1966) have shown theoretically that the potential $\Phi = 2Gm/c^2r$ (where G is the gravitational coupling constant, c the velocity of light, and m the total mass contained within radius r) of a static spherically symmetric system immersed in a region of zero mass density is bounded. These bounds result from solutions of the relativistic or post-Newtonian field equations under various assumptions regarding the equation of state. Examples of some of these theoretical potential limits are given in Table 1. Schwarzschild found the limiting value of unity for

TABLE 1

RELATIVISTIC POTENTIAL BOUNDS

Bound	Symbol	Constraints: Spherical Symmetry Plus	Upper Limit to $2Gm/c^2r$
Schwarzschild........	U_S	$\rho\equiv$Const.	1
Eddington............	U_E	$\rho\equiv$Const., p finite	0.888
Bondi I..............	U_B	ρ not increasing from center, $p\leq\rho c^2/3$	0.638*
Bondi II.............	U_A	Adiabatic stability $p\leq\rho c^2/3$	0.620*

* Instead of the proper radius, Bondi uses $\sqrt{(A/4\pi)}$, where A is the proper area.

the potential assuming that the interior of the spherical system consisted of an incompressible perfect fluid with constant proper density. Eddington derived the limiting value of $\frac{8}{9}$ under the additional assumption that the pressure be everywhere finite. Bondi has shown that 0.970 is a rigorous upper bound for the potential independent of any assumptions concerning the equation of state and subject only to the restriction that the density nowhere be negative. Other Bondi limits are given in Table 1.

In addition to these relativistic and post-Newtonian bounds suggested by theory, there is also observational evidence suggesting the existence of a potential bound for stable non-degenerate cosmic bodies (Wilson 1966). The maximum values of the mass/radius ratio that occur in those samples of stars, galaxies, and clusters of galaxies that have been measured are found to be nearly the same for each species of cosmic body. This bounding value is approximately $10^{23.6}$ g/cm, or $\Phi \leq 10^{-4.3}$. Table 2 gives the largest values of observed potentials (Allen 1963). The value (U_0) of this observed bound and its significance are uncertain. The same value of approximately 10^{23} g/cm may be derived for second-order clusters on the basis of their radii and cluster contents (de Vaucouleurs 1960; Abell 1961). Despite the fact that U_0 is markedly less than the theoretical limits, as might be expected in an expanding system, a nearly equal upper potential limit of systems that are aggregates of particles which are atoms, stars, or galaxies suggests a bound independent of the equation of state and in phenomenological correspondence with the predictions of theory.

TABLE 2

MAXIMUM OBSERVED GRAVITATIONAL POTENTIALS

System	Object	$\log_{10}(m/r)$ (g/cm)
Star.	V444 (Cyg A)	23.27
Galaxy.	M87	23.6
Cluster.	Coma	23.5

The above theoretical and observational results suggest the following *hypothesis:* there exists a constant *global* potential upper bound, U, such that for any sphere of proper radius r circumscribed about any point P as center, the total mass contained within the sphere will satisfy

$$\Phi = 2Gm/c^2r \leq U\,, \tag{1}$$

provided r is sufficiently large. This hypothesis, if true, would have interesting cosmological implications. The intent of this paper is to investigate the effects of this hypothesis on a class of relativistic cosmological models constructed under the assumption that the matter in the Universe may be approximated *in the large* by a uniform perfect fluid (Robertson 1933) whose density and pressure are functions of time only. In order for condition (1) to be consistent with the assumption of uniform density in the large as postulated for homogeneous models, it is necessary that the mean density in every sphere of radius r and center P also be bounded. This condition of bounded density is consistent with both observation and the properties assumed for homogeneous isotropic models.

