PUBLICATIONS AND PAPERS 1969

Albert G. Wilson

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I GRAVITATIONAL STUDIES

The direct observations underlying present theories of the nature of the gravitational interaction are almost exclusively observations of gravitational effects in regions of low potential, $2GM/c^2R < 10^{-6}$. This is true in the laboratory, in the solar system, and in the neighvorhoods of most observed binary stars. To determine whether present theories of the gravitational interaction are valid under markedly different conditions, a search has been made to find bodies and regions with high gravitational potentials. Good values of potentials are derived for stars from the properties of the light curves of eclipsing binary systems; for galaxies, from their rotational dynamics; for clusters, from the virial theorem. The potentials of other bodies such as radio galaxies, quasars, nuclei of galaxies, cannot be determined directly. Estimates of their potentials depend on distance and other assumptions that are at present unconfirmed.

Excluding these bodies, it is found for all objects with well determined gravitational potentials that the maximum potentials expressed in terms of atomic units (m_p, the proton mass and a_o, the Bohr radius) are of the order of 10^{39}, i.e. we may write

$$\frac{Ma_o}{m_pR} = \frac{k\ e^2}{Gm_pm}$$

where e and m are the charge and mass of the electron and k is a factor of the order of unity. Using the basic relations $a_o\alpha^2 = r_e$ and $e^2 = mc^2r_e$ gives

$$(1)\quad \frac{GM}{c^2R} = k\alpha^2$$

for the maximum observed potentials, where α is the Sommerfeld fine structure constant.

The empirical relation given by equation (1) is not readily accounted for in present theories of gravitation. However, it can be shown that equation (1) may be formally derived under the

Schwarzschild conditions of general relativity for a metric based on the limiting velocity of bound electrons instead of the limiting propagation velocity. The introduction of such a metric involves other modifications of gravitational theory that are currently being explored in collaboration with Prof. D.G.B. Edelen of Purdue University.

In 1969 it is planned to continue theoretical work on models incorporating potential bounds as well as developing more completely the observational picture of the limits governing gravitional bodies.

II GENERAL STRUCTURE THEORY

In 1968 through consulting activities on transportation systems and organization theory carried on in cooperation with the divisions, we became aware of basic parallels between these systems and certain structural systems being investigated in the laboratory. Of particular interest were parallels between the systems under study by the divisions and the crystal, molecular, and gravitational (noted above) systems being researched in D.A.R.L. We were led to consider the possibility of a theory of general structures that would probe the fundamental concepts common to various static and dynamic structures. The approach adopted took relationships between entities, rather than the entities themselves, as fundamental. Thus instead of the disciplinary viewpoint that categorizes structures and systems according to the substances out of which they are composed (atoms, crystals; beams, struts; cells, tissues; codes, languages; stars, clusters; vehicles, transportation networks), structures and systems are analyzed in terms of their relational ingredients (levels, hierarchies; channels, feedback loops; bounds, closure; inputs, outputs).

The first exercise in this study identified hierarchical structure as a ubiquitous organizational form common to a large number of different types of structure and systems -- both natural and artificial. The species, properties, and causes of hierarchical structure were explored in a three day interdisciplinary symposium attended by biologists, crystallographers, designers, informationalists, cosmologists, and managements specialists held at D.A.R.L. in

November. The proceedings of this conference will be edited and published in 1969.

Work for 1969 will also continue present research efforts on the geometrical and topological properties of crystaline and macroscopic static structures and the energetics, informational and motivational aspects of dynamic structures including transportation systems and social groupings. Homologues between natural and artificial structures of various scales will be used to parameterize complex structures.

Closure, Entity, and Level

Albert Wilson*

The manner of decomposition of a complex organism or structure into sub-components is arbitrary. With a scalpel in the dissecting room or with the knife of pure intellect, the decomposer has freedom to isolate many alternative sub-groupings. However, unless his knife follows the "natural interfaces," severing a minimum of connections in isolating the sub-components, his decomposition may prove to be confusing, uninteresting, and messy. Whereas all decompositions possess the kind of properties that are treated in classical set theory, those decompositions conforming to natural interfaces frequently reveal additional interesting properties. What we call the "natural interfaces" are identifiable either by the occurrence of a steep decrement in the number or strength of linkages crossing them, as developed by Simon (1962) in the concept of *near decomposibility*, or through the existence of some form of *closure*. The purpose of this note is to sketch how entity and level may be related to one or more forms of closure.

The most apparent from of closure is *topological* closure–the encompassing by (one or more) closed surfaces of a spatial neighborhood that coincides with or bounds the extension of a physical object. We thus perceive balls, donuts, strings, and sheets as topologically closed. In general, topological closure bestows finitude and convexity on objects and is a property of most entities that we differentiate by visual perception.

A second type of closure, associated with a neighborhood in time that coincides with or bounds the *duration* of an entity, may be called *temporal* closure. More abstract notions of closure may be employed to distinguish non-physical entities. Thus a *group* may be defined as a set of numbers, elements, or transformations that possess closure with respect to some *operation*. For example, the integers 0, 1, 2, 3, 4 form a group closed under addition modulo 5. This type of operational closure, when the number of elements is finite, joins temporal closure in being cyclical in the sense that some parameter follows a path that periodically returns to previously assumed values. Topological closure and cyclical closure can be related through various Fourier type transformations. Spatial representations (particles) and frequency representations (waves) may thus both be subsumed under the notion of closure. In addition isolation of entities may take the form of either physical separation or "detuning."

Not only may differentiatable entities and modules be described through the use of some form of closure or cyclical parameter, but many

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**Douglas Advanced Research Laboratories, Huntington Beach, California, 92647*

notions of *level* may also be differentiated through closure. For example, levels in control hierarchies such as industrial corporations are determined by subsystems identifiable through various feed-back loops which are mappable onto a set of closed cyclical parameters. In modular hierarchies (Wilson 1967) levels and modules share a set of topological closures and when the modules are homogeneous the levels become identical to the modules.

The example of hierarchical cosmic sub-structures (Wilson 1969) shows that levels may be distinguished by a characteristic time or frequency, which is to say that each level is temporally closed. This suggests that the properties of space and time are closure properties of structures, bringing to mind the basic idea of Leibniz that space and time have no independent existence, but derive from the nature of structures. Einstein's equivalence of dynamics and geometry contained in his field equations (e.g., matter density determines spatial curvature) is also consistent with Leibniz's view and a departure from the Newtonian idea that all structure exists within an independent framework of space and time. It may then be that from the various closures and partial closures of structures and systems, we infer the descriptions we call space and time.

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Hierarchical Structure in the Cosmos

Albert Wilson*

The primary focus of cosmological thought in the present century has been on interpreting the observations of the sample of the universe available to our telescopes in terms of a set of models based on various theories of gravitation; especially the General Theory of Relativity. The problem of the structure *of* the universe is customarily divorced from the problem of the structure *in* the universe. Theoretical cosmologists usually choose to explain the structure and behavior – past and future – of the universe with models that smooth out the distribution of matter in the universe, replacing the observed structured distribution of matter with a uniform homogeneous perfect fluid whose density varies in time, but not in space. However, the structure contained *in* the universe becomes difficult to relate to models constructed around smoothing postulates. This has resulted in separate theoretical approaches to the origin of the various structures in the universe. While most of these approaches have met with some success, they are inadequately related to one another and to cosmological theories.

The arbitrary separation of the structure and behavior of the universe from the structure and behavior of its contents may be expedient from the point of view of mathematical simplification, but it cannot be accepted as more than an exploratory strategy. The observational tests for discriminating between various cosmological models are difficult and marginal. Since several smoothed models are candidates for best fit to the observations, it is unfortunate that the large amount of information contained in the sub-structures of the universe cannot be used in testing these models. But until models that relate the properties of the sub-structures to the properties of the whole are employed, much information of potential cosmological value in sub-structure astronomical observations is not cosmologically useful.

*Douglas Advanced Research Laboratories, Huntington Beach, California, 92647.

So long as the cosmological problem has been approached through smoothing out the sub-structures, it is not surprising that little attention has been paid to the regularities that exist among the sub-structures. There are many features of the visible sample of the universe that suggest that the regularities in sub-structures which range over 40 orders of magnitude in size and 80 orders of magnitude in mass, are of central significance to the order and operation of the universe. The fact that these regularities may not be readily explainable in terms of existing physical theories, should not deter their examination. The object of this paper is to present an overview of the known structural regularities that link the properties of physical bodies across a hierarchy of levels from the atomic to the cosmic.

MODULAR HIERARCHIES

Because of the confusion created by the many uses of the term "hierarchy" some amplification concerning the sense in which hierarchy is used in astronomy and cosmology is needed. Astronomical usage, in general, employs "hierarchy" to mean a *set of related levels* where the levels may be distinguished by a size or mass parameter. Examples from the past include the hierarchy of spheres associated in ancient cosmographies with the various heavenly bodies beginning with the moon and continuing to the sphere of fixed stars, and the hierarchy of epicycles used by Ptolemy to account for observed planetary motions. Modern concepts of hierarchy in the cosmos began with the speculations of Lambert (1761) who extrapolated to higher order systems the analogy between a satellite system such as that of Jupiter and its moons and the solar system of the sun and its planets. Lambert speculated on a hierarchy consisting of a distant center about which the sun orbited as a satellite and an even more distant center about which the first center orbited, and on to more and more distant centers comprising larger and larger systems. To explain Olbers' and Seeliger's Paradox, Charlier (1908, 1922) posited a universe built up of a hierarchy of "galaxies." The first order galaxies were the familiar ones composed of stars, second order galaxies were aggregates of first order galaxies, third order of second order, and so on. Shapley (1930) pointed to the set of levels into which all matter appears to be organized extending from the sub-atomic particles to the "metagalaxies." Shapley's organization, like Charlier's, constructed the material bodies on any level from the bodies on the level next below. A hierarchy of this type which is of fundamental importance in astronomy we designate a *modular hierarchy*.

The central idea in a modular hierarchy is the *module* which is a structure or a system that may be regarded both as a *whole*, decomposible into sub-modules identified with a lower level, and as a *part* combinable into super-modules identified with a higher level. In astronomy, even though the modules on any level are not identical, the levels may be readily distinguished on the basis of the nature of the principal sub-modules out of which entities are directly composed. Thus, for organization in a modular hierarchy, open and globular star clusters and galaxies would be assigned the same level, all being aggregates of stars. Stars, planets, and moons, all built from atoms, would share the next lower level, while clusters of galaxies would be assigned the next level above. There are several other ways than that of a modular hierarchy for organizing cosmic bodies into levels. Some of these will be discussed later.

The term "module" being used here in this general sense need not be precisely defined, however, we may ascribe two fundamental properties to modules. First, a module possesses some sort of closure or partial closure (Wilson 1969). This closure may be topological, temporal, or defined by some operational rule as in group theory. Second, modules possess a degree of semi-autonomy with respect to other modules and to their context. These two properties appear to be common in all modular hierarchies.

In considering the origin of a modular hierarchy we may inquire at any level as to whether the size, the complexity, and the limits to the module are determined (1) totally by the properties of its sub-structures, (2) by its environment, or (3) by a combination of both module contents and context. And to these logical possibilities we must add a fourth: that the levels and modules in a hierarchical structure are determined by some principle or process that operates independently of all levels of the hierarchy. In this fourth case the *levels* of the modular hierarchy themselves become the *modules* on a single level of a meta-hierarchy. The various levels in the meta-hierarchy are an observable level, an energy or force level and a meta-relational level. As an example, we may think of the lines in the spectrum of an atom as an ordinary hierarchy (but not a modular hierarchy). The levels of the meta-hierarchy would be the spectral lines, the energy levels, and the mathematical law – such as the Balmer formula – that defines the sequence. It may be objected that this is but a representational hierarchy. But the essential point is that the levels are neither determined by the sub-levels nor the super levels, but by a set of eigen values that act as a causal meta-relation.

COSMIC-ATOMIC NUMERICAL RELATIONS

Let us now return to our specific example of a modular hierarchy: the levels of cosmic structure. Instead of assuming a two level model of the cosmos – the level of a homogeneous perfect fluid and the level of the universe as a whole – we shall attempt a multi-level view retaining the atomic, stellar, galactic, galaxy cluster and universe levels. Further, in view of the lacunae in our knowledge of physical processes governing "vertical" relations between levels, it is appropriate to work from observation toward theory. In doing this the steps we must take are somewhat analogous to those taken by Kepler and his successors in the investigation of planetary orbits. From the arithmetic ratios of various powers of the sizes and periods of planetary orbits, Kepler discovered his kinematical relations and from these later came Newton's formulation of the physical laws governing planetary motions. Thus while our ultimate goal is the formulation of the physical laws and processes governing the relations between the levels in the cosmic hierarchy, our immediate goal is much more modest. It is simply to display whatever quantitative regularities may exist between the fundamental measurements made on bodies at each cosmic level.

The properties of the arithmetic relations between fundamental atomic and cosmic constants is not new ground. It has received the attention of many leading physicists and astronomers. Eddington (1923, 1931a,b); Haas (1930a,b, 1932, 1938a,b,c); Stewart (1931); Dirac (1937, 1938); Chandrasekhar (1937); Jordan (1937, 1947); Schrödinger (1938); Kothari (1938); Bondi (1952); Pegg (1968); Gamow (1968); and Alpher (1968) all have developed the subject.

The central theme in the numerical approach to atomic-cosmic relations has been to identify quantitative equivalences between various dimensionless combinations of fundamental constants and whenever possible give them physical interpretations. The epistemological weakness in this approach is the shadow of chance coincidence that cannot be removed by any of the common tests of statistical significance. Confidence in the validity of the numerically indicated relations can only follow from successful predictions or the development of a consistent theoretical construct linked to well established physics.

The basic ingredients in the relational approach are the micro-constants, e, m_e, m_p, and h (the charge and mass of the electron, the mass of the proton, and Planck's constant) the meso-constants, c and G (the velocity of light and the gravitational coupling constant), and the macroparameters H and ρ_u (the Hubble parameter and the mean density of the universe). Recently determined valves of these constants are given in Table I. From these fundamental quantities several important dimensionless ratios may be formed. The values of the dimensionless quantities $\bar{\mu} = m_p/m_e$ (= 1836.12); $\alpha = 2\pi e^2/hc$ (= 1/137.0378); and $S = e^2/Gm_p m_e$ (= $10^{39.356}$) may

Table I.

Values of Fundamental Physical and Cosmic Constants

Constant	Value (c.g.s.)	$\log_{10}$ (value)	Reference
e	4.80298×10^{-10}	–9.318489	1
m_e	9.10908×10^{-28}	–27.040526	1
m_p	1.67252×10^{-24}	–23.776629	1
h	6.62559×10^{-27}	–26.178776	1
c	2.997925×10^{10}	10.476821	1
G	6.670×10^{-8}	–7.176	1
H^{-1}	13×10^{9} years	17.613 seconds	2
ρ_u	10^{-28}	–28	3
a_o	5.29167×10^{-9}	–8.276407	1
r_e	2.81777×10^{-13}	–12.550095	1
α^{-1}	137.0388	2.136844	1
S	2.265×10^{38}	39:356	
μ	1836.12	3.263901	

From top: charge on electron, mass of electron, mass of proton, Planck's constant, velocity of light, Newton's gravitational constant, inverse Hubble parameter, mean density of visible matter in universe, Bohr radius, radius of electron, inverse fine structure constant, ratio of Coulomb to gravitational forces, ratio of proton to electron mass.

1. Cohen and DuMond (1965), 2. Sandage (1938), and 3. Allen (1963) p. 261.

be established in the laboratory. These are respectively, the ratio of proton to electron mass, the Sommerfeld fine structure constant, and the ratio of Coulomb to gravitational forces.[1]

When the two macro-parameters H and ρ_u are introduced, three additional dimensionless quantities may be formed. The first of these is the "scale parameter" of the universe (the product of the velocity of light, c, and the Hubble time H^{-1}), divided by the electron radius, c/Hr_e. The second is the "mass of the universe" expressed in units of baryon mass (where the scale parameter is taken as the radius of the universe), $\rho_u c^3/H^3 m_p$. The third is the dimensionless gravitational potential of the universe $GM_u/c^2R_u = G\rho_u/H^2$. Using 75 km/sec/mpc as the present value of the Hubble parameter (Sandage 1968), and $10^{-28} g/cm^3$ for the mean density of matter in the universe (Allen 1963), we obtain:

$$c/Hr_e = 10^{40.64} \doteq 2\pi^2 S$$

$$G\rho_u/H^2 = 10^{0.05} \doteq 1.$$

$$\rho_u c^3/H^3 m_p = 10^{79} \doteq 2S^2$$

It is thus seen that to within small factors (whose exact value cannot be determined with the present precisions of ρ_u and H), the dimensionless cosmic quantities representing the potentital, size, and mass of the universe are closely equal to S^ν, where $\nu =$ 0, 1, and 2 respectively. The significant matter here is not the fact that the values differ from integral powers of S by factors

1 It has been recognized that S and α appear to be logarithmically related. As an example of an arithmetic equivalence presently lacking theoretical confirmation, we have $8\pi^2 S = 2^{1/\alpha}$ to within experimental uncertainties. If this equivalence is not a coincidence, it has several important implications. Bahcall and Schmidt (1967) have shown on the basis of 0 III emission pairs in the spectra of several radio galaxies with redshifts up to $\delta\lambda/\lambda = 0.2$ that α appears to have been constant for at least 2×10^9 years. The above equivalence, if non-coincidental, would imply that S has also been constant over this period. Hence if G has been changing with time, e^2 and/or m_p and m_e have also been changing, and if e^2 has been changing, so also has h and/or c. The gravitational constant may, indeed, be expressed in terms of other basic constants by the relation, $G = 8\pi^2 e^2/m_p m_e 2^{1/\alpha}$ (Wilson 1966).

as large as 2 or $2\pi^2$, but the fact that laboratory and observatory measurements of quite diverse phenomena when expressed in dimensionless form appear to approximate so closely some small power of the ratio of electric to gravitational forces. It is also interesting to note that the gravitational potential of the universe is near the Schwarzschild Limit, the theoretical maximum value for potential. These *quantitative* equivalences indicate that there probably exist basic causal *qualitative* relations between the structure of the universe and the properties of the atom and its nucleus (the question of the direction of causality being open).

So far the two levels represented by the atom and the universe as a whole have been shown to be derivable from integral powers of the basic dimensionless ratio S. Numerical relations of a similar type involving fractional powers of S were pointed out by Chandrasekhar (1937) to be related to other cosmic levels. Chandrasekhar formed the dimensional combination

$$M_\nu = \left(\frac{hc}{G}\right)^\nu m_p^{\,1-2\nu} \qquad (1)$$

having the dimensions of mass. He pointed out the case $\nu = 3/2$ occurring in the theory of stellar interiors, leads to $M_{3/2} = 5.76 \times 10^{34}$ grams, the observed order of stellar masses. This is also the upper limit to the mass of completely degenerate configurations.

But the Chandrasekhar relation (1) also gives the observed order of mass for other cosmic levels in addition to the stellar level although this is not justifiable theoretically. If values of ν of the form $(2 - 1/n)$ where n is an even integer 2, 4, 6, 8, . . . are selected, then the Chandrasekhar relation predicts a sequence of masses given in Table II that corresponds to those observed for the stellar, galactic, cluster, second order cluster, ...levels of cosmic bodies.[2]

Table II. Masses for Levels of Cosmic Bodies from the Chandrasekhar Relation

Level	n	ν	$\log_{10} M_\nu$ (grams)	$\log_{10} M_\nu$ (dimensionless)
stellar	2	3/2	34.766	58.543
galactic	4	7/4	44.523	68.299
cluster	6	11/6	47.775	71.552
2° cluster	8	15/8	49.401	73.178
3° cluster	10	19/10	50.377	74.153
.....	...	.....	.....	.....
Universe	∞	2	54.280	78.056

Using well known relations between fundamental constants, equation (1) may be rewritten in the form:

$$M_\nu = \left(\frac{2\pi m_e}{\alpha m_p} S\right)^\nu m_p = A^\nu S^\nu m_p \tag{2}$$

where $A = 0.4689$. Hence the masses of the bodies on various cosmic levels defined by $\nu = 1\frac{1}{2}, 1\frac{3}{4}, 1\frac{5}{6}, 1\frac{7}{8}, \ldots, 2$, are seen to be nearly equal to these respective powers of S times the proton mass.

