MATHEMATICS
BOOK 3

HAPPINESS

IS

MATHEMATICS

MATHEMATICS

BOOK THREE

MATHEMATICS

BOOK III

DISCRETE MATHEMATICS

TWO DIMENSIONAL CONFIGURATIONS

ZERO DIMENSION

- NATURAL NUMBERS

ONE DIMENSION

- SERIES, SEQUENCES
- CONTINUED FRACTIONS

TWO DIMENSIONAL

- ARRAYS
- YANGHUIS
- RHOMBOIDS
- MATRICES
- ZZ

ALSO SEE MATH BOOK II

AND N.B. MODULARIZATION

FRACTALS

ARRAYS

SPECIES OF ARRAYS

TWO DIMENSIONAL NUMBER PATTENS

MATRICES

DETERMINANTS

∠'s / . . .

SUM AND DIFFERENCE RHOMBOIDS LIST . . .

Δ's \ . . .

SET PARTITIONS: THE BELL TRIANGLE and "STEM CELLS"

YANGHUIS

GENERALIZATIONS OF PASCAL'S TRIANGLE

ARRAYS

NUMBER ~~MATRICES~~ ARRAYS

The first number ~~matrix~~ array is a two dimensional ~~matrix~~ set whose entries are the values, $\mathbf{W}_{N,S}$,where,

$$\text{1)} \qquad \mathbf{W}_{N,S} = \frac{\mathbf{N!}}{\mathbf{S!(N-S)!}}$$

$\mathbf{W}_{N,S}$ in Equation 1) is equal to the number of combinations of N distinct things taken S at a time. Or is the number of possible subgroups of S members in a group of N members.
If the value of 0! is taken as = 1, and the value of W as = 0 if S > N, then the matrix whose rows are the values of S and whose columns are the values of N is:

N =	1	2	3	4	5	6	7	8	9	10
S=1	1	2	3	4	5	6	7	8	9	10
S=2	0	1	3	6	10	15	21	28	36	45
S=3	0	0	1	4	10	20	35	56	84	120
S=4	0	0	0	1	5	15	35	70	126	210
S=5	0	0	0	0	1	6	21	56	126	252
S=6	0	0	0	0	0	1	7	28	84	210
S=7	0	0	0	0	0	0	1	8	36	120
S=8	0	0	0	0	0	0	0	1	9	45
S=9	0	0	0	0	0	0	0	0	1	10
S=10	0	0	0	0	0	0	0	0	0	1

THIS IS A
PASCAL
TRIANGLE

The number of **links** between **pairs** of objects in a group of N is given by the row S=2.
The number on **networks** involving X objects in a group of N is given by the row S=X.

THE HORIZONTAL SLICES

Note that the numbers in each row, S, are the differences between the numbers in the row S - 1 immediately below. That is, ΔS = S-1 for all S. And that every number $\mathbf{W}_{N,S}$ is the sum of the numbers in row S - 1 up to N -1. That is, the sum, $\mathbf{\Sigma}_{N,S}$ of the numbers in horizontal row S up to and including the value in column N, is given by equation 2)

$$\text{2)} \qquad \Sigma_{N,S} = \frac{\mathbf{(N+1)!}}{\mathbf{(S+1)!(N-S)!}}$$

Since the diagonal down-to-the-right slices [\\\\] are the same as the horizontal slices, equation 2) also applies to their sums.

THE VERTICAL SLICES

If single member sub-groups are included, the total number of sub-groups of all sizes, T, in a group of N members will be given by the sums of the vertical columns in the above table.

N=	1	2	3	4	5	6	7	8	9	10
T=	1	3	7	15	31	63	127	255	511	1023

where the values of T are given by the formula:

3) $$\mathbf{T = 2^N - 1}$$

Note that T is equal to the total number of **networks** of all sizes possible in a group of size N.

The sum of all the numbers in the matrix can be calculated by adding the numbers in the above T sequence. The sums of the values of T to the Nth column are given by,

4) $$\mathbf{\Sigma T_N = 2^{N+1} - (N+2) = 2T_N - N}$$

THE DIAGONAL UP-TO-THE-RIGHT SLICES [////]

The first row in the next table gives the sums, Σ[//], of the numbers in the up-to-the-right slices of the first ~~matrix~~ array. The second row, ΔΣ[//], gives their differences.

Σ[//] = ΣF	1	2	4	7	12	20	33	54	88	143
ΔΣ[//] = F		1	2	3	5	8	13	21	34	55

Note that the numbers in the ΔΣ[//] row are the **Fibonacci Numbers**, F.
The second ~~matrix~~ array will consist of a row of Fibonacci numbers, F, together with other rows giving sums and differences.