II. HOMOGENEOUS MODELS

a) Basic Equations

The bounded potential condition (1) will be applied to those homogeneous isotropic models with the usual assumption of simply connected covering spaces. Following Robertson (1933), we take coordinates such that the standard metric is

$$ds^2 = c^2dt^2 - R(t)^2du^2\,, \tag{2}$$

where

$$du^2 = d\chi^2 + \epsilon(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

is the line element on a three-dimensional space of constant curvature, $k = (1,0,-1)$. In equation (3),

$$0 \leq \theta \leq \pi, \qquad 0 \leq \phi < 2\pi$$

and

$$\epsilon(\chi) = \begin{cases} \sin(\chi), & 0 \leq \chi \leq \pi \\ \chi, & 0 \leq \chi \leq \infty \\ \sinh(\chi), & 0 \leq \chi \leq \infty \end{cases}$$

accordingly as the curvature is positive, zero, or negative. The forms assumed by the field equations in Robertson's notation are

$$\kappa\rho c^2 = -\lambda + 3(k + R'^2/c^2)/R^2\,, \tag{4}$$

$$\kappa p = \lambda - 2R''/(c^2R) - (k + R'^2/c^2)/R^2\,. \tag{5}$$

If $m(X,t)$ denotes the mass within the "coordinate sphere," $0 < \chi < X$ at time t; then, since the mean mass density $\langle p(t)\rangle$ is a function of time only,

$$m(X,t) = \langle\rho(t)\rangle R(t)^3 \int_0^X \epsilon(\chi)^2 d\chi \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi\,.$$

Since χ is a geodesic coordinate, the proper (geodesic) radius $r(X,t)$ of the coordinate sphere $0 \leq \chi < X$ at time t will be

$$r(x,t) = R(t) \int_0^X d\chi = R(t)X\,.$$

For positive, zero, and negative curvatures, the potentials on the surfaces $\chi = X$ are taken to be

$$\frac{2GM(X,t)}{c^2r(X,t)} = \frac{8\pi G\langle\rho(t)\rangle R(t)^2}{c^2}\begin{cases}[1-\sin(2X)/(2X)]/2\,, \\ X^2/3\,, \\ [\sinh(2X)/(2X)-1]/2\,, \end{cases} \tag{6}$$

where $0 \leq X \leq \pi$; $0 \leq X \leq \infty$; and $0 \leq X \leq \infty$ respectively.

b) Zero and Negative Curvature

By the global bounded potential condition (1), the right members of equation (6) will be $\leq U$ for all X in the allowable ranges.

In the cases of zero and negative curvature ($k = 0$ and $k = -1$) the allowable range for X is infinite. Since $R(t_0) = R_0 \neq 0$ at the present epoch (the subscript "0" will be used throughout to designate the value at the present epoch), it follows that the inequality (1) can be satisfied for all X if and only if $\langle \rho_0 \rangle \equiv 0$. It is also evident that $\langle \rho \rangle = 0$ for all t for which $R(t) \neq 0$. This leads to the conclusion that, for any finite potential bound U, the only universes with zero or negative curvatures that are isotropic and homogeneous have zero mean density. Furthermore, this conclusion holds regardless of the dynamical processes involved, since it was obtained without use of the field equations.

It must be noted that this conclusion may not be valid if the spaces are not simply connected. There exist eighteen different topological space forms for a three-dimensional space with zero curvature and infinitely many in the case of negative curvature. Some of these space forms are known to be closed, in which case the arguments of this section do not hold (Heckmann and Schücking 1962).