2. If equation (1) is valid for all ν of this sequence, then clusters of higher orders could exist until the ratio of consecutive cluster masses becomes less than two. The first pair for which this happens is $\nu = 31/16$ and $\nu = 35/18$, i.e., 6° and 7° clusters. Observationally, although 3° order clustering has been suspected (Wilson 1967), not even the existence of 2° order clustering has been satisfactorily established. While *even* values of n give masses in good agreement with cosmic levels, the *odd* values do not appear to correspond to any long lived objects. Nonetheless, if there exist two species of body, with masses $10^8 \odot$ and $10^{13} \odot$, such bodies would correspond to $n = 3$ and 5 respectively.

There are additional relations between the measurements of cosmic physics and microphysics. The largest gravitational potentials that have been observed for each of four species of cosmic bodies (stars, galaxies, clusters and 2° order clusters) are given in Table III. The potentials for each species are derived in physically distinct ways. For stars, from eclipsing binary observations; for galaxies, from rotational dynamics; for clusters, from the virial theorem; and for second order clusters, from angular diameters, distances and galaxy counts. It is interesting and somewhat surprising that the maximum in each case is nearly the same, a quantity of the order of 10^{23} grams/cm. If, instead of c.g.s. units, masses are expressed in baryon mass units and radii in Bohr radius units, the dimensionless ratio, $M/R \div m_p/a_o$, is in each case closely equal to 10^{39}. Thus, the upper bound for the gravitational potential of these species of cosmic bodies seems to be σS where σ is a factor of the order of unity not determinable from the present precision of the observational data.

Table III. Maximum Values of Potentials

System	$\log_{10} [M/R]$ (c.g.s.)	$\log_{10} [M/R]$ (dimensionless)
Stars	23.27	38.8
Galaxies	23.6	39.1
Clusters	23.5	39.0
Second-Order Clusters	23.2	38.7

From $M/R \leqslant \sigma S m_p/a_o$, substituting $e^2/Gm_p m_e$ for S and $e^2/m_e \alpha^2 c^2$ for a_o, we obtain

$$\frac{GM}{c^2 R} \leqslant \sigma \alpha^2 .$$

In other words, the dimensionless gravitational potential for these four species of cosmic bodies is bounded, not by the Schwarzschild limit, but by a bound α^2 times smaller. We thus see that not only the dimensionless microphysical quantity, S, but also the fine structure constant, α, emerges from cosmic measurement. (Another occurrence of α^2 in cosmic measurements derives from cluster redshifts (Wilson 1964).)

These results may be displayed graphically. Figure 1 is a small scale representation showing quantitative mass and size relations between atomic and cosmic bodies. The axes are logarithmic.

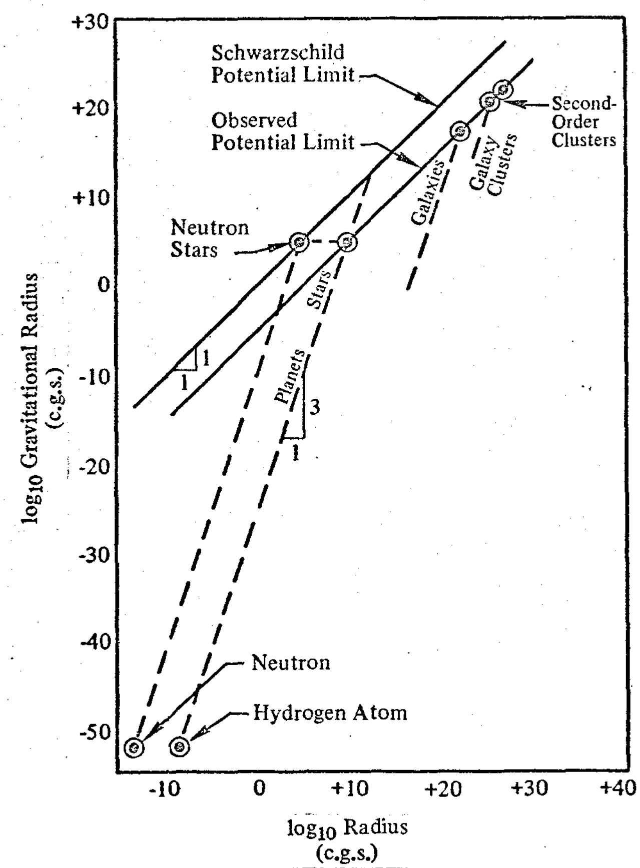

Figure 1. Mass and Size Relations Between Atomic and Cosmic Bodies

The abscissa represents the physical radius; the ordinate, the gravitational radius (GM/c^2). The upper 45 degree line is the Schwarzchild potential limit,

$$\frac{GM}{c^2 R} = \frac{1}{2},$$

the theoretical boundary separating the excluded region (upper left) from the allowable region for self-gravitating bodies. Such bodies as neutron stars, and presumably the universe itself lie on this limit. The lower 45 degree line is the observed or modular potential limit,

$$\frac{GM}{c^2 R} = \alpha^2,$$

marking the locations of the various cosmic bodies having the maximum observed potentials. All other stars, galaxies, clusters, etc., lie below this limit. The relation of the nucleus of the atom and the atom to the degenerate neutron star and the normal star is shown by the dotted lines of constant density (slope 3). Thus a neutron star has the largest mass with nuclear density allowed by the Schwarzschild limit. A normal main sequence star is seen to be limited to the same mass but is non-degenerate, lying on the line representing "atomic density." Thus, given the properties of the atom and the Schwarzschild limit, it is possible to derive the observed maximum mass for a star, but as with the Chandrasekhar relation, it is difficult to account for the locations on the diagram of the bodies of lower density (clusters, galaxies, etc.) and the fact that they are also bounded by the α^2 potential limit.

The parallel lines of equal density (slope 3) through the atom, planets and normal stars, the star clusters and galaxies, the clusters, etc., represent the levels of a modular hierarchy as previously described. These levels are thus definable by a discrete density parameter. Further, in consequence of the universal relation for gravitating systems, $\tau \propto \rho^{-1/2}$, relating a characteristic time to the density, the levels in the cosmic modular hierarchy are also definable in terms of a discrete *time* or *frequency* parameter. We shall return to this concept later.

MASS BOUNDS

In order to display the cosmic or upper portion of Figure 1 with more detail and to make comparisons with observations, the logarithms of observed masses (M) and potentials (M/R) of planets, stars, globular star clusters, galaxies, and clusters of galaxies have been plotted in Figure 2. The masses and potentials (Allen 1963) include maximum and minimum observed values and other representative values selected to show the domains occupied by the respective cosmic species.

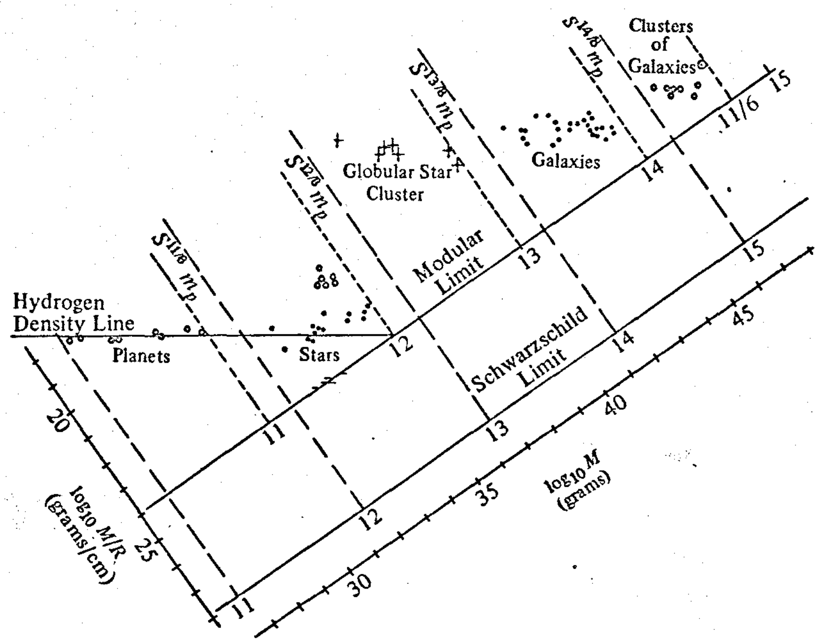

Figure 2. Mass Bounds of Cosmic Bodies

However, because of observational bias toward brightest and largest objects, the minimum observed values are not as representative of actual minimum values as the maximum observed values are of actual maximum values. Figure 2 is related to Figure 1 by an affine transformation (Figure 1 has not only been dialated, but has also been subjected to shear, reflection and rotation transformations). In Figure 2, the lines of constant density are shown horizontally so as to display the levels into which cosmic bodies fall when viewed as a modular hierarchy.

The supergiant stars lying above the mean stellar density level are shown as open circles, while the white dwarfs lying below the level near the modular potential limit are shown as dashes. The Schwarzschild Limit, $M/R = c^2/2G$ and the modular (or observed) limit, $M/R = Sm_p/a_o$ have a slope of 2/3 with respect to the horizontal equi-density lines. The short-dashed and long-dashed lines perpendicular to the Schwarzschild and modular limits are lines of constant mass. The set of short-dashed lines, extending only to the modular limit represent the sequence of masses $M_\nu = S^\nu m_p$, showing values of ν = 11/8, 12/8, 13/8, 14/8, and 11/6. The set of long-dashed mass lines, extending to the Schwarzschild Limit are located so as to pass through a sequence of points on the Schwarzschild Limit that have the same gravitational energy as the intersections of the $S^\nu m_p$ mass lines with modular limit. The pairs of intersections marked 14, 13, 12, . . .lie on lines of constant gravitational energy, $GM^2/R = S^\nu m_p(\alpha c)^2$. For identification, corresponding upper and lower bound intersections with the modular and the Schwarzschild Limits are marked with the *numerators* of the exponent ν. That is, 14 on the Schwarzschild Limit marks the lower bound of galaxies and corresponds to the upper bound $S^{14/8}m_p$ intersection with the modular limit.

The values of mass given by the Chandrasekhar relation (1) in Table II are the correct order of magnitude for the masses of stars, galaxies, and clusters. In Figure 2 it can be seen from the set of short-dashed lines of constant mass that the sequence of masses $S^{\nu}m_p$ are close in value to least upper bounds of the masses of planets, stars, globular star clusters, galaxies, and clusters of galaxies. Numerical comparisons of maxima are given in Table IV. In addition, the set of long-dashed lines are seen to be lower bounds, while probably not greatest lower bounds nonetheless close to the actual observed minimum values of the masses of the respective species of cosmic bodies. Numerical comparisons of minima are also given in Table IV where the lower bounds are the upper bounds diminished by $10^{3.9}m_p$. It can be shown that this value of maximum-minimum mass differential may be derived from "ν sequences" of maximum

Table IV. Observed and Calculated Mass Limits

Mass Limit	Planets	Stars	Globular Clusters	Galaxies	Galaxy Clusters
MAXIMUM					
	Jupiter	VV Cephei A	M22	M31	Local Super Cluster
Observed	30.279	35.225	40.14	44.8	48.3
Model	30.338	35.258	40.176	45.096	48.376
$S^{\nu}m_p$	$\nu = 11/8$	$\nu = 12/8$	$\nu = 13/8$	$\nu = 14/8$	$\nu = 11/6$
MINIMUM					
	Mercury	R CMa B	M5	NGC6822	
Observed	26.509	32.340	37.3	41.9	
Model	26.4	31.4	36.3	41.2	

All masses are given in $\log_{10}$ (grams). Upper bounds are given by $S^{\nu}m_p$, lower bounds by $S^{\nu}10^{-3.9}m_p$.

masses and gravitational energies, with the minimum mass being the least allowed by the Schwarzschild Limit for a given gravitational energy.

THE COSMIC DIAGRAM

The good agreement between the observed values for the masses and sizes of various species of cosmic bodies and the values given by sequences involving simple expressions containing fundamental physical constants indicates the probable validity of the gross features of the sequences. However, systematic errors and incompleteness in the observational data and the uncertainties intrinsic in establishing observationally least upper bounds and greatest lower bounds render it impossible, in the absence of a rigorous physical theory, to predict the exact form of the expressions and the values of the small factors (such as the 2π's, etc.) that should be included. We might, as an analogy, think of our discerning Kepler's Third Law in the form: periods squared are proportional to orbital diameters cubed without knowing the important constant of proportionality, $G(M_1 + M_2)$.

In the spirit of focusing on the major patterns that emerge from the present body of observations that are not likely to be seriously altered by refinements in observation, or even by discovery of new bodies, we represent the gross features of the structure in the universe in Figure 3. In this stylized representation, the cosmos is mapped on a rectangle whose length is the logarithm of the mass, $S^{\nu} m_p$, and whose hieght is the logarithm of the extension, $S^{\eta} a_o$. The masses and radii of various sub-components are related to values of ν and η. The hydrogen atom, mass m_p, and radius a_o, is located at the origin at H with $\nu = 0$, $\eta = 0$. The mass and radius of the universe are represented by the values $\nu = 2$, $\eta = 1$ at U. The modular and Schwarzschild potential limits are the upper and lower 45° lines respectively. The remaining observed bodies in the universe lie roughly within the three hatched bands, whose slope is that of constant density terminating at the modular limit. The bodies on the lowest and longest band have density of the order of one g/cm^3 and include asteroids, satellites, planets, and stars. This band terminates on the modular limit at $\nu = 3/2, \eta = 1/2$. With little mass overlap of the first sequence, the next sequence of bodies (star clusters and galaxies) begins near $\nu = 3/2$ and falls along an equi-density band reaching the modular limit at $\nu = 7/4$, $\eta = 3/4$. Above this point the observational uncertainties do not permit a definitive picture. It is not clear whether there exist two (or more) sequences of clusters of galaxies or only one.

A cluster sequence terminating at $\nu = 11/6, \eta = 5/6$ together with a second sequence of higher order clusters terminating at $\nu = 15/8$, $\eta = 7/8$ (as shown in Figure 1 and Figure 2) may fit observations better than the single sequence extending to $\nu = 15/8$, $\eta = 7/8$ shown in Figure 3. The resolution of this structure as well as whether still higher levels of clustering exist must be decided on the basis of future observations.

From the point of view of hierarchies, the levels occupied by cosmic bodies may be described either as *modular levels* (in the sense defined earlier), or as levels defined by a density

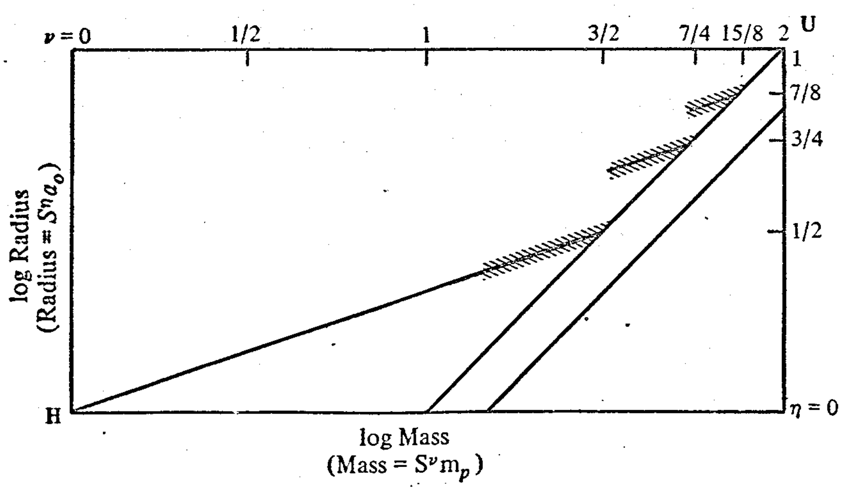

Figure 3. Cosmic Diagram

parameter, or its equivalent frequency parameter. In addition the structure may be "sliced" differently and the cosmic bodies may be allotted to distinct levels defined by a mass parameter. These levels are broad but on the scale of Figure 2 appear to be distinct.

INTERPRETATIONS

An intrinsic difficulty in relating empirical results (such as those displayed in Figures 2 and 3) to current physical theories is that numbers of the magnitude of S are not contained in any classical equations of physics. This difficulty has been expounded by Dirac (1938), Jordan (1947) and others. Eddington (1931) made attempts to derive the fundamental dimensionless constants from first principles, not, however, with complete success in reproducing the observed values. A theoretical understanding of the various observed relations between the different levels of cosmic structure – atoms, stars, galaxies, . . .the universe – is thus likely to come only after new theories of such concepts as time, degeneracy, and informational content of structure are available. At the present stage only some *speculative* suggestions can be made.

For example, the existence of *two* potential limits, the Schwarzschild and the modular, implying that the same extension ratio (the α^2 ratio of atomic to nuclear dimensions) holds between non-degenerate and collapsed configurations at stellar, galactic and cluster levels, suggests that through a generalization of the concept of degeneracy, the theorectical validity of equation (1) for all levels might be established. One might speculate that configurations at every level possess a collapsed or close packed state, and an extended state α^{-2} times larger. An alternate approach may be that the reflection of the α^2 ratio into higher levels of cosmic-structure is a cosmogonic vestige from a universe in a highly collapsed state. But whatever the cause of the modular limit, it must be regarded as an important observational feature to be accounted for by cosmological theories.

A second speculative suggestion is that in the sequence of powers of S that map observed mass configurations, we are encountering a resonance phenomenon. However, the fundamental and the overtones are exponentially related instead of being related in the manner of Pythagorean harmonics. This suggests kinship to the logarithmic time derived by Milne (1935) in his kinematic relativity. If we take as the basic gravitational frequency, the inverse Schuster period, $f_o = (Gm_p)^{1/2}/2\pi a_o^{3/2}$, then the overtones are given by

$$f_\nu = \frac{(GS^\nu m_p)^{1/2}}{2\pi(S^{\nu-1} a_o)^{3/2}} = f_o S^{3/2-\nu} \qquad (3)$$

where $\nu = 3/2, 7/4, 15/8, \ldots$.

Numerically, $f_{3/2} = f_o$, the frequency associated with the hydrogen-stellar line of Figure 3, corresponds to a period of about two hours; $f_{7/4}$, the galactic line corresponds to 10^6 years; $f_{15/8}$, the cluster line corresponds to 85 x 10^9 years; and f_2 corresponds to 10^{15} years. The cluster value is close to the period derived by Sandage for an oscillating universe. Viewed as a Hubble time, it corresponds to a value of H = 74.13 km/sec/mpc, in close agreement with the observed value of H = 75.3 km/sec/mpc derived from cluster distances (Sandage 1968).