N	1	2	3	4	5	6	7	8	9	10	11
Σ_3F	1	4	11	25	51	97	176	309	530	894	1490
Σ_2F	1	3	7	14	26	46	79	133	221	364	596
ΣF	1	2	4	7	12	20	33	54	88	143	232
F	1	1	2	3	5	8	13	21	34	55	89
ΔF	0	1	1	2	3	5	8	13	21	34	55
Δ_2F	1	0	1	1	2	3	5	8	13	21	34
Δ_3F	-1	1	0	1	1	2	3	5	8	13	21
Δ_4F	2	-1	1	0	1	1	2	3	5	8	13
Δ_5F	-3	2	-1	1	0	1	1	2	3	5	8
Δ_6F	5	-3	2	-1	1	0	1	1	2	3	5
Δ_7F	-8	5	-3	2	-1	1	0	1	1	2	3

The formulae for the values of the terms in the Fibonacci ~~matrix~~ array are as follows:

F $f(n) = f(n-2)+f(n-1)$
ΔF $\Delta F(n) = F(n-1)$
$\Delta_2 F$ $\Delta_2 F(n)=\Delta F(n-1)=F(n-2)$
. . . .
$\Delta_k F$ $\Delta_k F(n)=F(n-k)$

ΣF $f(n) = f(n-2)+f(n-1) + 1$
$\Sigma_2 F$ $f(n) = f(n-2)+f(n-1) + n$
$\Sigma_3 F$ $f(n) = f(n-2)+f(n-1) + W_{n,2}$
$\Sigma_4 F$ $f(n) = f(n-2)+f(n-1) + W_{n,3}$

Formulae for sequences are of several types:

Type in which the Nth term involves only N e.g. Equations 3) and 4) EXPLICIT
Type in which the nth term is defined in terms of proceeders e.g. Equation for F RECURSION
Mixtures of proceeders and term number e.g. Equation for $\Sigma_2 F$

PRIMES

IN THE NEGATIVE STANDARD
PASCAL
ALL ONES
AT ROW 8, 16, 32
2, 4,

IN THE NEGATIVE PRIME PASCAL
ALL PRIMES AT 8, 16,
2, 4,

A
PRIME ~~YANGHUI~~
~~OF PRIME~~
YANQHVI

Σ-Δ
~~ARRAYS~~

Σ

1
2 2
3 4 3
5 7 7 5
7 12 14 12 7
11 19 26 26 19 11
13 36 45 52 45 36 13
17 49 81 97 97 81 49 17
19 66 130 178 194
23 85
29 108

Δ

1 — 1
2 2 — 4
3 0 3 — 6
all prime 5 3 3 5 — 16
7 2 0 2 7 — 18
11 5 2 2 5 11 — 36
13 6 3 0 3 6 13 — 44
17 7 3 3 3 3 7 17 — all prime 8 — 60
19 10 4 0 0 0 4 10 19 — 66
23 9 6 4 0 0 4 6 9 23 — 84
29 14 3 2 4 0 4 2 3 14 29 — 104
31 15 11 1 2 4 4 2 1 11 15 31 — 128
37 16 4 10 1 2 0 2 1 10 4 16 37 — 140
41 21 12 6 9 1 2 2 1 9 6 12 21 41 — 184
43 20 9 6 3 8 1 0 1 8 3 6 9 20 43 — 180
47 23 11 3 3 5 7 1 1 7 5 3 3 11 23 47 — all prime 16 — 200

1-2-3 ~~YANGHUI~~

Σ

37
94
106
[illegible]

1
2 2
3 4 3
4 7 7 4
5 11 14 11 5
6 16 25 25 16 6
7 22 41 50 41 22 7
8 29 63 91 91 63 29 8
9 37 92 154 182 154 92 37 9

Δ

1
2 2
3 0 3
4 3 3 4
5 1 0 1 5
6 4 1 1 4 6
7 2 3 0 3 2 7
8 5 1 3 3 1 5 8
9 3 4 2 0 2 4 3 9
10 6 1 2 2 2 2 1 6 10
11 4 5 1 0 0 0 1 5 4 11
12 7 1 4 1 0 0 1 4 1 7 12
13 5 6 3 3 1 0 1 3 3 6 5 13
14 8 1 3 0 2 1 1 2 0 3 1 8 14
15 6 7 2 3 2 1 0 1 2 3 2 7 6 15
16 9 1 5 1 1 1 1 1 1 1 1 5 1 9 16
17 7 8 4 4 0 0 0 0 0 0 0 4 4 8 7 17
18 10 1 4 0 4 0 0 0 0 0 0 4 0 4 1 10 18
19 8 9 3 4 4 4 0 0 0 0 0 4 4 4 3 9 8 19
20 11 1 6 1 0 0 4 0 0 0 0 4 0 0 1 6 1 11 20
21 9 10 5 5 1 0 4 4 0 0 0 4 4 0 1 5 5 10 9 21
22 12 1 5 0 4 1 4 0 4 0 0 4 0 4 1 4 0 5 1 12 22
23 10 11 4 5 4 3 3 4 4 4 0 4 4 4 3 3 4 5 4 11 10 23
24 13 6 1 1 1 0 1 0 0 4 4 0 0 1 0 1 1 1 7 1 13 24
25 11 12 6 6 0 0 1 1 1 0 4 0 4 0 1 1 1 0 1 6 6 12 11 25

Σ_2 4
Σ_4 6 16
Σ_8 26 60
Σ_{16} 106 200

4 × (4 − 0) = 46
4 × (16 − 1) = 60
4 × (60 − 10) = 200

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1

1
1
2
3 Fibonacci Numbers
5
8
13
21

PRIME YANGHUI (Continued)