Two possibilities thus exist for open simply connected homogeneous universes with bounded potentials. They may be empty of all matter—which contradicts observation—or they may be hierarchically structured. Charlier (1922) has shown that a hierarchal universe with infinite orders of clustering has a vanishing mean density.

c) Positive Curvature

A similar argument cannot be made in the case $k = +1$, since the function $S(X) = [1 - \sin(2X)/(2X)]$ occurring in equation (6) is bounded for all X. Accordingly, in order to discuss the implications of bounded potentials for this case, we must use the properties of the field equations. For $k = +1$ we consider only the two customary topological cases: *spherical* space for which $0 \leq X \leq \pi$ and *elliptical* space for which $0 \leq X \leq \pi/2$. At the values $X = \pi$ and $X = \pi/2$, the so-called spherical and elliptic *horizons*, the potential $2Gm(X,t)/[c^2 r(X,t)]$, takes on the same value, namely, $\kappa c^2 \langle \rho(t) \rangle R(t)^2/2$. In the following it will be convenient to introduce the quantity

$$\Phi^*(t) = \kappa c^2 \langle \rho \rangle R^2 , \quad (7)$$

equal to twice the horizon value of the potential. By equations (6), (7), and the definition of $S(X)$, the inequality (1) becomes

$$\Phi^*(t) S(X)/2 \leq U .$$

In spherical space $S(X)$ assumes the maximum value of 1.217 at $X = 2.245$ radians, and hence

$$\Phi^*(t) \leq 1.644U . \quad (8a)$$

In elliptical space $S(X)$ assumes the maximum value of unity at the "end point" $X = \pi/2$ and hence

$$\Phi^*(t) \leq 2U . \quad (8b)$$

In order to make use of these bounds on Φ^*, we write the field equations (4) and (5) with $k = +1$ in terms of Φ^* and its time derivative:

$$\Phi^* - 3 = 3R'^2/c^2 - \lambda R^2 , \quad (9)$$

$$\Phi^* + \Phi'^*/H = 3\kappa p R^2 , \quad (10)$$

where $H(t) = R'/R$. From equations (5), (9), and (10), we may derive

$$-\Phi'^*/(2H) = \lambda R^2 + 3R'^2 q/c^2 , \quad (11)$$

where q is the "deceleration parameter," $-RR''/R'^2$.

For all potential bounds less than or equal to the Schwarzschild limit, ($U_S = 1$), in both spherical and elliptical cases the left member of equation (9) is strictly negative in some neighborhood of the present epoch. Hence, the cosmological constant, λ, must be *greater than zero*. The Schwarzschild limit taken as a global potential bound is accordingly inconsistent with homogeneous $k = +1$ models with vanishing or negative cosmological constant such as closed Friedmann models.

If we introduce the additional assumption that physically realizable pressures lie in the range $0 \leq p \leq \rho c/3$, the upper bound being photon gas pressure (Sandage 1961), by equation (10) the potential Φ^* must satisfy the inequalities

$$0 \leq \Phi^*/2 \leq -\Phi'^*/(2H) \leq \Phi^* . \quad (12)$$

… must satisfy the inequalities

$$0 \leq \Phi^*/2 \leq -\Phi'^*/(2H) \leq \Phi^* . \qquad (12)$$

Substitution of the right-hand inequality in expression (11) and adding equation (9) gives

$$(1 + q)R'^2/c^2 \leq 2\Phi^*/3 - 1 . \qquad (13)$$

This inequality requires that $q < -1$ whenever $\Phi^* < \frac{3}{2}$. Hence by expression (8a), q will be less than -1 for spherical space whenever $U < 0.912$. The Eddington limit, U_E, and smaller limits including the observed U_0 limit, all satisfy this requirement. Similarly by expression (8b), q will be less than -1 for elliptic space whenever $U < 0.75$. In addition, since $H_0 > 0$, it follows from expression (12) that $\Phi_0'^* < 0$.

It may thus be concluded that, ~~except possibly for hierarchical universes of infinite order,~~ *the only non-empty, homogeneous, isotropic cosmological models with simply connected topologies and a global bounded potential less than or equal to* 0.75 *are those with* $k = +1$, $\lambda > 0$, $q_0 < -1$, and $\Phi_0' < 0$.