If we take this equivalence between the $\nu = 15/8$ cluster gravitational time and the observed cluster Hubble time, as additional corroboration of the valid representation of the cosmic diagram, then we infer that the visible sample of the universe, the "realm of the galaxies and clusters" is not the $\nu = 2$ universe. The observations at the limits of our telescopes are describing the $\nu = 15/8$ sub-structure and not the universe. Characteristic times of the order of 10^{10} years are those associated with the cluster level sub-structure. The characteristic gravitational time of the $\nu = 2$ universe, on the other hand, is of the order of 10^{15} years. The appearance of a time of this magnitude brings to mind the controversy that waged in cosmology following the publication of James Jeans (1929) estimate of the dynamic age of the galaxy at 10^{13} years. The adherents of the "short time-scale," held the age of the universe to be but a few eons while those who subscribed to the "long time-scale," required an age of the order of 10^{13} years or greater. Since the galaxy could not be older than the universe, the issue was settled against Jeans. But if the few eons refers not to the universe but to the cluster level sub-structure, there is no *a priori* reason why the galaxy cannot be older than the cluster level sub-structure.

If the cosmic diagram suggests some form of resonance as the process of morphogenesis, then as sand collects at the nodes on a vibrating drum head, matter concentrates at nodes corresponding to the set of frequencies $S^{3/2-\nu} f_o$. This raises many physical questions. Most importantly, what is it that is pulsating or vibrating at these frequencies – some substratum, matter itself, or what? Analogies to familiar equations suggest that from the cosmic diagram, we have a set of eigen values representing mass levels, energy levels, or frequencies that are solutions to some "cosmic wave equation." Perhaps the first step toward a physical theory would be to derive such an equation.

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Hierarchical Structure in the Cosmos

Albert Wilson*

The primary focus of cosmological thought in the present century has been on interpreting the observations of the sample of the universe available to our telescopes in terms of a set of models based on various theories of gravitation; especially the General Theory of Relativity. The problem of the structure *of* the universe is customarily divorced from the problem of the structure *in* the universe. Theoretical cosmologists usually choose to explain the structure and behavior – past and future – of the universe with models that smooth out the distribution of matter in the universe, replacing the observed structured distribution of matter with a uniform homogeneous perfect fluid whose density varies in time, but not in space. However, the structure contained *in* the universe becomes difficult to relate to models constructed around smoothing postulates. This has resulted in separate theoretical approaches to the origin of the various structures in the universe. While most of these approaches have met with some success, they are inadequately related to one another and to cosmological theories.

The arbitrary separation of the structure and behavior of the universe from the structure and behavior of its contents may be expedient from the point of view of mathematical simplification, but it cannot be accepted as more than an exploratory strategy. The observational tests for discriminating between various cosmological models are difficult and marginal. Since several smoothed models are candidates for best fit to the observations, it is unfortunate that the large amount of information contained in the sub-structures of the universe cannot be used in testing these models. But until models that relate the properties of the sub-structures to the properties of the whole are employed, much information of potential cosmological value in sub-structure astronomical observations is not cosmologically useful.

*Douglas Advanced Research Laboratories, Huntington Beach, California, 92647.

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HIERARCHAL STRUCTURE IN THE COSMOS

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The primary focus of cosmological thought in the present century has been on interpreting the observations of the sample of the universe available to our telescopes in terms of a set of models based on various theories of gravitation, especially the General Theory of Relativity. The problem of the structure *of* the universe is customarily divorced from the problem of the structure *in* the universe. Theoretical cosmologists usually choose to explain the structure and behavior -- past and future -- of the universe with models that smooth out the distribution of matter in the universe, replacing the observed structured distribution of matter with a uniform homogeneous perfect fluid whose density varies in time, but not in space. ~~But~~ However, the structure contained *in* the universe becomes difficult to relate to models constructed around smoothing postulates. This has resulted in separate theoretical approaches to the origin of the various structures in the universe. While most of these approaches have met with some success, they are inadequately related to one another and to cosmological theories.

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The observational tests for discriminating between various cosmological models are difficult and marginal. Since several smoothed models are still candidates for best fit to the observations, it is unfortunate that the large amount of information contained in the sub-structures of the universe cannot be used in testing these models. But until models are used that relate the properties of the sub-structures to the properties of the whole, much information of potential cosmological value in sub-structure astronomical observations is not cosmologically useful.

So long as the cosmological problem has been approached through smoothing out the sub-structures, it is not surprising that little attention has been paid to the structural regularities that exist among the sub-structures. There are many features of the visible sample of the universe that suggest that the regularities in sub-structures which range over 40 orders of magnitude in size and 80 orders of magnitude in mass, are of central significance to the order and operation of the universe. The fact that these regularities may not be readily explainable in terms of existing physical theories, should not deter their examination. The object of this paper will be to present an overview of the known structural regularities that link the properties of physical bodies across a hierarchy of levels from the atomic to the cosmic.

Modular Hierarchies

Because of the confusion created by the many uses of the term "hierarchy" some amplification concerning the senses in which hierarchy is used in astronomy and cosmology is needed. Astronomical usage, in general, employs "hierarchy" to mean a set of related levels where the levels may be distinguished by a size or a mass parameter. Examples from the past include the hierarchy of spheres associated in ancient cosmographies with the various heavenly bodies beginning with the moon and continuing to the sphere of fixed stars, and the hierarchy of epicycles used by Ptolemy to account for observed planetary motions. Modern concepts of hierarchy in the cosmos began with the speculations of J. H. Lambert (1761) ~~(Ref. 1)~~ who extrapolated to higher order systems the analogy between ~~a~~ the satellite system ~~such as that~~ consisting of Jupiter and its moons and the solar system consisting of the sun and its planets. Lambert speculated on a hierarchy consisting of a distant center about which the sun itself orbited as a satellite and an even more distant center about which the first center orbited, and on to more and more distant centers comprising larger and larger systems. To explain Olbers' and Seeliger's Paradox, C. V. L. Charlier (1908, 1922) ~~(Ref. 2) early in the 20th century~~ posited a universe built up of a hierarchy of "galaxies." The first order galaxies were the familiar ones composed of stars, second order galaxies were aggregates of first order galaxies, third order of second order, etc. H. Shapley (1930) ~~in 1930 (Ref. 3)~~ pointed to the set of levels into which all matter appears to be organized extending from the sub-atomic particles to the "metagalaxies." Shapley's organization, like Charlier's, constructed the material bodies on any level from the bodies on the level next below. A hierarchy of this type which is of fundamental importance in astronomy we shall designate a modular hierarchy.

The central idea in a modular hierarchy is the module which is a structure or a system that may be regarded both as a whole, decomposible into sub-modules identified with a lower level, and as a part combinable into super-modules identified with a higher level. In astronomy, even though the modules on any level are not identical, the levels may be readily distinguished on the basis of the nature of the ~~smallest~~ principal sub-modules out of which entities are directly composed. Thus, for organization in a modular hierarchy open and globular star clusters and galaxies would be assigned the same level, all being aggregates of stars. Stars, planets, and moons, all built from atoms, would share the next lower level, while clusters of galaxies would be assigned the next level above. There are several other ways besides ~~the~~ that of a modular hierarchy ~~of~~ for organizing cosmic bodies into levels. Some of these will be discussed later.

The term "module" being used here in a general sense ~~as here~~ ~~is~~ need not be precisely defined. However, we may ascribe two fundamental properties to modules. First, a module possesses some sort of closure or partial closure (~~Ref. A~~) (see page __). This closure may be topological, temporal, or defined by some operational rule as in group theory. Second, modules possess a degree of ~~relational~~ semi-autonomy with respect to other modules and to their context. These two properties appear to be common in all modular hierarchies.

In considering the origin of a modular hierarchy we may inquire at any level as to whether the size, the complexity, and the limits to the module ~~at any level~~ are determined (1) totally by the properties of its sub-structures, (2) by its environment, or (3) by a combination of both module contents and context. And to these logical possibilities we must add a fourth: that the levels and modules in a hierarchical structure are determined by a meta-relational or transcendental principle that defines the ontological possibilities. In this fourth case the <u>levels</u> of the modular hierarchy themselves become the <u>modules</u> on a single level of a meta-hierarchy. The various levels in the meta-hierarchy are an observable level, ~~are~~ an energy or a force level, and a meta-relational level. As an example, we may think of the lines in the spectrum of an atom as an ordinary hierarchy (but not a modular hierarchy). The levels of the meta-hierarchy would be the spectral lines, the energy levels, and the mathematical law -- such as the Balmer formula -- that defines the sequence. It may be objected that this is but a representational hierarchy. But the essential point is that the levels are neither determined by the sub-levels, nor the super levels, but by a set of eigen values that act~~ing~~ as a causal meta-relation.

A. G. Wilson
4/10/69

Cosmic-Atomic Numerical Relations

Let us now return to our specific example of a modular hierarchy: the levels of cosmic structure. Instead of assuming a two level model of the cosmos -- the level of a homogeneous perfect fluid and the level of the universe as a whole -- we shall attempt a multi-level view retaining the atomic, stellar, galactic, galaxy cluster and universe levels. Further, in view of the lacunae in our knowledge of physical processes governing "vertical" relations between levels, it is appropriate to work from observation toward theory. In doing this the steps we must take are somewhat analogous to those taken by Kepler and his successors in the investigation of planetary orbits. From the arithmetic ratios of various powers of the sizes and periods of planetary orbits, Kepler discovered his kinematical relations and from these later came Newton's formulation of the physical laws governing planetary motions. Thus while our ultimate goal is the formulation of the physical laws and processes governing the relations between the levels in the cosmic hierarchy, our immediate goal is much more modest. It is simply to display whatever quantitative regularities may exist between the fundamental measurements made on bodies at each cosmic level.

The properties of the arithmetic relations between fundamental atomic and cosmic constants is not new ground.

It has received the attention of many leading physicists and astronomers. Eddington (1925, 1931a, b), Haas (1930a, b, 1932, 1938a, b, c) Stewart (1931), Dirac (1937, 1938), Chandrasekhar (1937), Jordan (1937, 1947), Schroedinger (1938), Kothari (1938), Bondi (1952), Pegg, (1968), Gamow (1968) and Alpher (1968) all have developed the subject.

The central theme in the numerical approach to atomic-cosmic relations has been to identify quantitative equivalences between various dimensionless combinations of fundamental constants and whenever possible give them physical interpretations. The epistemological weakness in this approach is the shadow of chance coincidence that cannot be removed by any of the common tests of statistical significance. Confidence in the validity of the numerically indicated relations can only follow from successful predictions on the development of a consistent theoretical construct linked to well established physics.

The basic ingredients in the relational approach are the micro-constants, e, m_e, m_ρ, and h (the charge and mass of the electron, the mass of the proton, and Planck's constant) the meso-constants, c and G (the velocity of light and the gravitational coupling constant), and the macro-parameters H and ρ_u (the Hubble parameter and the mean density of the universe). From these fundamental quantities several important dimensionless ratios may be formed. The values of the dimensionless quantities $\mu = m_\rho/m_e$ (= 1836.12) [illegible], $\alpha = 2\pi e^2/hc$ (= 1/137.0378), and $S = e^2/Gm_\rho m_e$ (= $10^{39.356}$) may be established in the laboratory. These are respectively, the ratio of proton to electron mass, the Sommerfeld fine structure constant, and the ratio of Coulomb to gravitational forces.*

When the two macro-parameters H and ρ_u are introduced, three additional dimensionless quantities may be formed. The first of these is the "scale parameter" of the universe (which is the product of the velocity of light, c, and the Hubble time H^{-1}), divided by the electron radius, c/Hr_e. The second is the "mass of the universe" expressed in units of baryon mass (where the scale parameter is taken as for radius of the universe), $\rho c^3/H^3 m_p$. The third is the dimensionless gravitational potential of the universe $GM_u/c^2 R_u = G\rho_u/H^2$. Using 75 km/sec/mpc, Sandage (1968), as the present value of the Hubble parameter, and 10^{-28} g/cm³, Allen (1963), for the mean density of matter in the universe, we obtain:

*It has been recognized that S and α appear to be logarithmically related. As an example of an arithmetic equivalence presently lacking theoretical confirmation, we have $8\pi^2 S = 2^{1/\alpha}$ to within experimental uncertainties (Wilson 1966). If this equivalence is not a coincidence, there are several important implications. Bahcall and Schmidt (1967) have shown on the basis of O III emission pairs in the spectra of several radio galaxies with redshifts up to $\delta\lambda/\lambda = 0.2$ that α appears to have been constant for at least 2×10^9 years. The above equivalence, if non-coincidental, would imply that S has also been constant over this period. Hence if G has been changing with time, e^2 and/or m_p and m_e have also been changing, and if e^2 has been changing, so also has h and/or c. The gravitational constant may, indeed, be expressed in terms of other basic constants by the relation, $G = 8\pi^2 e^2/m_p m_e 2^{1/\alpha}$.

delta

$c/Hr_e = 10^{40.64} \doteq 2\pi^2 S$; $\rho_u c^3/H^3 m_p = 10^{79} \doteq 2S^2$; $G\rho_u/H^2 = 10^{0.05} \doteq 1$. It is thus seen that to within small factors (whose exact value cannot be determined with the present precisions of ρ and H), the dimensionless cosmic quantities representing the potential, size, and mass of the universe are closely equal to S^{ν}, where $\nu = 0, 1,$ and 2 respectively. The significant matter here is not the fact that the values differ from integral powers of S by factors such as 2 or $2\pi^2$, but the fact that laboratory and observatory measurements of quite diverse phenomena when expressed in dimensionless form appear to approximate some power of the ratio of electric to gravitational forces. It is also interesting to note that the gravitational potential of the universe ~~is at~~ near the Schwarzschild Limit, the theoretical maximum value for potential. These quantitative equivalences indicate that there probably exist basic causal qualitative relations between the structure of the universe and the properties of the atom and its nucleus (the question of the direction of causality being open).

So far only the two levels represented by the atom and the universe as a whole have been shown to be derivable from integral powers of the basic dimensionless ratio S. Numerical relations of a similar type involving fractional powers of S were pointed out by Chandrasekhar (1937) to be related to other cosmic levels. Chandrasekhar formed the combination

$$M_{\nu} = \left(\frac{hc}{G}\right)^{\nu} m_p^{1-2\nu} \qquad (1)$$

having the dimensions of mass. He pointed out the case $\nu = 3/2$ occurring in the theory of stellar interiors, leads to $M_{3/2} = 5.76 \times 10^{34}$ grams, the observed order of stellar masses. This is also the upper limit to the mass of completely degenerate configurations.

~~The~~ But the Chandrasekhar relation (1) ~~though not justifiable theoretically,~~ also gives the observed order of mass for other cosmic levels in addition to the stellar level, though this is not justifiable theoretically. If values of ν of the form $2 - 1/n$, where n is an even integer 2, 4, 6, 8,...., are selected, then as given in Table II the Chandrasekhar relation predicts a sequence of masses, corresponding to those observed for the stellar, galactic, cluster, second order cluster,....levels of cosmic bodies.*

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*If equation (1) is valid for all ν of this sequence, then clusters of higher orders could exist until the ratio of consecutive cluster masses becomes less than two. The first pair for which this happens is $\nu = 31/16$ and $\nu = 35/18$, i.e., 6° and 7° order clusters. Observationally, though 3° order clustering has been suspected (Wilson, 1967), not even the existence of second order clustering has been satisfactorily established. While even values of n give masses in good agreement with cosmic levels, the odd values do not appear to correspond to any long lived objects. Nonetheless, if there exist two species of ~~quasar~~ object, with masses 10^8 ⊙ and 10^{13} ⊙, such bodies would correspond to n = 3 and 5 respectively.

Using well known relations between fundamental constants, equation (1) may be rewritten in the form:

$$M_\nu = \left(\frac{2\pi m_e}{\alpha m_p} S\right)^\nu m_p = A^\nu S^\nu m_p$$

where A = 0.4689. Hence the masses of the bodies on various cosmic levels defined by $\nu = 1\ 1/2,\ 1\ 3/4,\ 1\ 5/6,\ 1\ 7/8, \ldots 2$, are seen to be nearly equal to these respective powers of S times the proton mass.

There are additional relations between the measurements of cosmic physics and microphysics. In Table III are given the largest gravitational potentials that have been observed for each of four species of cosmic body, stars, galaxies, clusters and 2° order clusters. The potentials are derived in physically distinct ways for each species. For stars, from eclipsing binary observations; for galaxies, from rotational dynamics; for clusters, from the virial theorem; and for second order clusters, from angular diameters, distances and galaxy counts. It is interesting and somewhat surprising that the maximum in each case is nearly the same, a quantity of the order of 10^{23} grams/cm. If, instead of cgs units, masses are expressed in baryon mass units and radii in first Bohr radius units, the dimensionless ratio, $M/R \div m_p/a_o$, is in each case closely equal to 10^{39}. Thus the upper bound for the gravitational potential of these species of cosmic bodies seems to be σS where σ is a factor of the order of unity not determinable from the present precision of the observational data.

From $M/R \leq \sigma S m_p/a_o$, substituting e^2/Gm_pm_e for S and $e^2/m_e\alpha^2c^2$ for a_o, we obtain $GM/c^2R \leq \sigma\alpha^2$. In other words, the dimensionless gravitational potential for these four species of cosmic bodies is bounded, not by the Schwarzschild limit, but by a bound α^2 times smaller. We thus see that not only the dimensionless microphysical quantity, S, but also the fine structure constant, α, emerges from cosmic measurement.

These results may be displayed graphically. Figure 1 is a small scale representation showing quantitative mass and size relations between atomic and cosmic bodies. The axes are logarithmic. The abscissa represents the physical radius, the ordinate, the gravitational radius (GM/c^2). The upper 45° line is the Schwarzschild potential limit, $GM/c^2R \leq \frac{1}{2}$, the theoretical boundary separating the excluded (upper left) from the allowable region for gravitating bodies. Such bodies as neutron stars, and presumably the universe itself lie on this limit. The lower 45° line is the observed or modular potential limit, $GM/c^2R = \alpha^2$, marking the locations of the various cosmic bodies having the maximum observed potentials. All other stars, galaxies, clusters, etc., lie below this limit. The relation of the nucleus of the atom and the atom to the degenerate neutron star and the normal star is shown by the dotted lines of constant density (slope 3). Thus a neutron star has the largest mass with nuclear density allowed by the Schwarzschild limit. A normal main sequence star is seen to be limited to the same mass but is non-degenerate, lying on the line representing "atomic density." Thus, given the properties of the atom and the Schwarzschild limit, it is possible to derive the observed maximum mass for a star, but as with the Chandrasekhar relation, it is difficult to account for the locations of the bodies of lower density (clusters, galaxies, etc.,) and the fact that their masses are also bounded by the α^2 potential limit.

The parallel lines of equal density (slope 3) through the atom, planets and normal stars; the star clusters and galaxies; the clusters; etc., represent the levels of a modular hierarchy as previous defined. These levels are thus definable by a discrete density parameter. Further, in consequence of the universal relation for gravitating systems, $\tau \propto \rho^{-1/2}$, relating a characteristic time to the density, the levels in the cosmic modular hierarchy are also definable in terms of a discrete <u>time</u> or <u>frequency</u> parameter. We shall return to this concept later.

MASS BOUNDS

In order to display the cosmic or upper portion of Figure 1 with more detail and to make comparisons with observations, the logarithims of observed masses (M) and potentials (M/R) of planets, stars, globular star clusters, galaxies, and clusters of galaxies have been plotted in Figure 2. The masses and potentials ~~plotted~~ (Allen 1963) include maximum and minimum observed values and other representative values selected to show the domains occupied by the respective cosmic species. However, because of observational bias toward brightest and largest objects, the minimum observed values are not as representative of actual minimum values as the maximum observed values are of actual maximum values. Figure 2 is related to Figure 1 by an affine transformation, ~~Note that~~ Figure 1 ~~has~~ not only having been dialated, but also subjected to shear, reflection and rotation transformations. In Figure 2, the lines of constant density are shown as horizontal so as to display horizontally the levels into which cosmic bodies fall when viewed as a modular hierarchy.