8
all primes — 47 23 11 3 3 5 7 1 1 7 5 3 3 11 23 47 —— #16 200 ——
53 24 12 8 0 2 2 6 0 6 2 2 0 8 12 24 53 —— 214 1/2 2 = 107 odd
59 29 12 4 8 2 0 4 6 6 4 0 2 8 4 12 29 59 —— 248
61 30 17 8 4 6 2 4 2 0 2 4 2 6 4 8 17 30 61 —— 268
67 31 13 9 4 2 4 2 2 2 2 2 2 4 2 4 9 13 31 67 —— 272
71 36 18 4 5 2 2 2 0 0 0 0 0 2 2 2 5 4 18 36 71 —— 280
73 35 18 14 1 3 0 0 2 0 0 0 0 2 0 0 3 1 14 18 35 73 —— 292
79 38 17 4 13 2 3 0 2 2 0 0 0 0 2 2 0 3 2 13 4 17 38 79 —— 320
83 41 21 13 9 11 1 3 2 0 2 0 0 2 0 2 3 1 11 9 13 21 41 83 —— 372 — #24
89 42 20 8 4 2 10 2 4 2 2 2 0 2 2 2 1 2 10 2 4 8 20 42 89 —— 368
97 47 22 12 4 2 8 8 1 1 0 0 2 2 0 0 1 1 8 8 2 4 12 22 47 97 — 408
101 50 25 10 8 2 6 0 7 0 1 0 2 0 2 0 1 0 7 0 6 2 8 10 25 50 101 — 424 27

continue to 32

~~NEG~~ ARRAY OF SUMS from Prime Yanghui

135
47 88
15 31 56
5 10 22 34
1 4 6 16 18 36 44 60 66 84 104 128 140 184 180 200 214 248 268 272 280 292 320 372
3 2 10 2 18 8 16 6 18 20 24 12 44 -4 20 14 34 20 4 8 12 28 52
1 8 8 16 10 8 10 12 2 4 12 32 48 24 6 20 14 16 4 4 #15 drop 24
7 0 8 6 2 2 2 10 2 8 20 16 24 18 14 6 2 12 0 #25 drop
7 8 2 4 0 0 8 8 6 12 4 8 6 4 8 4 10 12 12 4 24 16 4
1 6 2 4 0 8 0 2 6 8 4 2 2 4 4 6 2 0 8 20 8 12
5 4 2 4 8 4 2 4 2 4 2 0 2 0 2 4 2 8 12 12 4
1 2 2 4 0 6 2 2 2 2 2 2 2 2 2 2 6 4 0 8
1 0 2 4 6 4 0 0 0 0 0 0 0 0 0 4 2 4 8
1 2 2 2 2 4 0 0 0 0 0 0 0 0 4 2 2 4
1 0 0 0 2 4 0 0 0 0 0 0 0 4 2 0 2
1 0 0 2 2 4 0 0 0 0 0 0 4 2 2 2
1 0 2 0 2 4 0 0 0 0 0 4 2 0 0
1 2 2 2 2 4 0 0 0 0 4 2 2 0
1 0 0 0 2 4 0 0 0 4 2 0 2
1 0 0 2 2 4 0 0 4 2 2 2 280 292 320 372 368 408 424
1 0 2 0 2 4 0 4 2 8 12 28 52 -4 40 16
1 2 2 2 2 4 4 4 4 16 24 56 44 24
1 0 0 0 2<0 12 0 12 8 32 16 20
1 0 0 2 2 12 12 4 24 16 4