III. THE FUTURE STATE OF HOMOGENEOUS UNIVERSES

a) Conditions in the Neighborhood of the Present

We may investigate allowable evolutionary paths of homogeneous models with bounded potentials by using the conclusions of the previous section as initial conditions and investigate the behavior of equations (9) and (10) subject to the assumed bounds on the potential and the pressure. Since $\Phi_0'^* < 0$ and $\Phi_0^* < 2$, for U_S (or any smaller bound) there will exist some time interval T containing the present, during which the left member of equation (9) will remain strictly negative. Accordingly the inequality

$$-g(t)^2 \equiv (\Phi^* - 3)/\lambda = 3R'^2/(\lambda c^2) < 0 \qquad (14)$$

holds for t on the interval T. The function $g(t)$ defined by equation (14) is real valued on the interval T, permitting the change of variables

$$R(t) = g(t) \cosh [w(t)] . \qquad (15)$$

Whereby equation (14) becomes

$$[(g'/g) \cosh w + w' \sinh w]^2 = (c^2\lambda/3) \sinh^2 w . \qquad (16)$$

Hence equation (15) will be a solution of equation (14) provided the function $w(t)$ satisfies the relation

$$w' = \pm c\sqrt{(\lambda/3)} - g' \coth (w)/g . \qquad (17)$$

Equation (17) may be used to eliminate w' from the derivative of equation (15), giving

$$R' = \pm c\sqrt{(\lambda/3)}\, g \sinh w . \qquad (18)$$

Since R_0' and g_0 are positive, we may have either

$$R' = c\sqrt{(\lambda/3)} g \sinh w, \qquad \text{with } w_0 > 0 ,$$

or

$$R' = -c\sqrt{(\lambda/3)} g \sinh w, \qquad \text{with } w_0 < 0 .$$

The evolutionary behavior of R' during the interval T is thus controlled by the functions $w(t)$ and $g(t)$. To ascertain the behavior of R' and R, we must determine whether $g(t)$ and $w(t)$ are increasing or decreasing in the neighborhood T. Differentiating equation (14), we get

$$g' = g\Phi'^*/(2\Phi^* - 6) . \qquad (19)$$

Since $\Phi_0'^* < 0$, $(\Phi_0^* - 3) < 0$, and $g_0 > 0$, the ~~value~~ function $g(t)$ must be an *increasing* function of t in the neighborhood T.

In order to investigate $w(t)$, we substitute equation (19) and the relation $H = \pm c\sqrt{(\lambda/3)} \tanh w$, (obtained from eqs. [15] and [18]), in equation (17):

$$w'(t) = \pm c\sqrt{(\lambda/3)}\{1 + \Phi'^*/[2H(3 - \Phi^*)]\} .$$

For the physically permissible range discussed in § II, the pressure, which may be written as $\eta\langle\rho\rangle c^2/3$ in terms of a dimensionless parameter η defined on the interval $0 \leq \eta \leq 1$, affords $3\kappa p R^2 = \eta\Phi^*$. Using this relation and equation (10), one may show that the permitted values of $w'(t)$ must lie in the interval

$$c\sqrt{(3\lambda)}\, \frac{(3 - 2\Phi^*)}{(3 - \Phi^*)} \leq \pm w' \leq c\sqrt{(3\lambda)}\, \frac{(2 - \Phi^*)}{2(3 - \Phi^*)}, \qquad (20)$$

where the left member corresponds to $\eta = 1$ and the right member to $\eta = 0$. Both the left- and right-hand expressions in Φ^* will be positive for $\Phi^* < \frac{3}{2}$. This is the same condition as that governing the validity of the bounds given in the last paragraph of § II. Hence, if these same global bounds are valid for all t in T, $+w'$ will always be bounded above and below by two positive numbers. It follows from the sign convention used for R' in equation (18) and from $g > 0$, $g' > 0$ in T that R' is *increasing* in T and that the values of R and R' are independent of the choice of sign, $-w'$ or $+w'$. Hence throughout T, R'' is strictly positive and q negative.