The supergiant stars lying above the mean stellar density level are shown as open circles, while the white dwarfs lying below the level near the modular potential limit are shown as dashes. The Schwarzschild Limit, $M/R = c^2/2G$ and the modular (or observed) limit, $M/R = Sm_p/a_o$ have a slope of 2/3 with respect to the horizontal equi-density lines. The short-dashed and long-dashed lines perpendicular to the Schwarzschild and modular limits are lines of constant mass. The set of short-dashed lines, drawn only to the modular limit, represent the sequence of masses, $M_\nu = S^\nu m_p$, with values of ν = 11/8, 12/8, 13/8, 14/8, and 11/6 being shown. The set of long-dashed mass lines, extending to the Schwarzschild Limit are located so as to pass through a sequence of points on the Schwarzschild Limit that have the same gravitational energy as the intersections of the $S^\nu m_p$ mass lines with modular limit. The pairs of intersections marked 14, 13, 12, etc., lie on lines of constant gravitational energy, $GM^2/R = S^\nu m_p(\alpha c)^2$. For identification corresponding upper and lower bound intersections with the modular and the Schwarzschild Limits are marked with the numerators of the exponent ν. Thus 14 on the Schwarzschild Limit marks the lower bound of galaxies and corresponds to the upper bound or intersection $S^{14/8}m_p$ with the modular limit.

The values of mass given by the Chandrasekhar relation (1) in Table II are the correct order of magnitude for the masses of stars, galaxies, and clusters. In Figure 2 it can be seen from the set of short-dashed lines of constant mass that the sequence of masses $S^\nu m_p$ are close in value to least upper bounds to the masses of planets, stars, globular star clusters, galaxies, and clusters of galaxies. Numerical comparisons are given in Table IV. In addition, the set of long-dashed lines are seen to be lower bounds, while probably not greatest lower bounds, nonetheless close to the actual observed minimum values of the masses of respective species of cosmic bodies. Numerical comparisons are given in Table IV where the lower bounds are the upper bounds diminished by $10^{3.9} m_p$. It can be shown that this value of maximum-minimum mass differential may be derived from the sequences of maximum masses and gravitational energies, with the minimum mass being the least allowed by the Schwarzschild Limit for a given gravitational energy.

The Cosmic Diagram

The good agreement between the observed values for the masses and sizes of various species of cosmic bodies and the values given by sequences involving simple expressions containing fundamental physical constants indicates the probable validity of the gross features of the sequences. However, systematic errors and incompleteness in the observational data and the uncertainties intrinsic in establishing observationally least upper bounds and greatest lower bounds render it impossible, in the absence of a rigorous physical theory, to predict the exact form of the expressions and the values of the small factors (such as the 2π's, etc.) that should be included. (We might, as an analogy, think of our discerning Kepler's Third Law in the form $(\text{periods})^2$ are proportional to $(\text{orbital diameters})^3$, without knowledge of the form of the important constant of proportionality $G(M_1 + M_2)$.)

In the spirit of focusing on the major patterns that emerge from the present body of observations that are not likely to be seriously altered by refinements in observation, or even by discovery of new bodies, we represent the gross features of the structure in the universe in Figure 3. In this stylized representation, the cosmos is mapped on a rectangle whose length is the logarithm of the mass, $S^{\nu}m_p$, and whose height is the logarithm of the extension, $S^{\eta}a_o$. The masses and radii of various sub-components are related to values of ν and η. The hydrogen atom, mass, m_p, and radius, a_o, is located at the origin $\nu = 0$, $\eta = 0$. The mass and radius of the universe are represented by the values $\nu = 2$, $\eta = 1$ at U. The modular and Schwarzschild potential limits are the upper and lower 45° lines respectively. The remaining observed bodies in the universe lie roughly within the three hatched bands, whose slope is that of constant density terminating at the modular limit. The bodies or the lowest and longest band have density of the order of one g/cm^3 and range from asteroidal bodies, through satellites and planets to stars extending to the modular limit of $\nu = 3/2$, $\eta = 1/2$. With little mass overlap of the first sequence the next sequence of bodies, star clusters and galaxies, begins near $\nu = 3/2$ and falls along an equi-density band reaching the modular limit at $\nu = 7/4$, $\eta = 3/4$. Above this point the observational uncertainties do not permit a definitive picture. It is not clear whether there exist two (or more) sequences of clusters of galaxies or only one.

A cluster sequence terminating at $\nu = 11/6$, $\eta = 5/6$ together with a second sequence of higher order clusters terminating at $\nu = 15/8$, $\eta = 7/8$ (as shown in Figure 1 and Figure 2) may fit observations better than the single sequence extending to $\nu = 15/8$, $\eta = 7/8$ shown in Figure 3. The resolution of this structure as well as whether still higher levels of clustering exist must be decided on the basis of future observations.

From the point of view of hierarchies, the levels occupied by cosmic bodies may be described either as modular levels in the sense that the objects on the lowest or (3/2, 1/2) level are modules whose sub-components are atomic or molecular; while the occupants of the (7/4, 3/4) level are modules whose sub-components are stars, etc.; or described as levels defined by a density parameter, or its equivalent in self-gravitating systems, a frequency parameter. In addition the structure may be "sliced" differently and the cosmic bodies may be allotted to distinct levels defined by a mass parameter. These levels are broad but on a larger scale appear to be distinct as indicated in Figure 2.

An intrinsic difficulty in relating empirical results, such as those displayed in Figures 2 and 3, to present physical theories is that numbers, the magnitude of S, are not contained in any classical equations of physics. These difficulties have been expounded by Dirac (1938), Jordan (1947) and others. Eddington (1931) has made attempts to derive the fundamental dimensionless constants from first principles, not, however, with complete success in reproducing the observed values. A theoretical understanding of the various observed relations between the different levels of cosmic structure -- atoms, stars, galaxies,......the universe -- is thus likely to come only after new theories of such concepts as time, degeneracy, and informational content of structure are available. At the present stage only some <u>speculative</u> suggestions can be made.

For example, the existence of <u>two</u> potential limits, the Schwarzschild and the modular, implying that the same extension ratio, ~~as~~ (the α^2 ratio of atomic to nuclear dimensions) holds between ~~stable, extended~~ non-degenerate, and collapsed configurations at stellar, galactic and cluster levels, suggests that through a generalization of the concept of degeneracy the theoretical validity of equation (1) for all levels might be established. One might speculate that configurations at every level possess a stable collapsed, or close packed state, and a stable extended state α^{-2} times larger. An Alternate~~ly~~ approach may be, the reflection of the α^2 ratio to higher levels of cosmic-structure ~~may be~~ as a cosmogonic vestige from a universe in a highly collapsed state. But whatever the cause of the modular limit, it must be regarded as an important observational feature to be accounted for by cosmological theories.

A second speculative suggestion is that in the sequence of powers of S that map observed mass configurations we are encountering a resonance phenomenon. However, the fundamental and set of overtones instead of being related in the manner of Pythagorean harmonics are exponentially related suggesting kinship ~~analogous~~ to the logarithmic time derived by Milne (1935) in his kinematic relativity. If we take as the basic gravitational frequency, the inverse Schuster period, $f_o = (Gm_p)^{1/2}/2\pi a_o^{3/2}$, then the overtones are given by

$$f_\nu = (GS^\nu m_p)^{1/2}/2\pi(S^{\nu-1}a_o)^{3/2} = f_o S^{3/2-\nu},$$

where $\nu = 3/2, 7/4, 15/8 \ldots 2$.

Numerically, $f_{3/2} = f_o$, the frequency associated with the hydrogen-stellar line of Figure 3, corresponds to a period of about two hours; $f_{7/4}$, the galactic line corresponds to 10^6 years; $f_{15/8}$, the cluster line corresponds to 85×10^9 years; and $\nu = 2$, corresponds to 10^{15} years. The cluster value is close to the period derived by Sandage for an oscillating universe and viewed as a Hubble time corresponds to a value of $H = 74.13$ km/sec/mega parsec, cf. Sandage's (1968) observed value for H at 75.3 km/sec/mega parsec derived from cluster distances.

If we take the equivalence between the $\nu = 15/8$, cluster gravitational time and the observed cluster Hubble time, as additional corroboration of the valid representation of the cosmic diagram, then we infer that the visible sample of the universe, the "realm of the galaxies and clusters" is not the $\nu = 2$ universe. The observations at the limits of our telescopes are describing the $\nu = 15/8$ sub-structure, and not the universe. Characteristic times of the order of 10^{10} years are those associated with the cluster level sub-structure. The characteristic gravitational time of the $\nu = 2$ universe, on the other hand, is of the order of 10^{15} years. The appearance of a time of this magnitude brings to mind, the controversy that waged in cosmology following the publication of James Jeans (1929) estimate of the dynamic age of the galaxy at 10^{13} years. The adherents of the "short time-scale," held the age of the universe to be but a few eons while those who subscribed to the "long time-scale," required an age of the order of 10^{13} years or greater. Since the galaxy could not be older than the universe, the issue was settled against Jeans. But if the few eons refers not to the universe but to the cluster level sub-structure, there is no a priori reason why the galaxy cannot be older than the cluster level sub-structure.

If the cosmic diagram suggests some form of resonance as the process of morphogenesis, then as sand collects at the nodes on a vibrating drum head, matter concentrates at nodes corresponding to the set of frequencies $S^{3/2-\nu} f_o$. But This ~~view~~ raises many physical questions. Most importantly, what is it that is pulsating or vibrating at these frequencies -- some substratum, all matter itself, or what? *From the cosmic [illegible]* Analogies to familiar equations suggest themselves. We have a set of eigen values ~~and eigen functions~~ representing mass levels, energy levels, or frequencies that are solutions to some "cosmic wave equation." Perhaps the first step toward a physical theory is to derive this equation.

TABLE I

Constant	value (c.g.s.)	$\log_{10}$ (value)	Reference
e	4.80298×10^{-10}	-9.318489	1
m_e	9.10908×10^{-28}	-27.040526	1
m_p	1.67252×10^{-24}	-23.776629	1
h	6.62559×10^{-27}	-26.178776	1
c	2.997925×10^{10}	10.476821	1
G	6.670×10^{-8}	-7.176	1
H^{-1}	13×10^{9} years	17.613 seconds	2
ρ_u	10^{-28}	-28	3
a_o	5.29167×10^{-9}	-8.276407	1
r_e	2.81777×10^{-13}	-12.550095	1
α^{-1}	137.0388	2.136844	1
S	2.265×10^{38}	39.356	
μ	1836.12	3.263901	

Values of Fundamental Physical and Cosmic Constants: (from top) charge on electron, mass of electron, mass of proton, Planck's constant, velocity of light, Newton's gravitational constant, inverse Hubble parameter, mean density of visible matter in universe, Bohr radius, radius of electron, inverse fine structure constant, ratio of Coulomb to gravitational forces, ratio of proton to electron mass.

1) Cohen and DuMond (1965), 2) Sandage (1968), 3) Allen (1963)

TABLE II

Level	n	ν	$\log_{10} M_\nu$ (grams)	$\log_{10} M_\nu$ (dimensionless)
stellar	2	3/2	34.766	58.543
galactic	4	7/4	44.523	68.299
cluster	6	11/6	47.775	71.552
2° cluster	8	15/8	49.401	73.178
3° cluster	10	19/10	50.377	74.153
.....	...	.....	.....	.....
Universe	∞	2	54.280	78.056

Masses for levels of cosmic bodies from the Chandrasekhar relation $M_\nu = (hc/G)^\nu m_p^{1-2\nu}$.

TABLE III

MAXIMUM OBSERVED GRAVITATIONAL POTENTIALS

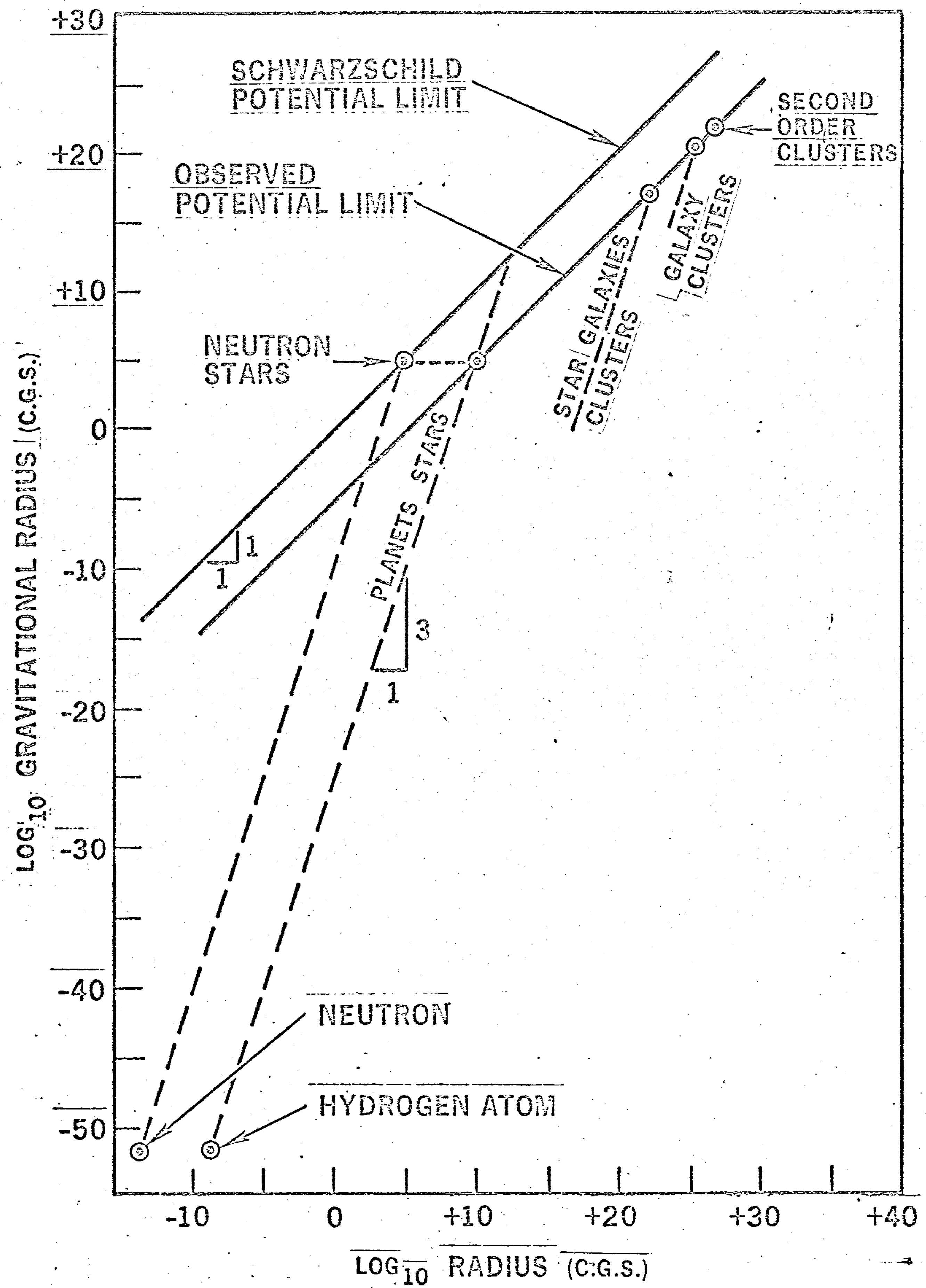
SYSTEM	$\log_{10}[M/R]$ (c.g.s.)	$\log_{10}[M/R]$ (dimensionless)
STARS	23.27	38.8
GALAXIES	23.6	39.1
CLUSTERS	23.5	39.0
SECOND-ORDER CLUSTERS	23.2	38.7

TABLE IV

	PLANETS	STARS	GLOBULAR CLUSTERS	GALAXIES	GALAXY CLUSTERS
MAXIMUM					
	JUPITER	VV CEPHEI A	M22	M31	LOCAL SUPER CLUSTER
OBSERVED	30.279	35.225	40.14	44.8	48.3
MODEL	30.338	35.258	40.176	45.096	48.376
$S^{\nu} m_p$.	$\nu = 11/8$	$\nu = 12/8$	$\nu = 13/8$	$\nu = 14/8$	$\nu = 11/6$
MINIMUM					
	MERCURY	R CMa B	M5	NGC6822	
OBSERVED	26.509	32.340	37.3	41.9	
MODEL	26.4	31.4	36.3	41.2	

OBSERVED AND CALCULATED MASS LIMITS

All masses are given in Log_{10} (grams). Upper bounds are given by $S^{\nu} m_p$, lower bounds by $S^{\nu} 10^{-3.9}\ m_p$.



SCHWARZSCHILD POTENTIAL LIMIT
OBSERVED POTENTIAL LIMIT
SECOND ORDER CLUSTERS
GALAXY CLUSTERS
STAR CLUSTERS
GALAXIES
NEUTRON STARS
PLANETS
STARS
NEUTRON
HYDROGEN ATOM
1
1
3
1
+30
+20
+10
0
-10
-20
-30
-40
-50
-10
0
+10
+20
+30
+40
LOG10 GRAVITATIONAL RADIUS (C.G.S.)
LOG10 RADIUS (C.G.S.)