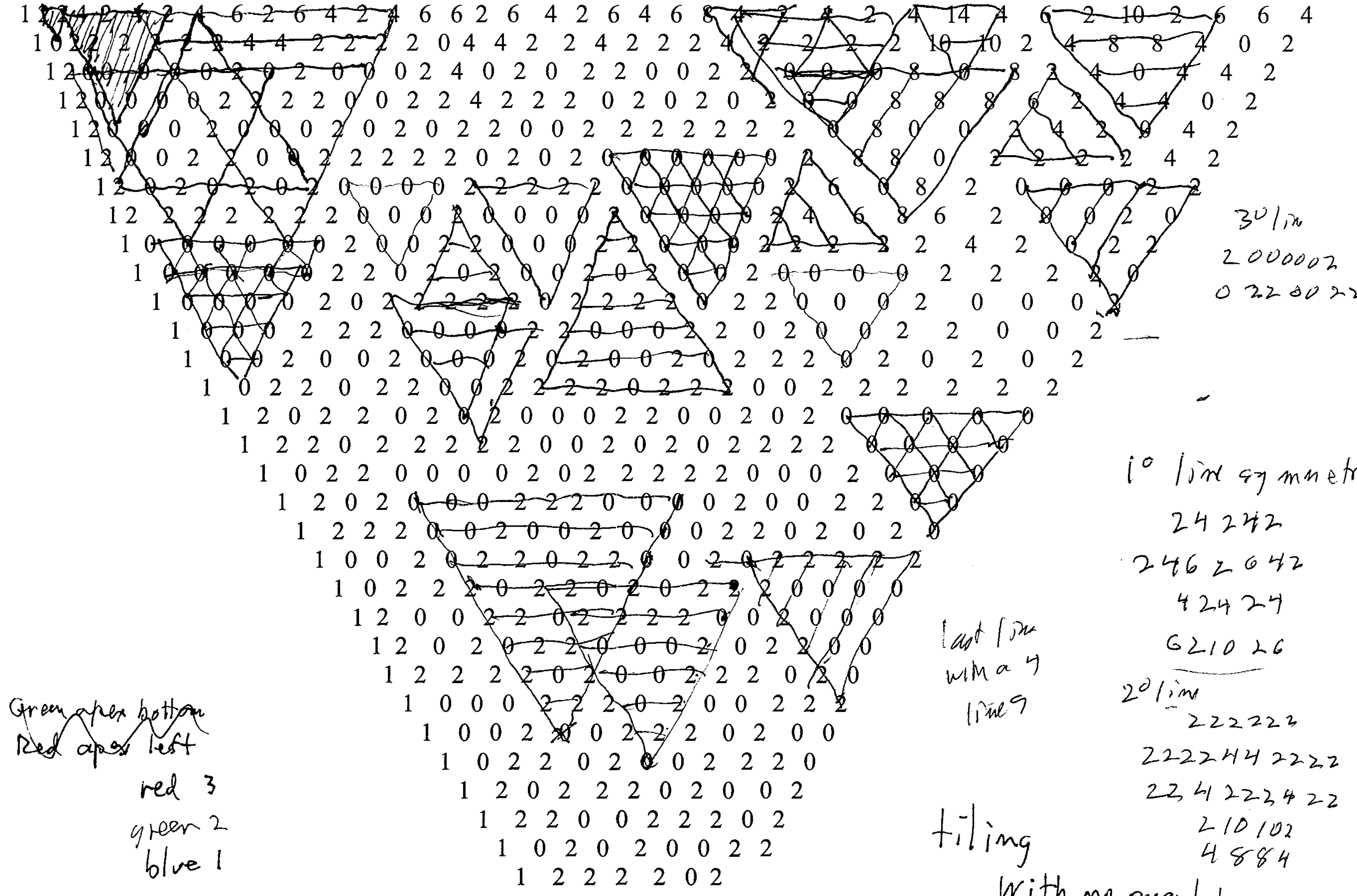
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1

Counting the apex 1 as row zero, the horizontal rows are designated by R. The ////diagonals are designated by D beginning with the left set of 1's as D = 0. The left–3, down–1 diagonals are designated by F. Each entry in the triangle $W[R,D] = R!/D!(R-D)!$ The sum of the entries along diagonal D to row R is $(R+1)!/(D+1)!(R-D)!$ The entries in a given row, R, are the coefficients of the binomial expansion, $(a + b)^{R.}$ The sum of the numbers in each row is $2^{R.}$ The total value of all the numbers in the first H rows is $2^{H} - 1$ The sums of the entries in F diagonals are the Fibonacci numbers. Symmetry allows \\\\ diagonals to be used as //// diagonals. Same with F.

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4
1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2
1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2
1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2
1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2
1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2
1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2
1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0
1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2
1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0
1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2
1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2
1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2
1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2
1 2 0 2 2 0 2 0 2 0 0 0 2 2 0 0 2 0 2 0 0 0 0 0
1 2 2 0 2 2 2 2 2 0 0 2 0 2 0 2 2 2 2 0 0 0 0
1 0 2 2 0 0 0 0 2 0 2 2 2 2 2 0 0 0 2 0 0 0
1 2 0 2 0 0 0 2 2 2 0 0 0 0 2 0 0 2 2 0 0
1 2 2 2 0 0 2 0 0 2 0 0 0 2 2 0 2 0 2 0
1 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 2 2 2
1 0 2 2 2 0 2 2 0 2 0 2 2 2 0 0 0 0
1 2 0 0 2 2 0 2 2 2 2 0 0 2 0 0 0
1 2 0 2 0 2 2 0 0 0 2 0 2 2 0 0
1 2 2 2 2 0 2 0 0 2 2 2 0 2 0
1 0 0 0 2 2 2 0 2 0 0 2 2 2
1 0 0 2 0 0 2 2 2 0 2 0 0
1 0 2 2 0 2 0 0 2 2 2 0
1 2 0 2 2 2 0 2 0 0 2
1 2 2 0 0 2 2 2 0 2
1 0 2 0 2 0 0 2 2
1 2 2 2 2 0 2 0
1 0 0 0 2 2 2
1 0 0 2 0 0
1 0 2 2 0
1 2 0 2
1 2 2
1 0
1