b) Asymptotic Values for Large t

From equation (10) and the definition of η, we have that $\Phi'^* = -(1 + \eta)\Phi^* H$. Substituting this quantity in equation (11) we obtain

$$R'' = c^2\lambda R/3 - (1 + \eta)c^2\Phi^*/6R\,. \tag{21}$$

Since R'' is positive in T, the first term of the right member must be larger than the second throughout T. Further, with R increasing and Φ^* decreasing, R'' will remain positive for all future time, and T is unbounded above. It follows that for all $t > t_0, R'(t)$ and $R(t)$ will be exponentially increasing functions. The Hubble parameter, $H(t) = c\sqrt{(\lambda/3)}\tanh[w(t)]$, is monotonically increasing and will approach the value $c\sqrt{(\lambda/3)}$ asymptotically for all allowable states of the model. This fact establishes $3H_0^2/c^2$ as a lower bound for the cosmological constant. ($H_0 = 100$ km/sec/Mpc $\sim \lambda > 3.5 \times 10^{-56}/\text{cm}^2$). The pressure and density decrease to zero as t increases without limit. It follows from equation (5) that $R''/R \to \lambda c/3$ and $q \to -1$. If $\Phi_0^* < 3/2$, then $q_0 < -1$ and the asymptotic value is approached from *below*.

Possible homogeneous models with $k = +1$ and $\lambda > 0$ are: (*a*) one that expands monotonically without limit from a singular value (Lemaitre); (*b*) one that expands from a non-zero critical radius (Lemaitre-Eddington); (*c*) a static model (Einstein); (*d*) one that expands asymptotically to a finite radius; (*e*) one that contracts from infinity to a minimum finite radius, then expands monotonically without limit; (*f*) one that oscillates between zero and a finite radius (Bondi 1952).

The behavior of R and R' for large t rules out all models except (*a*), (*b*), and (*e*). These three cases are indistinguishable on the basis of their future paths. Thus, while the future is uniquely determined under the above assumptions of bounded potential and pressure, the question remains which of the three permitted homogeneous models with positive cosmological constant represents the past history of the Universe.

While the use of homogeneous relativistic models to represent the present and future state of the Universe may afford a valid approximation to the physical situation; the validity of indefinite extrapolation to the past when other forces than gravitation played major roles is open to question. Additional information such as that contained in the natural aggregates—the elements, stars, galaxies, clusters—must be used to discriminate between past histories.

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Wilson

DOUGLAS AIRCRAFT COMPANY
IRAD LINE ITEM DESCRIPTION No. __________

Title:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

- [X] Basic
- [ ] Applied
- [ ] Development

Objectives and Approach:

The primary objective of the project on gravitational potentials is to study the nature of gravitational interactions under as diverse and extreme physical conditions as are available to observation. The Newtonian Theory of the gravitational interaction was based on gravitational phenomena observed to occur in regions of small potential ($GM/c^2R \ll 10^{-6}$) such as interplanetary space. Modifications in Newtonian Theory were found to be necessary to account for phenomena occurring in regions of higher potential. Several theories [e.g., those of Einstein (Reference 1), Dicke (Reference 2), and Kirkwood (Reference 3)] have been advanced to account for the phenomenon of the advance in the perihelion of Mercury. The object of the present project is to locate and explore the properties of the regions of highest gravitational potential to determine whether anomolous phenomena exist which would require further modification of gravitational theories.