MASS BOUNDS OF COSMIC BODIES

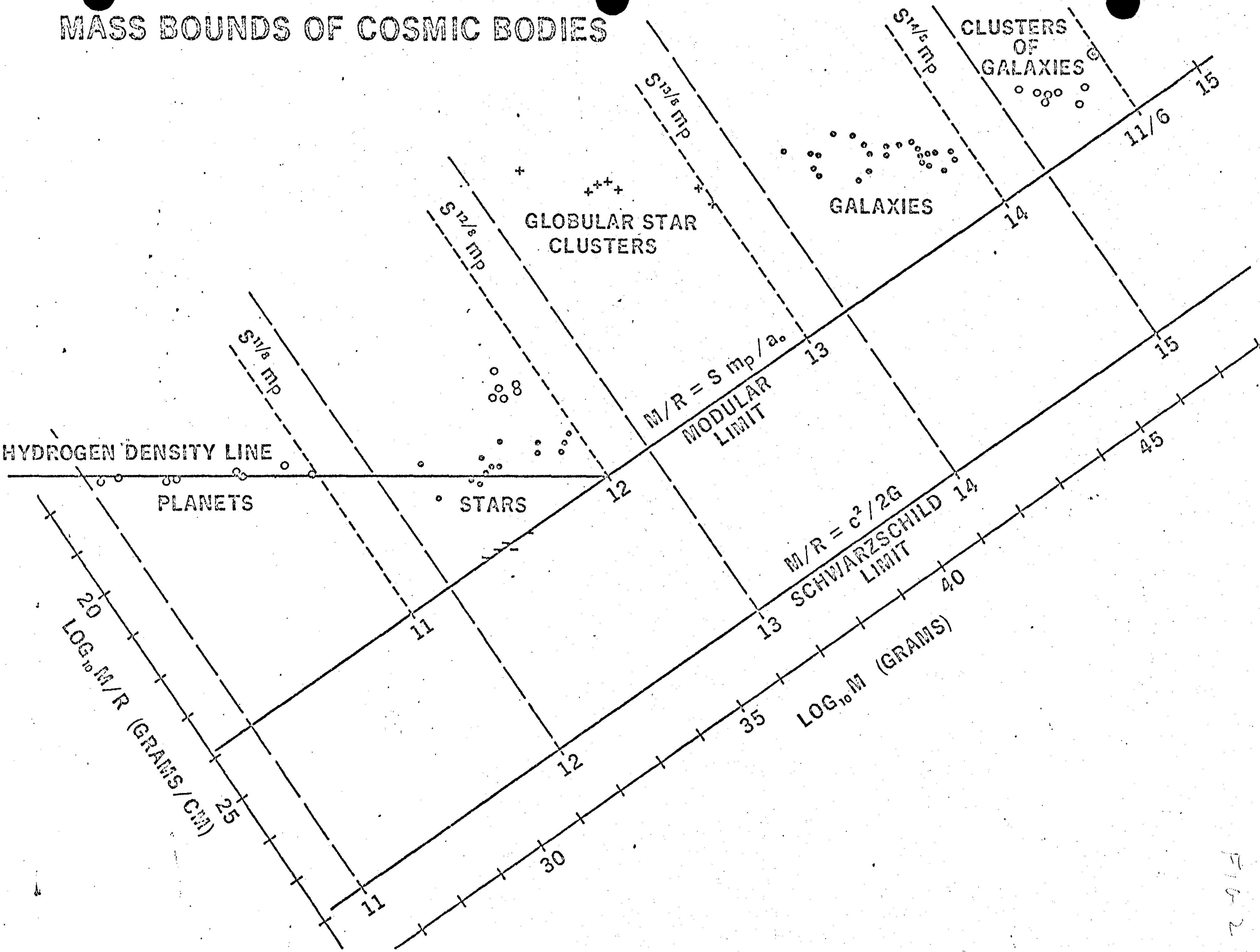
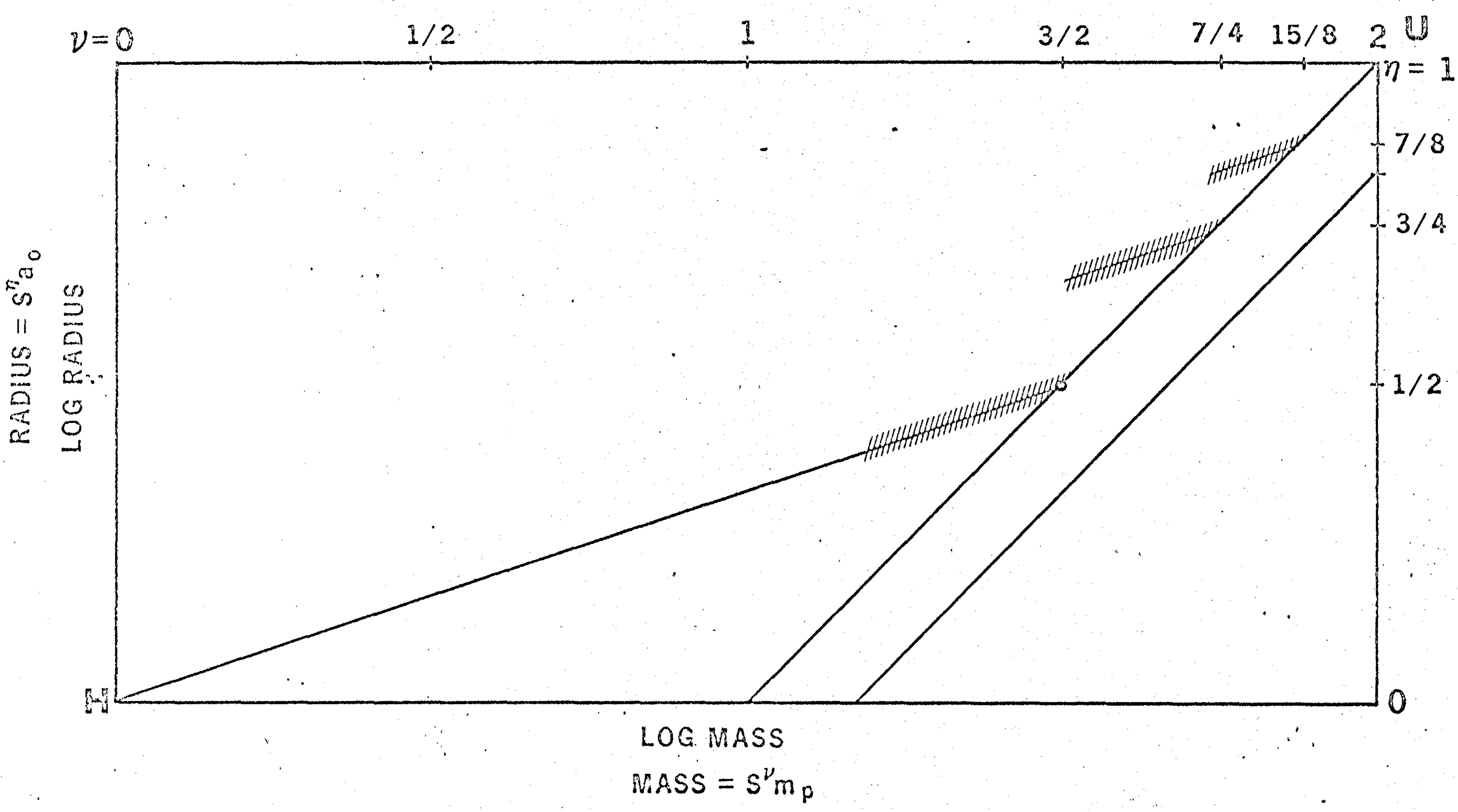


FIG 2

COSMIC DIAGRAM



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HIERARCHICAL STRUCTURE IN THE COSMOS

Albert Wilson
Douglas Advanced Research Laboratories

Well, I wasn't quite clear from Dr. Harrison whether if I talked about the structures and the Cheshire cat classification of cosmologies whether I would be giving the grin cosmology or the cat cosmology. So he tells me I am giving the cat cosmology.

Well, we will try to restore structure to the universe, although I think you can appreciate why the structure had been removed. The difficulty of doing anything with a complex problem like this, getting a mathematical solution is quite a trick, and I feel that the results that Dr. Kaufman reports are exceptionally interesting to come up with and have an actual theoretical sequence of structured bodies in the universe. I feel this is a remarkable achievement.

But let's address the question what is the information which has been thrown away by the theoretician? They are always complaining there are not enough check-points to test the various cosmological theories, but I think they should put up or shut up and really look at what the informational content of the various structure is. They talk about fluctuations in structure, regularity and so on. But to what extent are there regularities? What are the relationships between the various bodies that we do observe?

Before I dare approach this topic, since we have some philosophers of science present, I have to define what I am going to talk about. I am going to use the term "modular hierarchy." By a modular hierarchy I mean essentially what Simon defines in his paper on the architecture of complexity. We will start with a set of elements which we call modules, and these interact in some way, either communicate or force fields or some kind of interaction, and after an aggregate of so many have been accumulated a higher order module appears and so on. This larger one can be a subsystem of a higher level.

Now, if these modules are homogeneous, then the word "level" and the word "module" mean the same thing. For example, if I have a brick and we are going to build houses or walls with the brick, we are talking about the brick level or module. That doesn't make any difference. But if we are building a second order module out of bricks and tile and a mixture, then level and module are distinguishable.

Now, we should talk about different types of interactions. I want to use the word "horizontal interaction." We are talking about interactions on the same level; that is, between modules on the same level. This sort of thing would be a horizontal interaction. A vertical interaction or vertical communication would be between a module of an order I and the module I + 1. We will have then both vertical and horizontal interactions.

Physics is concerned almost exclusively with horizontal interactions. Newton's third law is talking about a horizontal interaction. We do not have too many vertical interactions or principles at the present time.

Now, one feature of a module, it in some sense is closed. Now, the usual sense of closure when you think of a module is topological closure like a brick, or, for example, something like that, in which there is some spatial parameter that returns to its own value. When we come to levels it may either be closed or partially closed or they may be only bounded.

Now I would like to return to a point that Professor Smith made yesterday and illustrate what I mean by the difference between an open and bounded level and a closed level. If we take as our first order module just any -- element, and we build up in any way a -- out of such element, our rule of closure would be something like this: the vertices minus the edges plus the number of faces is equal to the unit. Now, we can continue to add and every time you add something this is still preserved and we can go on in an unlimited way until we come to something besides this rule of closure, partial closure, that will terminate this aggregation. A discontinuity in surface or interface or some physical limitation that will terminate this.

Now, we will call this the sum of the parts. We will call this the whole and actually the sum of the parts is equal to the whole. Now, I am using this in the didactic sense and not in a serious way.

If we make a system in which the whole is greater than the sum of the parts, sticking to these two definitions we will have an emergent level. And I am giving this just as an illustration. We really need an illustration for the difference between a hierarchy and an aggregate which has an emergent feature and one which has not an emergent feature. There is nothing that emerges out of this. If we keep on adding and continue on around the world and have covered everything and we are now coming into putting the last tile in the last space, we increase this by one and get our two. What is the emergent property? The emergent property is we have introduced a new dimension, a third dimension in space. So we can think of the dimensionality of space as an emergent property from the closure rule of this sort. That is turning the whole thing around backwards and this is the way Leibnitz looked at the world.

Newton thought of structure as existing in space. Leibnitz said that space is defined by structure and if we have a set of descriptors such as Cartesian coordinates and all of the usual things we use to define space, these are just handy devices, but what is done fundamentally is the structure and the type of structure that exists defines the space.

Well, if Leibnitz had prevailed instead of Newton, maybe we wouldn't be going through this detour. The general theory is really through the concept of the presence of matter having the geometric property -- being associated with curvature -- we are coming back to a Leibnitzian view of what space is. But we call this the Newtonian detour. This serves not only to define space from structure, but to illustrate two types of aggregates, one of which has a feature of emergents and one which doesn't.

Now, with this bit of background, one more remark:

Professor Gerard said the difference between the brain of man and the brain of a chimpanzee is the number of units present. But I would feel it might be related to something like these two types of closure. Maybe you just don't add units; maybe you have finally put that last tile in place and made a closure and reached an emergent property.

Now, with this let's turn to cosmology or at least a description of the cosmos. I want to use a Gestalt cosmology -- I guess that is a terrible redundancy -- and look at the universe, all of the levels, simultaneously. As Professor Harrison so beautifully explained, cosmologists today have really considered only two levels. They are considering the universe as a whole and some kind of a fluid that is one level below which is the construction end of this. It may be a galaxy or they may smooth it out to having properties of a free fluid. The cosmologist's view is a two-level view. We want to try to view all of the levels simultaneously. To do that we have to start with these numbers that we defined as the properties of atomic and nuclear particles.

Now, the history of this subject is an interesting one. It starts, as far as I can find, although I may be wrong or others may have better references, a fellow named Arthur Hooves in the 1920's first compared the fundamental atomic dimensions -- constants, with constants that occur in cosmology. You recall the three-dimensional list constants that we encounter, the ratio of coulombs to gravitational force where E is the charge, G is Newton's gravitational constant, and the subscript C for the mass of the proton -- I will use "S" to designate that ratio, and to the best of that measured value this is something like $10^{39.356}$. The weakness here is in the gravitational constant.

Now, a second dimensional constant, the Summerfelt ion structure constant where H is constant and C is the velocity of light and the latest value of the reciprocal of this given by Cohen is 137-point, something of that size.

Now, it has been known for a long time that this is sort of logarithmically related. These two constants are logarithmically related. In fact, we have enough accuracy here to see what it is. But there is enough accuracy to write S in terms of alpha and it turns out that S is equal to 2^1 over alpha pi squared, fitting both of these. Maybe that is true and maybe it isn't. If it is true, it is important because recently through the study of the fine structure in the spectra, this constant has been proved to be constant at least for the last 200 million years by observing the spectra of distant galaxies. If this is constant and this is true, then this is a constant.

The third dimensional constant is this: the ratio of the mass of the proton to the mass of the electron.

Now, from cosmology we have two fundamental parameters; one, the Hubbel parameter which is the present rate at which the universe is expanding. In terms of Dr. Harrison, this is $\dot{R}$ over R and in other observable cosmologies is the mean density of the universe.

Now, Haas noted, and Eddington followed this up shortly -- I don't know who was first; I think Haas was -- they pointed out that the radius, the dimensionality of radius which is the velocity of light times the inverse of the Hubbel constant, divided by the radius of the electron is again a number of this order. The mass of the universe, of which I take this radius, in terms of the mass of the proton is this 10^{39} squared. So these are known usually as Eddington's numbers and they possibly have some significance to the structure of the cosmos. There are papers scattered through the literature on this subject, but nothing has been done to develop it to the point where we have any kind of a theory. The latest paper was by George Gomhoff. The last paper he wrote before he died was in the Proceedings of the National Academy. It was called "Cosmic Numerology," and he stressed the importance of trying to get something out of this.

Now, Sacar in 1937 noted from theory of cellular structure that you have a term of this sort; mu is just an integer. This is the dimension of mass. He noted that interstellar structure -- theory of interstellar structure -- mu should have the value of three halves. This would give the mass of a star. If mu were 7/4, you would get the mass of a galaxy. This turned out to be 10^{35} grams and this about 10^{47} grams. Very close to the maximum values that are actually observed for these objects.

If you put in 2, you get the Eddington number of barium in the universe, 10^{78} - 23. The mass of the protein comes out 10^{55} grams. This is a summary of the numerology, if you will, concerning these numbers which relate the atomic dimension and mass to the cosmic dimension and masses.

Now, there is an interesting result. May we have the first slide, please.

Now, if we investigate all of the objects, the name of the object being in the left-hand column, star, galaxy, cluster of galaxies, and what some people call a second-order cluster, the gravitational potentials are given about 10^{23} grams per centimeter. This is the same as Professor Kaufman was referring to as the binding energy per gram. But if we express these in -- terms where masses are expressed in the term of barium and -- we again get this dimensionless number showing up for each of these bodies in the universe. Now, the reoccurrence of this number may be meaningless. We may be seeing several numbers of this order or we may be seeing the same number. DeRoc felt that the probability that you are going to get a number of that magnitude, of 10^{39}, showing up by chance in so many places is very low.

Say we have a big roulette wheel with lots of numbers on it and spin it; the probability of getting this thing every time is very low or you have very, very few numbers on that wheel and that is probably what it is. There are only a few important basic dimensional numbers of this sort in cosmic and atomic structure.

This number which is the mass of an object divided by the proton mass, divided by the radius over the Bohr radius is something on the order of 10^{39}. This we can assume if DeRoc's reasoning is to correct to within a -- within a number of the order of units could be pi or something of that type. Then we can solve this with a dimensionalized gravitational potential substituting for a naught and then this comes out to be the finished structure constant squared by direct subtraction in this result. In other words, where these bodies -- the potential that occurs is given by this expression and not by the expression GM over C squared R less E 1/2 which is -- limit. This comes from a solution of the -- conditions and general relativity.

The easiest way to see it without worrying about general relativity is if you want to consider that C is approaching velocity, 2GM over R is escape velocity. Then if you -- but we do not find that this is the observed bounds. This is the one that seems to bind or bound the body in the universe. This, of course, is known as a degenerate body.

Next slide.

There is an adage in making slides that you shouldn't put so much information on one slide. In this sense this is a terrible slide. We have the whole universe here in logarithmic scale, gravitational radius, CGS units.

This is alpha squared or module limit, and the upper one is the Schwartzchild limit. Eddington, Swikey and others pointed out that if you take a degenerate body having the density of the nucleus of the atom and subject and make it as large as possible but subject to the Schwartzchild limit, you will get a mass of 10^{35} grams which is the maximum stellar mass. The relationship between this modular limit and the Schwartzchild limit is simply the ratio bounds of the nuclear dimension and the atomic dimension as shown on that slide. So this is talking about maximum values for different objects that occur in the universe.

Now, there is one other interesting thing about these. If we write the matter of a star in dimensionalized terms, the maximum mass would be about 10^{59} and the 5th root of that gives the galaxy mass on the order of $10^{11.8}$ and the 4th root of that gives the cluster mass of about $10^{2.9}$. The cube root of that gives the second order cluster matter of 10.

Question: does the hierarchy stop or go on? Well, if it gets to 2 it has to stop. So if a sequence of that type exists between the different levels we can say second order -- if we grant second orders exist as Abel and his colleagues hold which is contradicted by Swikey and his colleagues, not only does the second order but the third order exist, they would have radii something on the order of 100 million parsecs. That is something, of course, very speculative. But we get a pyramid-type of structure here. But this does not fill out the universe. There is some left over, 10^{55} grams.

Now, let's take the whole picture and put in just not the maximum but what I have done in the next slide is turn the slide that Michele Kaufman showed us on its side. It is essentially the same parameters. Call this the cosmic diagram. A log of the mass of the body and a log of the potential of the body.

Now, the Schwartzchild limits in this diagram is this line. The line is turned this way. I wanted these levels to be horizontal. This is the modular alpha squared limit. We find that most of the bodies are distributed along three levels, possibly four, that we can now observe. This level goes clear off here. We can think of it as a density or we can think of it as a time. If it is a time, it is atomic vibrational time. We run into meteroids and astroids and we get up here to satellites. These are planets, Earth, Saturn, Jupiter. Then we get into the stars, suns, the super -- the gray and the white dwarfs. Then we stop. We reach the alpha squared limit. No more beyond. The other level up here, this is essentially the same objects that Kaufman had. Clusters, galaxy clusters, globular clusters, M-87 and second order clusters.

Now, if we go back to Sacar and take his numbers here and put them on this diagram, we see that the limiting mass for the stars is right here where these are the numerators. The denominators are all 8. The three halves would be 12/8. The upper mass of the star would be as Sacar gives $S^{12/8}$ times M. This is the 7/4 limit, and this would be the 15/8 limit. What we find here is this is the second harmonic or overtone. If we take as a fundamental S, then that is the star's, the S for the wavelength, S time, a naught is the radius. This is the second harmonic. This is the fourth; this is the sixth; and this is the eighth. The odd harmonics do not seem to show up, the third harmonic, fifth harmonic, eleventh and thirteenth are interesting, though. This has a characteristic time on the order of 1 second which may mean something.

This has a characteristic mass on the order of 10^8 or 10^7 solar mass. The object that may be missing -- 13 for quasars and 11 for pulsars. We don't have but one piece of data that fits here; so that should not be viewed with anything but high disbelief.

Now, the direction of expansion on this diagram is extentionally like this and if stars and other objects have increased their radius in time, they have moved off in this direction and they may have been -- the galaxy may at some time have been bounded by this density limit and moved off and clusters moved off and so on. I think that is about all that I have on there.

I remember Professor Harrison said when he started out he drew a diagram and on the left end was the micro-range and on the right end the cosmic-range, and he said we had no confidence in either extreme. This diagram represents a combination of the two extremes. It is all right to match impedances.

Now, do we have a possible way of approaching this configuration? We can do some speculation here. I want to say one thing: I feel that this information is something that the cosmologist has to take into account because these are gross features. We are not talking about any deltas or epsilons here. These are gross features and new observations and reobservations are not going to change this. You can argue about whether something would fit 13 or something else better. I wouldn't get into that. But in the gross features I think Sacar has fitted very well.

Now, a very interesting set of bounds occurs here. All of the planets seem to be bounded. These are lines of constant mass. All of the stars in this group, globular clusters, galaxies here, higher order clusters there.

Next slide, please.

This gives the observations versus the model comparison. The model says that this is the maximum cold body. This is Jupiter, the maximum mass carefully measured per star at the present time is this. This is what our model shows should be the limit. M to T is the maximum globular cluster in our galaxy that has been measured. This is the limit. The M-87 -- there may be other measures of masses for some of the radial objects, but this one depends on an assumption of the luminosity and mass -- this is the comparison and this is for clusters. These are minimums which are derived from lower harmonics, as you saw in the previous diagram.

That is all for that slide. Thank you.