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4
1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2
1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2
1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2
1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2
1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2
1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2
1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0
1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2
1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0
1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2
1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2
1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2
1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2
1 2 0 2 2 0 2 0 2 0 0 0 2 2 0 0 2 0 2 0 0 0 0 0
1 2 2 0 2 2 2 2 2 0 0 2 0 2 0 2 2 2 2 0 0 0 0
1 0 2 2 0 0 0 0 2 0 2 2 2 2 2 0 0 0 2 0 0 0
1 2 0 2 0 0 0 2 2 2 0 0 0 0 2 0 0 2 2 0 0
1 2 2 2 0 0 2 0 0 2 0 0 0 2 2 0 2 0 2 0
1 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 2 2 2
1 0 2 2 2 0 2 2 0 2 0 2 2 2 0 0 0 0
1 2 0 0 2 2 0 2 2 2 2 0 0 2 0 0 0
1 2 0 2 0 2 2 0 0 0 2 0 2 2 0 0
1 2 2 2 2 0 2 0 0 2 2 2 0 2 0
1 0 0 0 2 2 2 0 2 0 0 2 2 2
1 0 0 2 0 0 2 2 2 0 2 0 0
1 0 2 2 0 2 0 0 2 2 2 0
1 2 0 2 2 2 0 2 0 0 2
1 2 2 0 0 2 2 2 0 2
1 0 2 0 2 0 0 2 2
1 2 2 2 2 0 2 0
1 0 0 0 2 2 2
1 0 0 2 0 0
1 0 2 2 0
1 2 0 2
1 2 2
1 0
1

PRIME1.WPD June 1, 2004



```
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4
1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2
1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2
1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2
1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2
1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2
1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2
1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0
1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2
1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0
1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2
1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2
1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2
1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2
1 2 0 2 2 0 2 0 2 0 0 0 2 2 0 0 2 0 2 0 0 0 0 0
1 2 2 0 2 2 2 2 2 0 0 2 0 2 0 2 2 2 2 0 0 0 0
1 0 2 2 0 0 0 0 2 0 2 2 2 2 2 0 0 0 2 0 0 0
1 2 0 2 0 0 0 2 2 2 0 0 0 0 2 0 0 2 2 0 0
1 2 2 2 0 0 2 0 0 2 0 0 0 2 2 0 2 0 2 0
1 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 2 2 2
1 0 2 2 2 0 2 2 0 2 0 2 2 2 0 0 0 0
1 2 0 0 2 2 0 2 2 2 2 0 0 2 0 0 0
1 2 0 2 0 2 2 0 0 0 2 0 2 2 0 0
1 2 2 2 2 0 2 0 0 2 2 2 0 2 0
1 0 0 0 2 2 2 0 2 0 0 2 2 2
1 0 0 2 0 0 2 2 2 0 2 0 0
1 0 2 2 0 2 0 0 2 2 2 0
1 2 0 2 2 2 0 2 0 0 2
1 2 2 0 0 2 2 2 0 2
1 0 2 0 2 0 0 2 2
1 2 2 2 2 0 2
```

3° line
2000002
02200220

1° line symmetries
24242
2462642
42424
621026

2° line
222222
2222442222
224222422
210102
4884

last line with a 4 line 9

tiling
With no overlaps

~~Green apex bottom~~
~~Red apex left~~
red 3
green 2
blue 1

PRIME1.WPD June 1, 2004

31 37

2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167

1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4

1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2

1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2

1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2

1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2

1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2

1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2

1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0

1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2

1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0

1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2

1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2

1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2

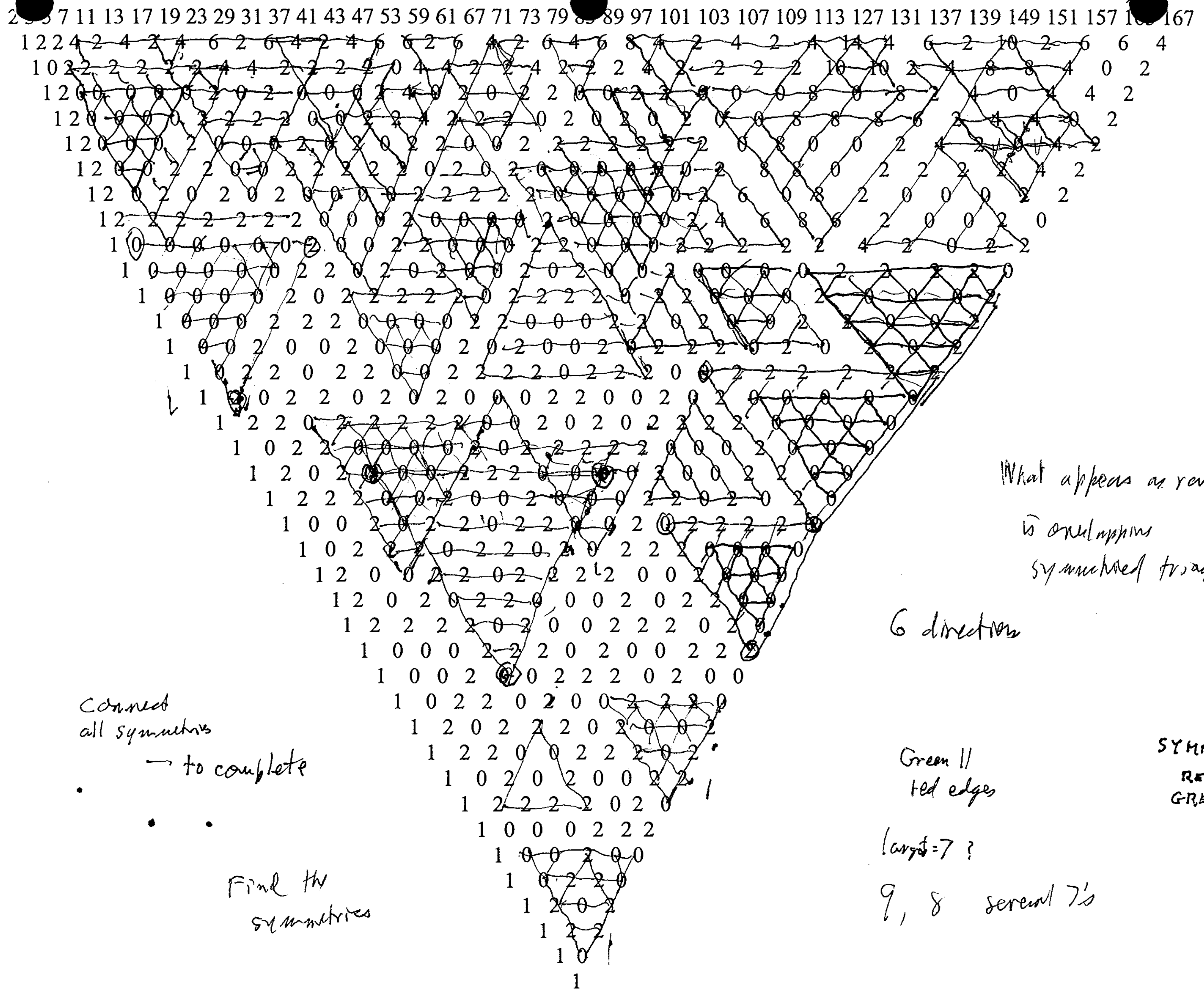
1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2