Since gravitational forces are of the order of 10^{-40}, the magnitude of Coulomb and other atomic forces, gravitational effects are studied only with great difficulty in the laboratory and regions of high gravitational potential are unavailable. Observation of the structure and motion of cosmic bodies provides the most useful source of data for the study of

(Continued)

Principal Investigator and Assisting Personnel: (see Appendix for Biographical Data)
P.I.:

This effort is a continuation of work described in:
Report __________ Calendar Year ______ Page ______
Facilities Utilized see Volume ______ Page ______
Related Government Documents:

	Calendar Year	Previous Year(s)
Est. Professional Man Years		
Estimated Other Expenditures		
Estimated Total		

Project Start Date
Estimated Completion Date
Est. % Completion This Date

____
Director

Title:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

Objectives and Approach: (Continued)

gravitation. A function of primary usefulness in cosmic gravitational studies is the dimensionless gravitational potential,

$$\Phi = \frac{2GM}{c^2 R}$$

where M is the mass of the body, R the metric radius, G the Newtonian gravitational coupling constant and c the velocity of light. It is fortunate that Φ for many astronomical bodies may be evaluated independently of knowledge of the distance to the body. This permits data to be accumulated that is free of the troublesome problems besetting astronomical distance scales and their calibration.

During the past two years the value of Φ for many astronomical bodies has been derived from new and published observations. Through the courtesy of the Mt. Wilson and Palomar Observatories use has been made of the 48 inch telescope and the B spectrograph on the 100 inch telescope to determine the potentials of several clusters of galaxies using the Virial Theorem (References 4 and 5). Search has been made for the bodies with largest Φ. It is planned to complete the present survey for regions of high potential in 1968.

Summary of Past Achievements:

The Schwarzschild solution to the field equations of the general theory of relativity, in addition to giving the three well known tests of the theory, predicts that the potential Φ be less than unity (Reference 1). The present survey of potentials of stars, galaxies, and clusters of galaxies confirms the existence of a potential bound, but finds that for stable non-degenerate bodies this bound is of the order of $10^{-4.3}$ (Reference 6). The existence of banded structure in the distribution of the redshifts of radio sources and rich clusters of galaxies (Reference 7) is predicted by the existence of this potential bound (Reference 8).

Edelen and Wilson (Reference 9) have developed the cosmological implications of the results suggested by these data. Under the assumption that the gravitational potential is globally bounded the only homogeneous closed cosmological models permitted are those with positive cosmic constant which expand for all future time with increasing rate. Whereas the future is determined, the past state of these models cannot be unequivocally decided. Cosmogonic models consistent with bounded potentials are being investigated.

(Continued)

MCDONNELL DOUGLAS CORPORATION

DOUGLAS AIRCRAFT COMPANY
IRAD LINE ITEM DESCRIPTION No. ______

Title:

GRAVITATIONAL POTENTIALS OF COSMIC BODIES

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June 7, 1968

ABSTRACT OF PROJECT

HOMOLOGY IN GRAVITATING STRUCTURES

A. G. Wilson

A search for the homologous properties of dynamical structures of a general nature has been initiated with the study of the properties of gravitating systems. These systems have the advantage of observable quantitative parameters possessing well known functional relationships. Stable non-degenerate gravitating bodies (stars, galaxies, clusters, hyperclusters) have been found to possess an homologous potential bound of the order of 10^{-5} of the Schwarzschild Limit. However, gravitational forces plus only a potential bound are not adequate to define the mass or size of these structures. An additional constraint, such as a density limit, is required. An answer for the cosmogonic problem of the origin of the particular cosmic bodies observed requires that some further homologous parameter be identified. Possible quantitative homologies in angular momenta are now being investigated.

Submitted to Nature
June 7, 1968

AN EMPIRICAL RELATION IN PULSAR PERIODS

Pulse periods of the four known pulsars have been determined to an accuracy of the order of one part in 10^7 (Ref. 1, 2). The pulse durations cannot be nearly so accurately measured, but the mean pulse durations of sets of superimposed pulses can be estimated to within three or four milliseconds (Ref. 3, 4). To within the accuracies of these estimates a single simple empirical relation appears to hold between the pulse periods, T, and the mean pulse durations, $\langle d \rangle$, for each of the four observed pulsars,

$$(1) \qquad AT = \langle d \rangle^2, \qquad \text{where } A = 10^{-3} \text{ seconds.}$$

A comparison of values is shown in the table where $\langle d_o \rangle$ is the approximate <u>observed</u> mean duration and d_c is the value given by Eq. (1). All values are in seconds.