Now could we go back and account for irregularities of this type existing between the various configurations. The thing that suggests itself, since we do have overtones here, is that we possibly are encountering some kinds of regional phenomena. We can define frequency in several ways dimensionally. We can define a frequency dimensionally in this way: M as the mass of the object and R as the radius. We can define a frequency in this way: we will call this U_1. The square root of GM over 2pi. R is V_2. V_4 if MC squared over 1/2. We can also write dimensionally two other frequencies: C cubed over GM. These are dimensional ways of defining a frequency.

Now, resonance in our experience is a morphogenic process. It does allow energy to accumulate in a system. I really don't know of another physical process besides resonance that does this. I don't have time to do this, but if we take a conventional type of resonance condition that one of these frequencies is equal to, N is 1, 2, 3 and so on, and overtones of the other, this condition implies that the potential is M over R. These objects are discretized or quantitized. It amplifies the radii and masses and density.

Dominic Igan, a mathematician, on the basis of the theory of general relativity, a few years ago derived a formula in which the same conditions came out of his reductions and we have with, let's say, not very much success but some success verified that a great many of the bodies in the universe do seem to follow this. That is a separate subject, but this is not something that is completely wild.

If you examine these sets of data like binary stars, things where the masses -- this type of quantization does appear to exist.

Now, to go to the gross features, we have to get into something. Here we are leaving and introducing something that there is no justification for at all. We have a second resonance condition, something of this sort: S to a power which satisfies Igan's value of some sort. This kind of condition would give the gross features as seen between the various horizontal levels and mass levels.

And a third type for which there is no reason except one could postulate this: alpha squared to some other set of Igan's values -- these would give the potential bounds, the Schwartzchild bounds, and modular bounds and so on.

So from this I feel that morphogenesis, the origin of structure, may possibly arise from some form of resonance and like when you have a giant drum covered with sand and you beat the drum the sand goes to the nodes, and that is an aggregating force or process, and just what the things are here that are vibrating in these cases we don't know. But this type of postulate will fit what we observe on the chart.

Now, I would like to turn to the subject of closure. We have topological closure which we started out with, objects belonging to the set if they are in a neighborhood of a set, of a point. But we have also another type of closure we can call temporal closure and temporal closure means not that we are in the neighborhood of a certain type, but of frequency. We think of time too much as a linear parameter. Time is not a linear parameter. Time is cyclical. So if we represent a spectrum of frequencies in the neighborhood, it would be wavelengths or frequency bands.

Now, these, of course, these two types of neighborhoods can be formally equated through some transformation and we can represent a closed object, an entity or module, either, by a spectrum or some topological form. Let's suppose that every object could have a representation in each of these modes, either the temporal or topological. Then what we have observed in the universe, a series of levels, is a temporal module and its subcomponents are levels which are modules of infinity. These can be thought of as spectral analysis of some kind of entity. This would be a metahierarchy of some sort. So we have certain frequencies or times that we define this unit or module with.

Now, this removes completely the question of holism and reductionism because we are not concerned whether this level worked up or this one down. The structure lies completely outside and I call that a metatatic, to just give a name to a different type of structure or module in which the relationship between the subcomponents is defined by some sort of Igan's value or some principle which lies outside the level.

Thank you.

DR. MENZEL: Very interesting and challenging presentation.

Your reference to the constant 137 reminds me of the last time that I saw Dr. Eddington which was in 1948 just a few months before his death. He and I happened to meet at the end of the last session of the

International Astronomical Union meeting in Zurich. As we walked out together to get our hats in the cloakroom, he reached up and said, "I always hang my hat on 137."

Well, this paper is now open for discussion.

Professor Whyte.

DR. WHYTE: May I ask two questions:

I am exceedingly interested in your relation GM over C squared. Is this new? Is this published?

DR. WILSON: Abstracts have been put in the Astronomical Journal.

FROM FLOOR: Has it been subject to criticism yet?

DR. WILSON: At meetings of the Society, yes.

FROM FLOOR: Anything interesting from that?

DR. WILSON: No. Nobody believes it.

That is the way it should be at this stage of the game.

DR. WHYTE: I have a further comment to make:

Second question, how many apparently independent places does the number 10^{39} arise?

DR. WILSON: Well, in the four cases that I gave. You see, the masses and the radii are determined in the cases of stars, galaxies and clusters and superclusters in quite independent ways. Our best values for stars are determined from eclipsing binaries. This depends on essentially Kepler's third law. The galaxies from rotation dynamics and the clusters from the --

DR. WHYTE: So there are four or five?

DR. WILSON: Yes.

DR. WHYTE: Is this the first time that the -- has been brought into relation to cosmic quantities?

DR. WILSON: To my knowledge. It is really the S that is involved. The reduction to alpha squared follows immediately from the relationship. We have the M over MP times a naught over R is equal to S or some factor times S. Then using a naught is equal to the radius electron over alpha squared and the radius electron is NA squared over C squared. These combine to give you GM over C squared R.

DR. WHYTE: So perhaps it is a combination of S and alpha.

I am very struck indeed by this because if it is indeed the first time the constant has been brought into relation to the cosmic concept, I consider this of greatest importance. I have meditated on -- structure constants in a rather different way than Eddington and I just want to give you one suggestion to meditate on: E_2 over H_C.

E_2 squared is the electrostatic effect. H is rotational. The combination of the two is the Carroll form.

DR. MENZEL: One other thing which I thought I noted from that diagram with the lines kind of askew on on it, it seemed to me that the hydrogen, the protein which was down here in the lower corner which was way below the stability limit for the stars, the difference between that and above it was about 10^{39}. I wasn't sure.

DR. WILSON: That is where this M over -- when you put it in terms of the MP.

DR. MENZEL: In other words, if you express the electrical potential instead of gravitational.

DR. WILSON: I really feel that the explanation of this is going to be that electrostatic forces are playing a role in the cosmogenesis of a structure.

FROM FLOOR: Just to this last point, if you take a body consisting of ionized gas and you want it to be gravitationally bound, you have electromagnetic interactions on a short range and -- the electromagnetic constant over the gravitational coupling constant gives you the number of particles in the star multiplied by the mass. This is the physics. When you take mu to be different from three halves, we have no -- that is it exactly.

DR. WILSON: Getting up to the level of the stars we are all right.

FROM FLOOR: Why does that repeat itself at a higher level?

DR. WILSON: The fact that the first time this showed up in physics was on a microscale shouldn't prejudice us. These numbers could have shown up anywhere first. We mustn't think because we have known about this measurement in the lab for four or five decades and we find it in the telescopic observations that we have to explain it in terms of the laboratory. This may have a more fundamental relationship.

I think there is a last slide. I will take a second to show you how these two constants do enter into the Gestalt picture.

This is the universe again in which the horizontal is the mass. You multiply the S by MP. The vertical is the radius. This is the hydrogen atom. These are where matter is bounded in the cosmos and these are the two observed potential limits. This separation here is where the alpha squared appears and the S and the various powers appear in both. These are the places we now can use S and alpha in structural relation in cosmic bodies.

DR. WHYTE: If alpha is involved, it must be electromagnetic.

DR. MENZEL: Any other questions?

FROM FLOOR: Is there general agreement as to the number of fundamental independent constants which are involved?

DR. WILSON: I don't think it has been predicted. I think it is now felt there are only four interacting forces. There are other fundamental relationships. So there would be information of basic forces. We know there would be a ratio. This, of course, is sort of the Eddington version of it.

DR. MENZEL: Multiplying these constants by factors like pi and so on doesn't change their order of magnitude, but if you take the square root of dimensionless things it does do something. I was wondering what right we have in our thinking in the way we think of it -- isn't the square root or perhaps the square of the constant something that might be more fundamental? The S squared perhaps is more fundamental or the number of bariums.

DR. WHYTE: E. P. Vigner showed in his first book after he introduced group methods in quantum mechanics that the energy and transaction rate involved in the hydrogen spectrum can be developed as a series in the finite structure constant. If you study the series you see quite clearly that a structure by nature, so to speak, which order to use and such-and-such. The first order will come out in relation to alpha squared. After the third part doesn't appear in any known physical effect. After the fourth $alpha^5$ represents the ordinary transaction between standard -- and so on.

DR. WILSON: There is one other place that alpha squared has shown up. This is quantitization of the values of the Redshift themselves. This is something that is now coming up again -- have current papers on this. It does seem the Redshift themselves are quantities. It is related to this parameter.

DR. MENZEL: Any other questions?

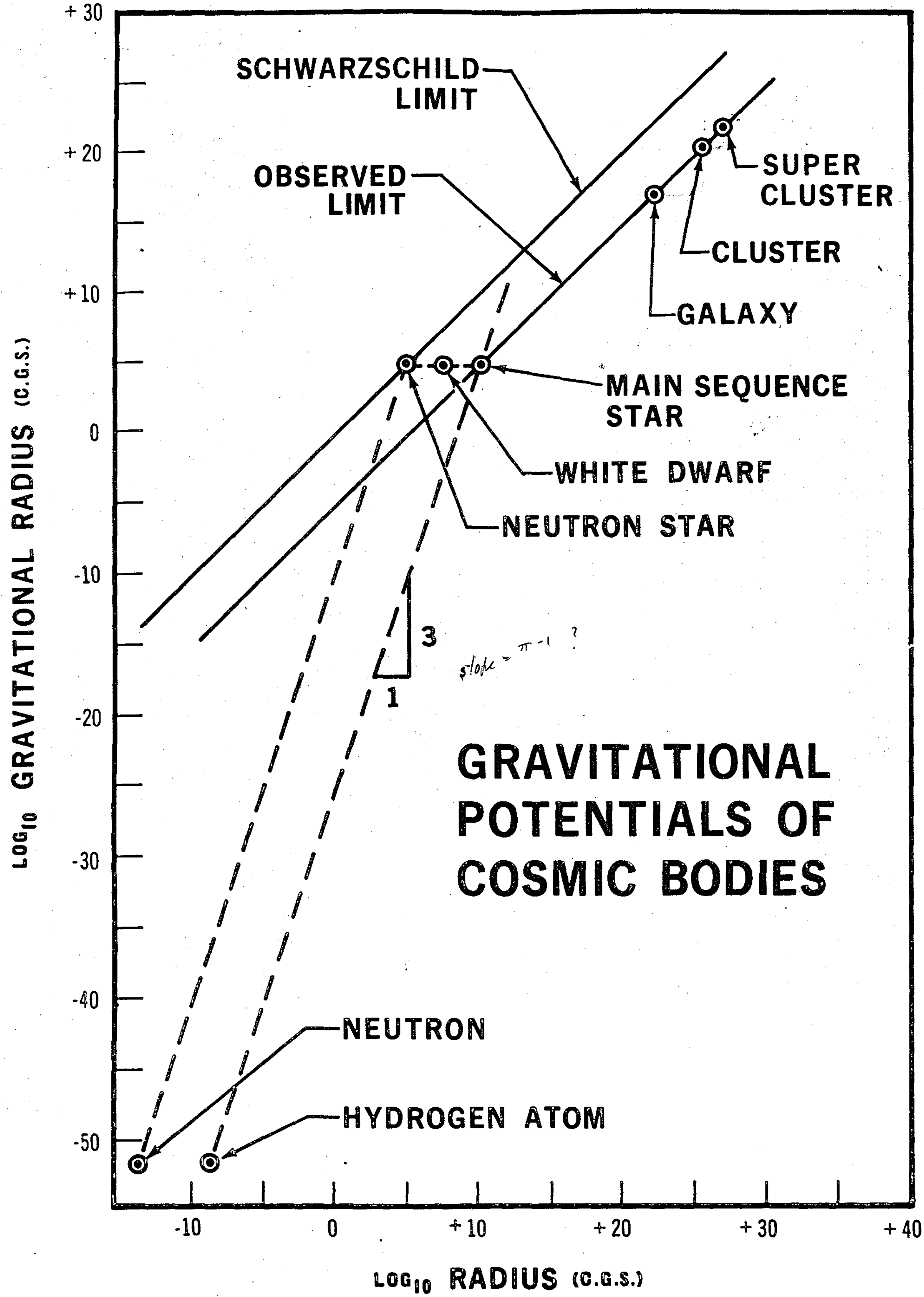
If not, then we come to the final paper, the

MAXIMUM OBSERVED GRAVITATIONAL POTENTIALS

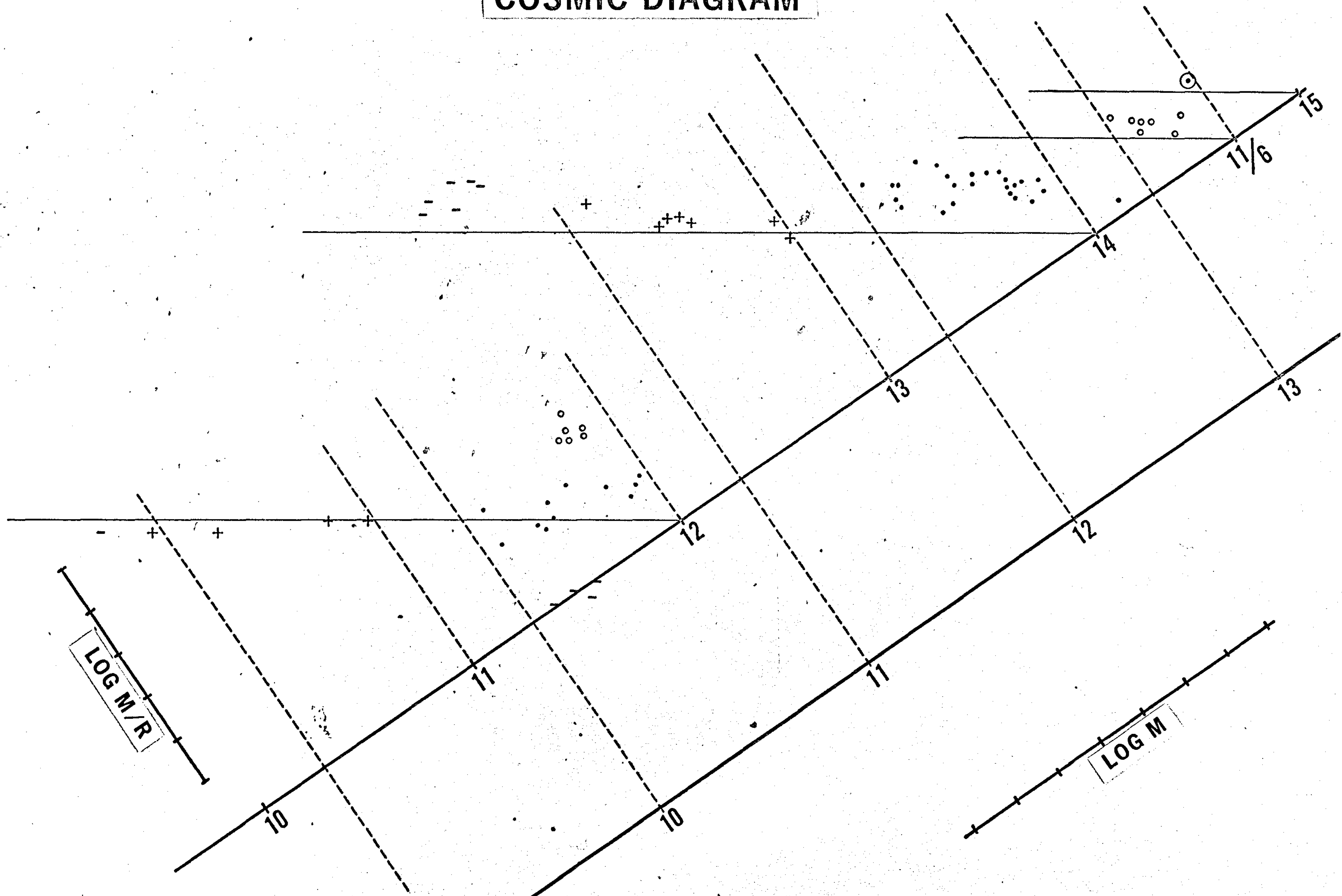
SYSTEM	$\log_{10}[M/R]$ (c.g.s.)	$\log_{10}[M/R]$ (dimensionless)
STARS	23.27	38.8
GALAXIES	23.6	39.1
CLUSTERS	23.5	39.0
SECOND-ORDER CLUSTERS	23.2	38.7

	PLANETS	STARS	GLOBULAR CLUSTERS	GALAXIES	GALAXY CLUSTERS
MAXIMUM					
OBSERVED	JUPITER	VVCEPHEIA	M22	M87	LOCAL
	30.279	35.225	40.14	45.9	48.3
MODEL	30.338	35.258	40.18	45.1	48.4
	$\upsilon = 11$	$\upsilon = 12$	$\upsilon = 13$	$\upsilon = 14$	$\upsilon = 11/6$
MINIMUM					
OBSERVED	MERCURY	RCMaB	M5	MGC6822	U.M.I.
	26.509	32.340	37.3	41.9	46.6
MODEL	26.782	31.702	36.6	41.5	46.5

Log_{10} grams


SCHWARZSCHILD LIMIT
OBSERVED LIMIT
SUPER CLUSTER
CLUSTER
GALAXY
MAIN SEQUENCE STAR
WHITE DWARF
NEUTRON STAR
3
1
GRAVITATIONAL POTENTIALS OF COSMIC BODIES
NEUTRON
HYDROGEN ATOM
+30
+20
+10
0
-10
-20
-30
-40
-50
-10
0
+10
+20
+30
+40
LOG10 GRAVITATIONAL RADIUS (C.G.S.)
LOG10 RADIUS (C.G.S.)

COSMIC DIAGRAM



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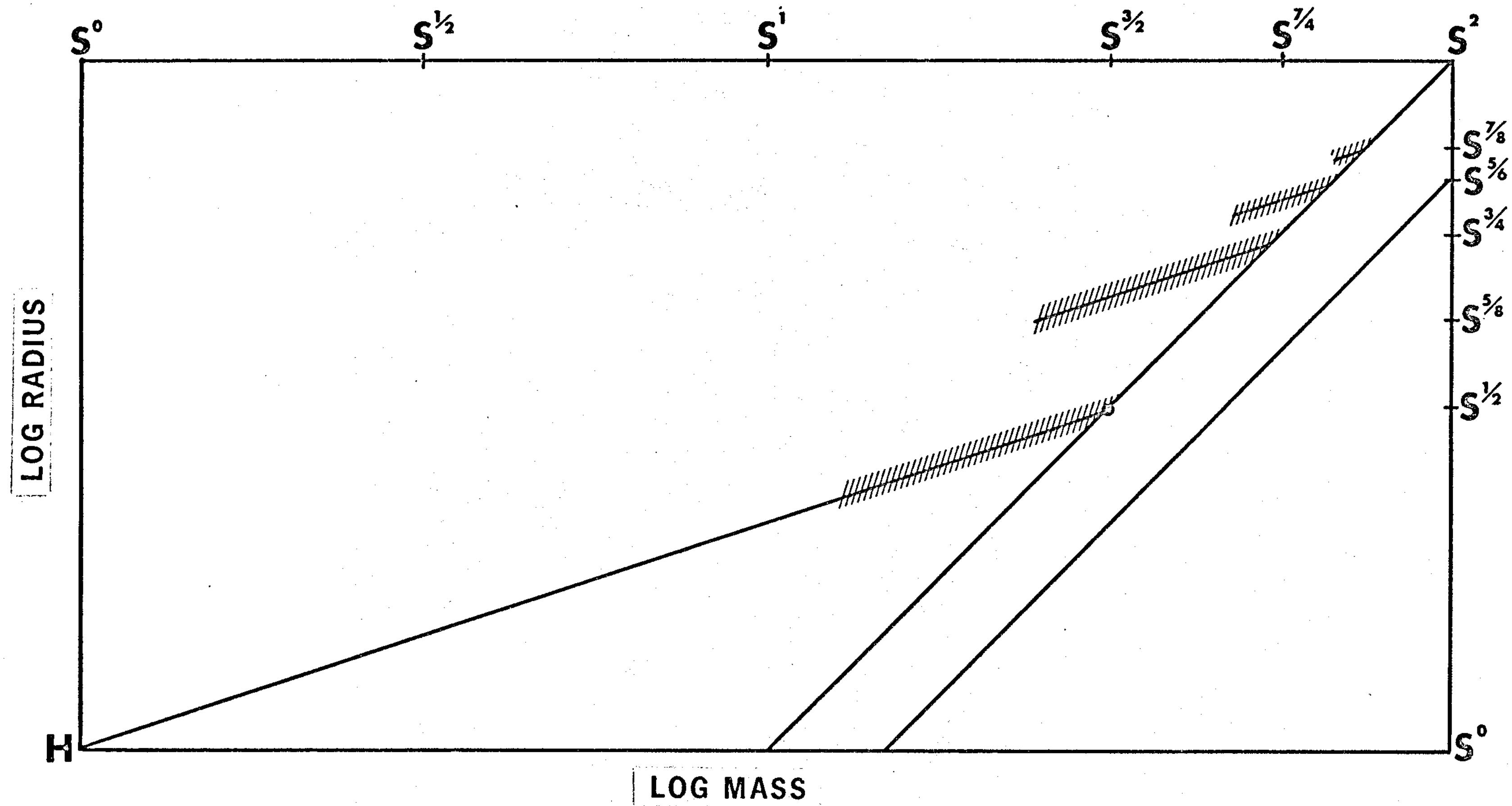

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57.42

A TAXONOMY OF HIERARCHIES

Albert Wilson

In 1968 two important conferences on the subject of heirarchy were convened. These conferences were independently inspired and conducted unknowingly of each other. The first conference was held in the summer in Alpbach, Austria, organized by Arthur Koestler and funded by three publishing companies. It was attended by 15 distinguished scientists and philosophers, including such notables as Jerome Bruner, Ludwig von Bertalanffy, Viktor Frankl, Jean Piaget, C. H. Waddington and Paul Weiss. The thrust of the conference was the search for viable alternatives for organizing biological thought, spurred by a general dissatisfaction with reductionist approaches and the inadequacy of neo-Darwinian ideas of evolution. The nature and origin of hierarchical structures became the central theme of the conference, although the book recording the proceedings came out with the title, BEYOND REDUCTIONISM (A. Koestler and J.R. Smythies, editors, Macmillan, 1969).