Theorem: every number belongs to
one or more symmetric triangles
order 3 or higher

Random
occurrence of
Symmetries

linear symmetries horizontal, vertical, slope
2 dim symmetries
size of symmetries # of elements
overlapping symmetries

Find the largest Δ pattern
with symmetry



```
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4
 1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2
  1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2
   1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2
    1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2
     1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2
      1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2
       1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0
        1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2
         1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0
          1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2
           1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2
            1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2
             1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2
              1 2 0 2 2 0 2 0 2 0 0 0 2 2 0 0 2 0 2 0 0 0 0 0
               1 2 2 0 2 2 2 2 2 0 0 2 0 2 0 2 2 2 2 0 0 0 0
                1 0 2 2 0 0 0 0 2 0 2 2 2 2 2 0 0 0 2 0 0 0
                 1 2 0 2 0 0 0 2 2 2 0 0 0 0 2 0 0 2 2 0 0
                  1 2 2 2 0 0 2 0 0 2 0 0 0 2 2 0 2 0 2 0
                   1 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 2 2 2
                    1 0 2 2 2 0 2 2 0 2 0 2 2 2 0 0 0 0
                     1 2 0 0 2 2 0 2 2 2 2 0 0 2 0 0 0
                      1 2 0 2 0 2 2 0 0 0 2 0 2 2 0 0
                       1 2 2 2 2 0 2 0 0 2 2 2 0 2 0
                        1 0 0 0 2 2 2 0 2 0 0 2 2 2
                         1 0 0 2 0 0 2 2 2 0 2 0 0
                          1 0 2 2 0 2 0 0 2 2 2 0
                           1 2 0 2 2 2 0 2 0 0 2
                            1 2 2 0 0 2 2 2 0 2
                             1 0 2 0 2 0 0 2 2
                              1 2 2 2 2 0 2 0
                               1 0 0 0 2 2 2
                                1 0 0 2 0 0
                                 1 0 2 2 0
                                  1 2 0 2
                                   1 2 2
                                    1 0
                                     1
```

What appears as random
is overlapping
symmetrical triangles

6 directions

Connect
all symmetries
→ to complete

Find the
symmetries

Green //
red edges

largest=7 ?