OBJECT	T	$\langle d_o \rangle$	d_c
CP. 0834	1.273764	0.035	0.0357
CP. 0950	0.253065	0.015	0.0159
CP. 1133	1.187909	0.035	0.0345
CP. 1919	1.337301	0.037	0.0366

The parameter A with the dimensionality of time defines a nearly constant <u>characteristic</u> <u>time</u> for objects of the pulsar class.

The correct identification of the observed periods with limiting periods ($\sqrt{3\pi/GP}$) for various classes of bodies is critical for formulation of the right pulsar model. The pulse period itself is too short for white dwarfs. The periods d and A are both consistent with the limiting period of neutron stars.

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MEMORANDUM

June 20, 1968

To: L. Larmore, G8

From: A. G. Wilson, G8-52

Subject: PULSARS

The attached sheets give a summary of the salient present knowledge concerning pulsars. Optical objects suspected of being associated with CP.1919 are probably spurious. No satisfactory theory has yet been evolved. Preliminary ideas range from rotating neutron stars, binary white dwarfs, to "little green men." The empirical relation, $d^2 = AT$ (note d) is a DARL contribution to the subject.

A. G. Wilson

AGW/jbg

A. G. Wilson 6/19/68

PULSARS -- A SUMMARY

A new large radio telescope operating at 81.5 MH_3 was put into use by the Mullard Radio Astronomy Observatory of the University of Cambridge in July of 1967. The aerial consists of a rectangular array containing 2048 full wave dipoles arranged in 16 rows of 128 elements. The array is 470 meters (E-W) by 45 meters (N-S). This telescope was built to investigate the angular structure of compact radio sources using the scintillation caused by the interplanetary medium. A weekly survey of the sky between the declination zones -08° and +44° using this new telescope resulted in the detection of four very weak pulsating signals at fixed declinations and right ascensions. Systematic investigations of these signals were started in November of 1967 and the first publication of discovery appeared in Nature, vol. 217, p. 709, February 24, 1968. The observed properties of these sources -- now called "Pulsars" -- are summarized in the table.

No distances have been determined, but the observing of a doppler shift reflecting the earth's orbital motion places the pulsars definitely outside of the solar system. From frequency dependence of signal retardation and the value of interstellar electron density,the pulsars are estimated to be over 150 light years distant.

The precise periods afford many applications -- determination of the A. V., galactic rotation and magnetic field, time service, space navigation, etc.

PULSARS

Designation	Position	Galactic Coordinates	Pulse Period	Pulse Duration	Mean Flux Density
	α (1950.0) δ (1950.0)	λ II b II	seconds	milliseconds	@ 81.5MH3
CP.0834	$08^h\ 34^m\ 07^s$ + 07° 00'	220°	1.273 764 200 ± 300	∿ 35 ± 4	0.3
CP.0950	$09^h\ 50^m\ 29^s$ + 08° 10'	230° 44°	0.253 065 000 ± 100	∿ 15 ± 4	0.8
CP.1133	$11^h\ 33^m\ 32^s$ + 17° 00'	240° 70°	1.187 909 280 ± 150	∿ 35 ± 4	0.3
CP.1919	$19^h\ 19^m\ 37^s$ + 21° 47'	56° 4°	1.337 301 092 ± 2	∿ 37 ± 4	0.4

NOTES:

a) Mean flux density in units 10^{-26} watts m^{-2} H_3 -1

b) The pulsars have been observed at frequencies from 75.3 to 2700 MH_3

c) The fine structure of the pulses follows in general the pattern: single pip (CP.0950), double pip (CP.0834 and CP.1133), and triple pip (CP.1919).

d) The pulse duration, d, seems approximately to follow the law $d^2 = AT$ where T is the pulse period and A is about one millisecond.

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