The second conference was held in Huntington Beach, California in the autumn, organized by Lancelot Law Whyte, Albert Wilson and Donna Wilson and was supported by the Douglas Advanced Research Laboratories of the McDonnell Douglas Corporation. There were some 50 attendees including Ralph W. Gerard, Mario Bunge, Chauncey Leake, Howard Pattee, Cyril Smith and John Platt. The thrust of this second conference was a comparison of hierarchies as encountered by physicists, biologists and computer scientists. (Unfortunately social hierarchies were only incidentally covered at both conferences.) This conference also resulted in the publication of a book, HIERARCHICAL STRUCTURES, (L.L. Whyte, A. Wilson and D. Wilson, editors, Elsevier, 1969). With the publication of these two books, the subject of hierarchy as a research area came of age.

Since 1969, there has been increasing interest in what hierarchies are and several conferences and symposia related to hierarchies have been held and new books published. (It would be useful to have the excellent bibliography compiled by Donna Wilson in 1969 brought up to date.)

At the outset of our discussion, I feel that it is worth re-stating the basic questions that confronted those who participated in the two inaugural conferences on hierarchy held nine years ago, mainly because most of these questions still require better answers than have so far been provided:

- What are we talking about when we use the word hierarchy?

  An essential structure that pervades the natural world and somehow also emerges in our own creations? Or an apparent structure imposed by our particular way of perceiving or by our need to structure our experiences?

- In talking about hierarchy are we talking about one thing or many things?

  In the sense of relation or structure?
  In the sense of function or purpose?
  In the sense of cause or origin?

- Do the similarities between the various specific hierarchies imply a structural commonality that is meaningful on some level of abstraction and, if so, can the reason for such structures be derived from some fundamental meta-principle--informational, combinatorial, topological, whatever?

Postponing final answers to these questions, it has been generally agreed that whether hierarchy is real or imposed, results from one or varied causes, the concept is a useful, and even needful, category for describing complex structures. But Ralph Gerard would remind us that the making of categories is both man's great intellectual strength and weakness: strength, since only by dividing the world into categories can he reason with it; weakness since he then takes the categories seriously.

Nine years ago it seemed proper to many that the first step in our attempt to get a handle on the concept of hierarchy was to get a good definition, and then we could proceed with a logical development of our inquiry. Others were opposed to this on the grounds that this would delimit our inquiry before we could obtain a reasonable notion of its extent. Inquiry, as well as decision making, should have both its expansive and contractive phases. In inquiry the expansive phase is performed through the operation of characterizing. It is only in the contractive phase, after the domain of inquiry has been fully surveyed, that we should attempt definition. Now nine years later, I am not sure that we are yet ready for a hard definition of hierarchy. There still remains too much to do by way of characterizing hierarchies.

Perhaps it is in order here to say to those who feel that the opposite of well-defined is fuzzy, that this is not necessarily so. The opposite of well-defined may often be incomplete. From Gödel's work we know that there exists the essential choice between total rigor and completeness. The question before us in our inquiry of how best to abstract the concept of hierarchy is 'what is the optimum way to truncate a necessarily incomplete list of characterizations of hierarchy'. The truncation will mark the switch from the expansive to the contractive phase.

What has actually happened since the inaugural conferences is that most researchers interested in hierarchies have not been concerned with the basic questions that were formulated to assure as holistic an approach as possible. Instead they have taken those particular hierarchies of immediate interest to them--bio, socio, systemic, etc,--and sought to develop theories covering their origins and properties. This is perhaps as it must be, and maybe even as it should be. But the concept of hierarchy as a unifying schema of great potential importance must not be lost sight of. The work done on specific hierarchical structures, or certain classes of hierarchies, e.g. those in living systems, should enable us to further our original goal if we are willing from time to time to return to a holistic view of the subject.

Let me re-iterate that the name of the game at this stage is abstracting. Our strategy will be to find the intersect or overlap of the characterization sets (C_i) of each system or structure which is considered in some way to be 'hierarchical'. If our open-ended list contains N members we consider first the intersect characteristic set,

$$I_N = C_1 \cap C_2 \cap C_3 \cap \cdots \cap C_N$$

If I_N is not a null set, we may base our definition of hierarchy on the characterizations contained in I_N. But I_N may be unduly restrictive providing an inadequate axiomatic base for developing derivative properties. In this case the base may be expanded by considering the set of N sets $\{I_{N-1}\}$ obtained by removing one of the N sets, C, from I_N. Each of these intersect sets may be tested as a base of definition of hierarchy, or we may proceed further and consider the N(N-1)/2 sets, $\{I_{N-2}\}$ obtained by removing two of the N sets C, from I_N employing all possible combinations, etc. Whatever set (or sets) is choosen for the definition base, there is an essential trade-off involved. Our selection will be based on the criteria of comprehensiveness, simplicity and precision. But as pointed out earlier precision is always purchased at the price of comprehensiveness. So we may anticipate at this point that hierarchical theory will be able to say a few general things about all (or most) hierarchies and say much more and much more precise things about particular hierarchies. The holistic approach has the goal of saying all that can be said at each level of comprehensiveness.

The first step is to consider the intersect set I_N, which is common to all hierarchical systems so far considered as such. We find that this set consists of the two properties:

- Discreteness
- Orderability

Which is to say that all things called hierarchies are composed of discrete elements, variously called levels, layers, strata, etc.; which are readily ordered by some criteria such as size, frequency, complexity, etc.

However, discreteness and orderability alone admit a great many arrangements to the class of hierarchy that we really do not wish to include, examples being such things as the natural numbers and the lines in line spectra. While both of these examples certainly consist of discrete elements that are readily ordered, as by magnitude and frequency respectively, they lack something else that we feel is essential to the notion of hierarchy. We thus, according to our strategy , reject a definition of hierarchy based on this set I_N, but we note that I_N is useful to us in eliminating at the outset those arrangements we do not wish to admit to the class of hierarchy.

Following our strategy, we next consider sets selected from $\{I_{N-1}\}$. Ralph Gerard feels that basic to the idea of hierarchy is the concept of subsumption. This is a generic term. Specific examples include subordinate, which is essentially the imposing of a 'pecking order' on the discrete elements of the hierarchy. This pecking order or bossing relationship is basic to the classical meaning of hierarchy--the heavenly hierarchy as visualized by the Neo-Platonist, Pseudo-Dionysius. Mario Bunge has formalized this particular notion of hierarchy as consisting of ' a set partially ordered by an antisymmetric relation of domination or command,' domination being the obverse of subordination. (He further feels that this is the only arrangement to which the term hierarchy should be applied. All others should be called, 'levels of organization'.) In this relation, all information flow is top-down, there being no two way communication between levels. But not even armies are purely antisymmetric.

level to level or part to part i.e. holonistic

A second common meaning of subsumption is that of subdivision. This concept brings before us the important question of the relation between parts and whole, adding the idea of containment to the basic characterizers of discreteness and orderability. Containment of course is itself an ordering relation, but it carries the further connotation either of each level being nested within higher levels, such as subprograms and subroutines within a computer program, or the classical nest of Chinese Boxes, or of the higher levels actually consisting of aggregates of lower levels. This latter type of hierarchy is so important in the inorganic world, that it should be given an explicit name. We shall here call such hierarchies modular hierarchies. Examples include the primary physical and cosmographic hierarchies of particles, atoms, molecules,..., stars, galaxies, clusters,...

Whole/part relations may be such that a) the whole is <u>less</u> than the sum of the parts. An example of a system of levels ~~or hierarchy~~ with this property would occur in what Messarovic calls a descriptive hierarchy. As one introduces more abstraction in each level of description there will be less and less information, but the essential bare bones of the structure become more apparent. The strategy of abstracting the concept of hierarchy is itself an hierarchy of this type. Or there is b) the Euclidean type in which the whole is <u>equal</u> to the sum of the parts. Strictly Euclidean hierarchies are rare in the real world, but a modular hierarchy perhaps more closely approximates the Euclidean case than other hierarchies. Then there is c) the hierarchies in which the whole is <u>greater</u> than the sum of the parts. Examples are hierarchies in living systems in which properties not contained an any of the subsystems 'emerge' at each subsequent higher level. (Even modular hierarchies possess this property in a limited way, but 'emergence' in modular hierarchies is usually traceable to properties of lower level systems; as temperature is an emergent property in an ensemble of molecules, but it is only a manifestation of the motions already present in the lower systems and thus not strictly emergent in the sense of being qualitatively innovative or unpredictable. It is felt by many that it is expressly this condition of <u>emergence</u> that is essential to what we shall wish to classify as a hierarchy.

The prediction of the properties of the aggregate from the properties of the components--and knowing when this is and is not possible--is one of the important reasons why we wish to study hierarchies in the first place. We need to know how to aggregate behavioral characteristics of the micro-systems involved in order to come up with the behavior of the macro-system. A case of critical interest today is, of course, the economy, where each business enterprise is individually proceeding by the making of decisions that make good sense from the microview of their management but which result in a macro-configuration which is moving toward making less and less sense in terms of quality of life, utilization of human and natural resources, inflation and pollution. We look to the development of a theory of hierarchical systems to help us to understand this so-far untractable problem.

A further type of subsumption is subsequent--an ordering in time. Pre-quantum-mechanical views of the nature of time required a single direction for its flow. This arrow of time imposed a one way ordering, like domination or subordination, on events. Thus sub-sequent maps onto much of what we said about sub-ordinate in the case of static hierarchies. We can think of produces, generates, begets and causes as creating temporal analogues to pecking-order, that is to say, domination, if not a strictly one way flow, is at least a one way net flow.

The second mode of temporal subsumption, corresponding to sub- ~~division~~ division also has its dynamic counterpart. If the first mode, temporal subordination, represents itself through causality and causal determinism, the second mode, temporal subdivision, represents itself through what John Platt has called hierarchical restructuring, i.e. a temporal pattern in which a new level with qualitatively new properties not germinal in the past states of the system, emerges out of existing levels through a catastrophic, or very short time process. The discrimination between causal deterministic processes and hierarchical restructuring is a most important one. We have to recognize that a process of discontinuous change--like emergence-- cannot be reductionistically derivable from a process of continuous change --like evolution.

Since many hierarchical structures have been created by man, we should have some direct insight into the factors favoring their origin. Many of these factors have to do with some sort of optimization process. I mention one example. In Herb Simon's classical paper, 'The Architecture of Complexity', he demonstrates how a hierarchical organization of piece work leads to a minimization of production time.

There are also cases in nature in which a clear picture of an optimization process can be postulated. Charlier showed how a continuing modular hierarchy could resolve both Olbers' and Seeliger's Paradoxes concerning the densities of radiation and gravity in the universe. While the relativistic reorganization of cosmological thought gave feasible alternatives to Charlier's resolution, the implications of general relativity for hierarchical universes are not yet fully resolved and ultimately some form of hierarchical universe may prove essential.

Paul Weiss holds that the essential characteristic of hierarchy is that of the imposition of constraints by one level on another. This is a special form of Gerard's subordination, a constraint being a partial container. Quoting Weiss's paper, 'One plus One does not equal Two':

> A structural level, or unit, in a structural hierarchy can be usefully defined as a three-dimensional system of structures or processes involving characteristic constraints imposed on the degrees of freedom of its elemental parts, so that the properties of the level are not the simple linear summation of the properties which the same parts display when isolated.

Here we see emphasis on the constraints imposed by the whole on its parts. There are also constraints imposed by the parts on the whole. This upward direction type is described by Albert Ando in 'Essays on the Structure of Social Science Models':

> Variables belonging to a higher ordered level are influenced by the variables in the lower order (more elemental) levels. When such a stratification exists, then we may say that the variables in the lower order levels are the causes of the variables in the higher order levels. This type of hierarchical structure provides the justification for ignoring the variables in the higher order levels when the object of an investigation is restricted to the behavior of variables in the lower orders.

Ando is describing the assumed configuration on which reductionism is based, the dependence of the higher level variables on the lower and the independence of the lower level variables from the higher. The Weiss and the Ando hierarchies give us two extreme species: The higher constraining the lower, to which we shall give the name Machian hierarchy and the lower influencing the higher, which we shall term a purely reductionist hierarchy. Whether purely Machian or purely reductionist hierarchies exist in nature is doubtful since we are well aware of the absurdities that arise when treating hierarchies as though they were purely of one type.

Another characteristic frequently encountered in certain types of hierarchies is <u>self-reference</u>, in which certain features are common to every level. Galileo remarked this self-referential property in the satellite system of Jupiter, which he had just discovered, and the Solar System. Two types of self-reference are encountered:

The first in which there is a mapping, an isomprphism or homeomorphism, between levels. e.g. In each cell of the body there is the information for the whole.

The second, in which there exists a subject/object containment iteration. A subject on the first level moves to a second level and views itself on the first level as an object. The subject moves to the third level and views the subject/object of the first two levels as new object, etc. This is descriptive of a property of <u>consciousness</u>. which is a Janus like sequence rising to a new level every time a subject can extract itself and view all earlier levels, on which it still exists, as objects. A calculus of self reference is developable from the system of H. Spencer Brown. (There is a sort of inverse of the consciousness hierarchy in the "Hutchins Method" of forming a Group.)

Finally, we may have hierarchies in which interactions have little respect for chain of command. Robert Rosen describes bio-organizations in the following way:

> A particular biological function, at any level tends to be distributed over much, if not all, of the entire system. i.e. The whole sequence of levels is simultaneously imposed on the same indivisible system. The recognition of levels in organisms operationally involves merely different descriptions. of the activities of the <u>same</u> system. Nothing that happens at any one level can be without consequence at all other levels.

We thus have hierarchies going from simple ladders with top-down, bottom-up or combined flows, each level interacting only with adjacent levels. We have systems in which each level directly acts with every level below it or with every level above it. And there is the a system with a richness of interaction so great that even the characteristic of discreteness disappears. IN the limit there are <u>holarchies</u> in which not only the whole contains every part, but every part contains the whole.

We may summarize the various types of hierarchies isolated through the intersect set strategy or by combinations of characteristics with a preliminary morphology.

- o Direction of flow
  - Reductionist, Machian or combination, f(t)
- o Ordering
  - by containment, time, size, complexity,...
- o Dynamic
  - static, evolving per causal determinism, evolving per innovative discontinuities (hierarchical restructuring).
- o Parts/Whole
  - Sum of the parts less than the whole — emergent
  - Sum of the parts equal to the whole — modular
  - Sum of the parts greater than the whole — abstraction
- o Communication
  - holon level to adjacent level only
  - each level to all below
  - each level to all above
  - all to all
- o Type of self reference
  - Mapping (cloning)
  - Consciousness
  - Hutchins - Iterated Conscious
  - none
- o Compactness
  - Discrete — hierarchy
  - Continuous — holarchy

Additional Material on Hierarchies from the Cal-Poly Meeting

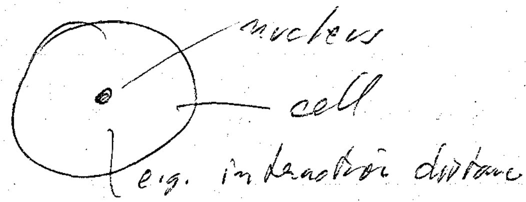
New Levels are created by making a distinction. (Goguen)

More on self-reference:
Recursion
Iteration
Oroboros
Consciousness

If a higher level goes lower levels get wiped out.
– Harmon

↑ possibility
↓ selection

Construct cellular hierarchies: with cell → nucleus of next level



This is a form of self-reference.

organelles are only in cells
star clusters are only in galaxies

hierarchy
heterarchy
holarchy
malarchy

A device for still enabling us to equate or join evolution and emergence

List of characteristics of hierarchies
Discreteness
Ordering
Subsumption / Subordination
Characteristic Time
Parts / Whole
Subsequent
Self Reference – Distinction
Emergence – Hierarchical Restrictions
Constraint

Small no. of prime explain a lot.

A. G. Wilson, 11/5/68

Causes

~~FORMS~~ OF HIERARCHY

Fold = ?

on of problem

of ob routines

A type of hierarchy very frequently encountered is the modular hierarchy - the hierarchy whose levels are identified with stable semi-autonomous modules that 1) are composed of lower level sub-modules, and that 2) are assembled into higher level super-modules. Familiar examples are molecules composed of atoms and assembled into crystals, words composed of letters assembled into sentences, platoons composed of squads assembled into companies.

The ubiquity of modular hierarchies and the commonality of their structure intriques us to inquire as to whether they differ only in the specific media in which they are cast or whether there exist several distinct types of modular hierarchies distinguishable through the details of their form and the causes of their origin. It is possible that the resemblance of specific modular hierarchies to one another is only superficial even though it is sufficient to produce a pattern that attracts our attention and causes us to establish a class we name modular hierarchies. Some patterns sufficiently regular to attract our attention may arise from chance - which means that the occurrences of the elements in the pattern are attributable to many different causes. It is only when a high percentage of the members in a class owe their presence to a very small number of causes that the class becomes epistemologically meaningful to us. Otherwise it leads to no economies of representation or relationship and any analogues perceived are likely to be misleading and superficial.

The effort of investigating modular hierarchies finds justification, therefore, on the premise that the large number of specific hierarchies that we encounter will be explainable by a small number of underlying principles. (This premise itself also is a modular hierarchy.) Since the immediate explanation of any specific modular hierarchy is to be given in terms of the known physical, chemical, biological, psychological, or social laws appropriate to the substantive ingredients of the hierarchy, the premise that the large number of different specific hierarchies are explicable in terms of a few basic principles implies the existence of a meta-law underlying or defining the forms of the laws of physics, psychology, sociology, etc.