9, 8 several 7's

SYMMETRIES
RED THREE
GREEN ONE

```
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167
 1 2 2 4 2 4 2 4 6 2 6 4 2 4 6 6 2 6 4 2 6 4 6 8 4 2 4 2 4 14 4 6 2 10 2 6 6 4
  1 0 2 2 2 2 2 2 4 4 2 2 2 2 0 4 4 2 2 4 2 2 2 4 2 2 2 2 10 10 2 4 8 8 4 0 2
   1 2 0 0 0 0 0 2 0 2 0 0 0 2 4 0 2 0 2 2 0 0 2 2 0 0 0 8 0 8 2 4 0 4 4 2
    1 2 0 0 0 0 2 2 2 2 0 0 2 2 4 2 2 2 0 2 0 2 0 2 0 0 8 8 8 6 2 4 4 0 2
     1 2 0 0 0 2 0 0 0 2 0 2 0 2 2 0 0 2 2 2 2 2 2 2 0 8 0 0 2 4 2 0 4 2
      1 2 0 0 2 2 0 0 2 2 2 2 2 0 2 0 2 0 0 0 0 0 0 2 8 8 0 2 2 2 2 4 2
       1 2 0 2 0 2 0 2 0 0 0 0 2 2 2 2 2 0 0 0 0 0 2 6 0 8 2 0 0 0 2 2
        1 2 2 2 2 2 2 2 0 0 0 2 0 0 0 0 2 0 0 0 0 2 4 6 8 6 2 0 0 2 0
         1 0 0 0 0 0 0 2 0 0 2 2 0 0 0 2 2 0 0 0 2 2 2 2 2 4 2 0 2 2
          1 0 0 0 0 0 2 2 0 2 0 2 0 0 2 0 2 0 0 2 0 0 0 0 2 2 2 2 0
           1 0 0 0 0 2 0 2 2 2 2 2 0 2 2 2 2 0 2 2 0 0 0 2 0 0 0 2
            1 0 0 0 2 2 2 0 0 0 0 2 2 0 0 0 2 2 0 2 0 0 2 2 0 0 2
             1 0 0 2 0 0 2 0 0 0 2 0 2 0 0 2 0 2 2 2 0 2 0 2 0 2
              1 0 2 2 0 2 2 0 0 2 2 2 2 0 2 2 2 0 0 2 2 2 2 2 2
               1 2 0 2 2 0 2 0 2 0 0 0 2 2 0 0 2 0 2 0 0 0 0 0
                1 2 2 0 2 2 2 2 2 0 0 2 0 2 0 2 2 2 2 0 0 0 0
                 1 0 2 2 0 0 0 0 2 0 2 2 2 2 2 0 0 0 2 0 0 0
                  1 2 0 2 0 0 0 2 2 2 0 0 0 0 2 0 0 2 2 0 0
                   1 2 2 2 0 0 2 0 0 2 0 0 0 2 2 0 2 0 2 0
                    1 0 0 2 0 2 2 0 2 2 0 0 2 0 2 2 2 2 2
                     1 0 2 2 2 0 2 2 0 2 0 2 2 2 0 0 0 0
                      1 2 0 0 2 2 0 2 2 2 2 0 0 2 0 0 0
                       1 2 0 2 0 2 2 0 0 0 2 0 2 2 0 0
                        1 2 2 2 2 0 2 0 0 2 2 2 0 2 0
                         1 0 0 0 2 2 2 0 2 0 0 2 2 2
                          1 0 0 2 0 0 2 2 2 0 2 0 0
                           1 0 2 2 0 2 0 0 2 2 2 0
                            1 2 0 2 2 2 0 2 0 0 2
                             1 2 2 0 0 2 2 2 0 2
                              1 0 2 0 2 0 0 2 2
                               1 2 2 2 2 0 2 0
                                1 0 0 0 2 2 2
                                 1 0 0 2 0 0
                                  1 0 2 2 0
                                   1 2 0 2
                                    1 2 2
                                     1 0
                                      1
```

upper 2 boundry

H

3, 6, 7, 9, 4,

V 1 1 2 4 1

ZERO TRIANGLES

4 or MORE

ZERO TRIANGLES

2 or 3

2 triangles

of the form

```
   2
  2 2
 2 0 2
2 2 2 2
```

CRC Encyclopedia of Mathematics

BELL NUMBERS

$$B_{n+1} = \sum_{k=0}^{n} B_k \binom{n}{k}$$

$$e^{e^x-1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n$$

1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115973

THE BELL STEM CELL

BELL
PARTITIONING
OF SETS
See Book IV

See ~~Al~~ Also
TIME NOTE BOOK on EVOLUTION [BIO-EVOLUTION]
LAWS of CHANGE NOTE BOOK on Evolution

The Stem Cell Question:
Uniqueness of Number?
of Sequences?
of patterns?

BELL STEM CELLS

THE SYSTEMATIC GENERATION OF STRUCTURE WITH DIVERSITY FROM IDENTICAL INITIAL CONDITIONS

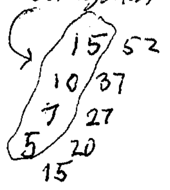
3 STEM CELLS

#1 0 1
1

#2 1 1
0

#3 1 0
1

The Bell Triangle is ~ A recursion formula
given one diagonal e.g.



Can find next # 52

1	2	3	4	5	6	7
1	2	5	15	52	203	877
1	2	4	16	64	256	
	2^1	2^2	2^4	2^6		

$2^{10} = 1024$
$2^9 = 512$
$2^8 = 256$
$2^7 = 128$
$2^6 = 64$
$2^5 = 32$
$2^4 = 16$
$2^3 = 8$
$2^2 = 4$
$2^1 = 2$
$2^0 = 1$

2^{n-4}

$2^{10} - 2^8 + 2^7 - 2^5 + 2^4 - 2^2 + 2^0 = 877$ 7
~~2~~ $+ 2^8 - 2^6 \quad + 2^4 - 2^3 - 2^0 = 203$ 6
$2^6 \quad - 2^3 - 2^2 = 52$ 5
$2^4 \quad - 2^0 = 15$ 4
$2^2 + 2^0 = 5$ 3
$2^1 = 2$ 2
$2^0 = 1$ 1