The concept of meta-law is as old as Plato, but it has not been fruitful or a popular concept in the 20th century. In a pragmatic culture the pursuit is too high risk for most tastes. None-the-less from time to time papers appear concerning, for example, the properties of the fundamental constants of nature and hint at relations between the microcosmos and the macrocosmos. Some of the best physicists have looked at this question - Schroedinger, Dirac, Eddington, Chandrasekhar, and most recently Gamow. Perhaps these gentlemen display their pragmatism by limiting their search for meta-concepts to infrequent incursions separated by years of solid more immediate research. I raise this only to remind us that in confronting the problem of hierarchy, in seeking relational concepts between the laws of various disciplines, at one end of the risk axis we are possibly quixotically assaulting what may turn out to be a windmill of superficial analogy, ~~or~~ at the other end we may possibly obtain ~~some~~ new glimpses of the heights and depths of the structure that surround us.

In order to classify the types of modular hierarchies, we may first inquire as to whether the size, the complexity, the limit of the module at any level is determined 1) by the properties of its sub-components, 2) by its environment, or 3) by a combination of both contents and context. And to these possibilities we must add a fourth, that the levels and modules in a hierarchical structure are determined by a meta-relational or transcendental structure that determines the ontological possibilities. In such a meta-structure, the <u>levels</u> in the basic hierarchy themselves become the modules on a single level of the meta-hierarchy, while their hierarchical representation in the material world becomes a second level of the meta-hierarchy.

As an example we may think of the energy levels in an atom as an ordinary hierarchy (but not a modular hierarchy). A meta-hierarchy would ~~have~~ consist of the levels of spectral lines, energy levels, and the abstract rule - such as the Balmer sequence that defines the levels.

We may object that this is not a real hierarchy but rather a representational hierarchy. But the essential point is that the levels are not determined by the sub-levels, or the super levels, or both in combination, but by a set of eigen values. We shall need names for these concepts.

June 1969
Hierarchical Structures

Preface

This book is based on the interdisciplinary symposium, "Hierarchical Structure in Nature and Artifact," held in November 1968 at Huntington Beach, California. The symposium was convened under the sponsorship of the Douglas Advanced Research Laboratories and the University of California, Irvine to bring together scientists, engineers, designers, and others interested in the function of hierarchical structures in nature, concept and design. Through placing in juxtaposition specific hierarchical systems from the inorganic, organic, conceptual, and artifact worlds, it was hoped to gain insight into the problems of levels, parts and wholes, and the origin of the various species of hierarchical structures.

For purposes of the symposium, the terms "hierarchical structure" and "hierarchy" were taken generally to mean *a set of ordered levels*. Whereas a more orthodox definition of "hierarchy" requires a *governing-governed* relation between levels, this attribute was intended only when specified. It was felt that this symposium, the first built around *hierarchy* as a unifying theme, should explore rather than define. Consequently, it was decided to postpone sharpening of terminology until the full variety of meanings given to the term "hierarchy" could be assimilated. For this reason, a standard terminology is not used throughout this book. However, this causes little confusion, since most of the authors are careful to amplify the meaning of the terms they introduce.

Beyond the questions of definition and classification, several basic problems concerning hierarchical structures were raised: do some or all of the hierarchies we discern in nature possess objective reality or are they subjective patterns derivative from the human mode of perception and conception; if levels are structural realities, can the origin of inorganic hierarchies be explained in terms of known physical laws without improbable *ad hoc* initial conditions; can a reductionist explanation be found for the levels of biological organization; do the similarities between the various species of hierarchies and level structures imply a structural commonality that is meaningful on some level of abstraction; if so, can the existence of such structures be derived from some fundamental meta-principle – informational, combinatorial, topological, or whatever. These and other relevant questions were approached during the symposium along a path leading from the specific to the general. While few answers were forthcoming, the new differentiations and syntheses developed by the participants gave the general feeling that the proceedings produced much of value to the embryo subject of hierarchical structure.

The material generated for and by this symposium on hiearchies appears here in the form of the papers invited to be read at the symposium and notes based on the ensuing discussions. Instead of publishing the verbatim discussion following the presentation of each paper, the editors invited those making substantial contributions to the discussion to prepare brief formal notes. These have been included at the end of each topical part. In addition to the papers and notes, a selected annotated bibliography coverning a sizeable portion of the existing literature on hierarchies has been included.

The editors hope that this volume will provide a useful overview for those who have an interest in the problems of levels, hierarchies, parts and wholes, reductionism, holisim, and general systems whatever the area of application. Finally, we also hope that the synoptic material covered in this book will further erode disciplinary overspecialization and lead to the creation of a new fraternity of communication.

□

Acknowledgements

The editors want to offer their sincere thanks to Dr. Lewis Larmore and the McDonnell Douglas Corporation for support of the symposium and this publication, and to Dean Ralph Gerard for the support of the University of California, Irvine. Cordial thanks are also given to the session chairmen, Victor Azgepetian, Ralph Gerard, Donald Menzel, and Paul Shlichta and to Mario Bunge, chairman of the post session workshop, for their personal contribution and for promoting lively and penetrating exchanges. We extend special thanks to Joe Gauger, Jeanne Gray, and Thornton Page for their assistance in logistics and coordination. We acknowledge our indebtedness to the Charles Eames Staff for their film, *The Powers of Ten*. We are pleased to acknowledge the splendid cooperation and support of Jim Eastman and his enthusiastic staff of McDonnell Douglas Astronautics Company – Western Division, with special gratitude to Robert Fisher and Robin Simpson in helping prepare the camera ready copy from which this volume was produced. Finally we want to thank the authors and all other participants who by their contributions have opened doors on the understanding of hierarchical structures.

□

Part I
Hierarchy in Concept

As humans, we belong to that component of nature given to organizing and structuring. We not only physically organize ourselves and our environment, but we also organize our perceptions of the physical world into abstract structures. When we project these abstractions back onto the physical world, their usefulness leads us to surmize that they reflect to some degree a structure possessing independent existence.

The human method of conceptualization discriminates entities, relations, processes, and levels as the ingredients of structure. The scientific study of structures and systems – natural, artificial, or abstract – has primarily been concerned with entities, relations, and processes largely ignoring the roles of levels and hierarchies because of their complexity. However, Lancelot Law Whyte in documenting the history of thought concerning hierarchical structure from Plato and Aristotle to the twentieth century establishes in the first paper of Part I the thesis that the study of hierarchies has now come of age. As we engage in the study and creation of structures and systems of larger complexity, the essential role of levels and hierarchies in complex situations is increasingly realized as is evidenced by the current expansion in the literature of many disciplines which treats this subject.

Mario Bunge in the second paper suggests some useful working definitions for the concepts of *hierarchy*, *level structure* and *level*. Bunge's basic definition is that of a level structure which is taken as a family of sets, having a relation between the sets that represents emergence or a novelty generating process. The emergence relation that holds *between* the sets does not hold *within* the set whose elements are taken to be qualitatively homogeneous systems. Bunge defines a *level* as a set having these properties and belonging to a level structure. If, instead of the emergence relation between sets or levels, there is an anti-symmetric dominance relation, the level structure is a *hierarchy*. Bunge developes the ontological and epistemological aspects of structures with these properties.

M. D. Mesarovic and D. Macko consider three concepts of hierarchy: (i) Hierarchies of description whose levels (called strata) are of description or abstraction; (ii) Multi-layer decision systems whose levels (called layers) are sequential events in a decision making process; and (iii) Multi-level multi-goal systems whose levels (called levels) are those of an organizational hierarchy. In the first concept there is autonomy of language and principle on each strata, but an asymmetrical interdependence of function between different strata. In the second concept each layer specifies constraints affecting the operation of subsequent layers. In the third concept interacting subsystems are structured to develop capability for tasks beyond the capacity of individual units.

Amplification of the discussion of the concepts of hierarchy is contained in four brief notes. Lancelot Law Whyte raises five primary questions pertaining to the properties and origin of structural hierarchies. Robert Rosen stresses that the interaction between the functional levels in a biological hierarchical system are reciprocal relations and not unidirectional, although the possibility of a pair of "bossing" relations operating in opposite directions exists. Albert Wilson describes the role of topological and temporal closure in defining levels in inorganic hierarchies. Marjorie Grene, in searching for a unifying concept in the different usages of hierarchy, suggests that levels are always governed by some form of ordering relation.

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Part II

Inorganic Hierarchical Structures

Two primary hierarchies that occur in the inorganic world are the hierarchy whose primary bonding derives from electrical forces and whose levels are molecules, crystals, and crystalline aggregates, and the hierarchy whose primary bonding derives from gravitational force and whose levels are stars, galaxies and clusters of galaxies. In the first paper of Part II, Cyril Smith discusses the levels of organization in the first hierarchy – the super-atomic world. The existence of levels depends on repeatably local interactions and connections among which discontinuities eventually occur to give rise to larger groupings. But since each level is what the observer sees at certain resolutions, Smith considers that the structures that emerge on a larger scale may be partly illusory. An assembly of elements will not form a coherent aggregate unless the parts interact in such a way as to modify their internal structure and energy. The interfaces between entities at various levels may coincide with actual physical discontinuities or they may be only surfaces at which the gradient of some property changes sign. Smith concludes with six general principles that appear to hold for many classes of hierarchical structures.

The other three papers discuss the large scale inorganic hierarchy, the domain of self-gravitating bodies. E. R. Harrison and Michele Kaufman consider the problem of origin of the levels of structure that are observed in the universe. Harrison reviews modern approaches to cosmology through gravitational theories treating "smoothed" universes, in which the various cosmic sub-structures are replaced by a hypothetical uniform perfect fluid whose density and motion conform to the averages for observed bodies. The difficulties of recapturing the observed structure from a homogeneous fluid in the time span of the accepted age of the universe are developed. Kaufman reports the explanation that she and Layzer derive for the origin of the various levels of self-gravitating bodies based on the role of electrostatic forces in a cold, compact, primordial universe. While their model is successful in producing a sequence of gravitating bodies, it runs into difficulty with the observation of the 3°K background radiation. Albert Wilson views the cosmic hierarchy as a structure he calls a modular hierarchy, i.e., a set of levels each characterized by an aggregate or module that is in turn decomposable into sub-modules associated with the next lower level and grouped into super-modules associated with the next higher level. He shows that among cosmic bodies, the modular levels may be characterized by a density parameter that appears to assume only discrete values. For gravitating systems, density parametization is equivalent to a time parametization, implying that each modular level may be associated with a discrete characteristic time.

In the two notes that follow, Robert Williams illustrates special cases of Euler's Law by aggregating geometrical polyhedra and demonstrates how dimension ($n + 1$) emerges from the operation of combining entities of n dimension. Paul Shlichta illustrates the existence of overlap in three examples from inorganic hierarchies – symmetry groups, polyhedra and crystal structures – and raises the question of using "tree-like" diagrams to study hierarchical structures.

Part III

Organic Hierarchical Structures

It is in the organic world that the concepts of level and hierarchy acquire added dimensionality and greater subtlety. While the structure of the inorganic world is such that models that do not employ levels, hierarchies, or vertical relations have been extremely successful in explaining and predicting phenomena, the basic properties of the organic world require models whose construction quickly leads into matters of levels and their relation to one another. Part III begins with an historical account by Chauncey Leake of the development of awareness of organic levels in man and nature. Starting with the differentiation between the individual and the group in prehistoric times, Leake traces the discovery of organs, cells, and sub-cellular units, leading to the recognition in modern times of the molecular level at the lower end of the organic hierarchy and the ecological level at the upper end.

Howard Pattee discusses some reductionist-holist aspects of hierarchical models of bio-organisms. From the perspective of the upper levels, controling constraints are taken for granted and the problem is to explain how the organism works. From the perspective of the lower levels with elements that obey the laws of physics, the problem is to show how the constraints that control the elements arise from a collection of elements and generate an integrated function or purpose. Pattee distinguishes structural, functional, and descriptive hierarchies and concludes that all hierarchical organizations require a balance between the number of degrees of freedom of their elements, the fixed constraints that function as a record, and flexible constraints that control.

Robert Rosen joins Pattee in pointing out that appropriate to every level of a hierarchy, there is a different system description or language. He goes on to formulate the problem of levels as the phenomenological specifying of macrosystem behavior in terms of suitable observables, the specification of microsystem dynamics, and the development of a formalism (like statistical mechanics) connecting the two. He concludes that hierarchical structures cannot be based solely on automata-theoretic descriptions since the mechanism to generate higher level descriptions is explicitly abstracted out of the description at the outset.

John Platt illustrates the importance of the concept of boundaries to hierarchical systems and develops several of their functional attributes: boundary coincidence for different properties making a "thing" perceivable; gradients and flows being parallel or perpendicular to boundaries; ratios of interconnections to gates (spatial and temporal, distributed or concentrated) limiting the ability of the system to sense and respond to the external world.

In summarizing the symposium, Ralph Gerard reemphasized the roles of boundaries and edges, gradients, integration, and function. The more highly integrated an organism, the larger are the forces operating down with respect to the forces acting upward. Evolution of systems is toward higher integration with an increase in the number of levels. With structure (the system component constant in time) and function (the reversible behavior) and evolution (the irreversable behavior) there is an evolutionary spiral or helix of structure determining function and function producing structure. Gerard also feels it is premature to differentiate "system" and "hierarchy", but it is most important to order by origin as well as to order by function. In the note that follows, Herbert Gutman outlines the argument that an understanding of the genesis of hierarchies in living systems must proceed from a fundamental clarification of the relationship of structure to function and of organic wholes to their parts.

□

Part IV
Hierarchy in Artifact

No longer is the natural order the sole source of *basic* scientific knowledge. Sophisticated human artifacts such as computers, communication networks, and space systems have joined molecules, stars, and bio-organisms as fruitful objects of study for the discovery of fundamental scientific relations and principles. The study of complex creations of technology frequently produces basic knowledge beyond that used in their design. For example, levels and hierarchies are often designed into man-made systems and organizations, but sometimes they emerge unplanned, as in the discovery that the flow of traffic through the Hudson River tunnels could be increased through the platooning of vehicles.

The examples of important and interesting hierarchical structures in social and technological systems could dominate this volume if they were to be adequately described. But space permits the selection of only two. In the first paper of Part IV, Fred Tonge discusses some of the hierarchies encountered in computer technology and information processing such as; file structures and the organization of computer memories; control hierarchies employing executive programs and user programs with sub-routines and sub-sub-routines. The structure of a program frequently provides an excellent analogue to administrative organization. The forms of general problem solving programs parallel decentralized, centralized, bureaucratic and roving managerial strategies and give insights into the advantages and limitations of each approach.

In the second paper, Robert Lucky discusses the problem of minimizing errors in data transmission codes. The concept of hierarchy enters through the use of concatenated codes or codes within codes to provide cross checks on the accuracy of transmitted data. The mathematical development possible in this subject offers one of the few quantitative approaches to a theory of hierarchical structure available at present.

In the notes that follow, Magoroh Maruyama discusses the levels that occur between the perception of patterns and patterns in social events. Bill Wells comments on the necessity to take into account the level structure of society in the dynamics of social change. In reminding us that the problem of how many parts come to be a unified whole is a 5,000 year old problem, Ronald Jones emphasizes the cultural significance of the study of structural hierarchies. ◘

Epilogue

A characteristic of the current renaissance in epistemology is the intellectual thrust toward a more comprehensive and interrelated picture of nature, man, and society. In adopting a broad multi-disciplinary approach to the theme of *hierarchy,* this symposium explored what was felt to be one promising path toward such a coordinated view. In retrospect, the basic question relevant to this goal is whether the apparent structural analogies, all labeled with the term hierarchy, do indeed converge toward a single representation. The point of departure of the symposium was the focus on structure and function as essence, with atoms, cells, stars, and codes taken only as alternative mediums for the expression of the essence. While the basic question is what, if any, properties of hierarchies are medium independent, an important corollary question is what analogous structural and behavioral patterns display confluences sufficient to allow the formulation of precise propositions valid over the set of specific hierarchies entering the confluence.

In answer to these basic questions, we may cite such propositions as: *A stable aggregate will form only if its elements interact in such a way as to modify their internal structure* and *Hierarchical organization requires a balance between the number of degrees of freedom of its elements, the number of fixed constraints which function as a record, and the number of flexible constraints which program its evolution.* These propositions are nearly medium independent and indicate that there do exist hierarchical concepts of broad applicability. The extent to which they may be precisely formulated remains to be seen. Less broadly, the symposium exhibited evidence that analogies between hierarchical phenomena within certain clusters of disciplines, especially the bio-social-computer cluster, took on greater richness indicating that more intensive and detailed study within such a confluence should prove fruitful.

A second predication of the symposium's multi-disciplinary approach to hierarchy was the usefulness of analogy, however tenuous. While analogies range from those rich and deep enough to become the basis for productive and predictive theories, to those too superficial to provide even specious illustrations; whatever their validity, analogies constitute a basic mode of epistemological exploration. Through the simultaneous consideration of two or more analogous specifics, we are enabled both to parameterize and generalize. Hence, in the initial stages of investigating any specific hierarchy, bold and broad use of the analogies between many hierarchies is productive.

We conclude that the broad multi-disciplinary approach to hierarchy should be continued in the future. A too rapid narrowing of the jointly considered subject area would remove opportunities to stimulate our intuitions concerning whatever principles of unification that may reside in the alternate realizations of common structural and functional organization. While improving the precision of formulation is always an important goal in science, it must not be confused with narrowing the domain of discourse. But ultimately the nature of the relation between the specificity of formulation and the extent of the domain of discourse is itself a problem of hierarchy.

Feb 1971
To Eng + Sci.

Discovery, Invention, Research, Through the Morphological Approach.
Fritz Zwicky. Macmillan, New York, 1969. xii + 276pp $6.95.

This is a translation of the author's Entdecken, Erfinden, Forschen im morphologischen Weltbild, first published by Droemer Knaur in 1966. It makes available in english the most comprehensive description to date of many of Zwicky's highly original epistemological ideas including the methodologies of negation and construction, systematic field coverage, and the morphological box, but only cursorily mentions Zwicky's theory of marks. The several types of morphological analysis are developed with illustrations that come mostly from Zwicky's own specialties, but since these are many, there is something for almost everyone.

The reactions to Zwicky's attempts to popularize the morphological method thirty years ago were highly polarized. On the one hand morphology was regarded as an almost tautological way of thinking that every rational person used but did not bother to formalize. On the other hand, morphology was considered to be a formalization of but a sub-set of the total analytical process that Zwicky used to make his inventions and discoveries. Unless one were equipped with an insightful intuition, deep knowledge in several specialties and broad general knowledge, morphology could not be made to work. In other words, in addition to the formal steps given by Zwicky for the morphological process, the step "first, become a genius" should be added. But Zwicky feels everyone <u>is</u> a genius and therefore the morphological method could be used by anyone.

His faith in the intellects of his fellow men may yet prove warranted. Recently the morphological method has been discovered by forecasters and long range planners and is being fruitfully applied in many problem areas. Most recent texts on forecasting include chapters describing the use of Zwicky's morphological matrices in futuribles. With increasing evidence that morphology is a useful tool in many hands, Discovery, Invention, Research should be read by all who anticipate they might have a problem of some sort to solve in the next few years. The book may be read eclectically with profit by those wishing an introduction to morphological methods; or may be read in its entirety with enjoyment by those who would like a behind-the-scenes glimpse into the thinking processes and personality of one of the 20th century's most original thinkers.

Albert Wilson

February, 1971.