$2^0 = 1$
$2^1 = 2$
$2^2 + 2^1 = 5$
$2^4 \quad - 2^0 = 15$
$2^6 \quad - 2^4 + 2^2 = 52$
$2^8 \quad - 2^6 + 2^4 - 2^2 - 2^0 = 203$

THE BELL TRIANGLE

A Bell Triangle is constructed on a triad of three initial numbers. These three numbers must be such that the third is equal to the difference of the first two. The first two numbers are on the top line of the triangle, their difference, the third number, on the second line:

```
1 2       1 0       1 1       0 1       2  5       3  3
 1         1         0         1         3          0
```

The rules for the construction of the triangle state that the last (right most) number on the top line is brought down to the line below the last entry. The third line in the case below:

```
1  2      1  0      1  1      0  1      2  5       3  3
 1         1         0         1         3          0
  2         0         1         1         5          3
```

The line above the bottom line is then filled in by a number such that the number in the bottom line is the difference of the two numbers in the line above.

```
1  2      1  0      1  1      0  1      2  5       3  3
 1  3      1 1       0 1       1  0      3  8       0  3
  2         0         1         1         5          3
```

This process is repeated until the top line is reached:

```
1  2  5   1  0 1    1  1 0    0  1  1   2  5 13    3  3  0
 1  3      1 1       0 1       1  0      3  8       0  3
  2         0         1         1         5          3
```

Again the right most number is brought to the bottom and the process repeated:

```
1  2  5     1  2  5     1  2  5          1  2  5  15
 1  3        1  3        1  3  10         1  3  10
  2           2  7        2  7             2  7
   5           5           5                5
   ↑
```

(The example immediately above is the original Bell Triangle. Other examples are based on alternate initial triads.)

Of particular interest are Bell Triangles whose initial triads are of the form, Of particular interest are Bell Triangles whose initial triads are of the form, These will be called stem cells

X 0	X X	0 X
X	0	X

where X is any positive integer. For example, the triangle,

1 0 1 1 0 1 1 0
1 1 0 1 1 0 1
0 1 1 0 1 1
1 0 1 1 0
1 1 0 1
0 1 1
1 0
1

Following the rules of triangle construction, at any of the red zeros marked below, there exists a choice. Instead of the **0**, a **2** could have been inserted. But once a **2** instead of a **0** is inserted the triangle takes off on a different course in which there is no longer any choice.

1 0 1 1 **0** 1 1 **0**
1 1 **0** 1 1 **0** 1
0 1 1 **0** 1 1
1 **0** 1 1 **0**
1 1 **0** 1
0 1 1
1 **0**
1

Two examples:

1 0 1 3 10 39	1 0 1 1 **2** 7
1 1 **2** 7 29	1 1 **0** 1 5
0 1 5 22	0 1 1 4
1 4 17	1 **0** 3
3 13	1 3
10	2

There is choice so long as **0** is chosen. Once **0** is not chosen there is no longer choice. In other words there is choice until you exercise choice.

E. T. BELL NUMBER RHOMBOID

674 674
151 523 Σ 151 523
37 114 409 37 114 409
10 27 87 322 10 27 87 322
3 7 20 67 255 3 7 20 67 255

---

1 2 5 15 52 203 1 2 5 15 52 203 877

---

1 3 10 37 151 1 3 10 37 151
2 7 27 114 2 7 27 114
5 20 87 5 20 87
15 67 Δ 15 67
52 52

BELL RHMB.WPD
4-9-24

E. T. BELL NUMBER HEXOID

674 523 409 322 255 **203** 674
203 151 114 87 67 **52** 151 523
255 **52** 37 27 20 **15** 37 114 409
322 67 **15** 10 7 **5** 10 27 87 322
409 87 20 **5** 3 **2** 3 7 20 67 255
523 114 27 7 **2** **1** **1** **2** **5** **15** **52** **203**
674 151 37 10 3 **1** ◘ **1** 3 10 37 151 674
203 **52** **15** **5** **2** **1** **1** **2** 7 27 114 523
255 67 20 7 3 **2** 3 **5** 20 87 409
322 87 27 10 **5** 7 10 **15** 67 322
409 114 37 **15** 20 27 37 **52** 255
523 151 **52** 67 87 114 151 **203**
674 **203** 255 322 409 523 674

Counter Clockwise Σ
Clockwise Δ
Toward Center Δ

BELLHEX2.WPD
4-9-24

E. T. BELL NUMBER HEXOID

674 523 409 322 255 **203** 674
203 151 114 87 67 **52** 151 523
255 **52** 37 27 20 **15** 37 114 409
322 67 **15** 10 7 **5** 10 27 87 322
409 87 20 **5** 3 **2** 3 7 20 67 255
523 114 27 7 **2** **1** **1** **2** **5** **15** **52** **203**
674 151 37 10 3 **1** ▣ **1** 3 10 37 151 674
203 **52** **15** **5** **2** **1** **1** **2** 7 27 114 523
255 67 20 7 3 **2** 3 **5** 20 87 409
322 87 27 10 **5** 7 10 **15** 67 322
409 114 37 **15** 20 27 37 **52** 255
523 151 **52** 67 87 114 151 **203**
674 **203** 255 322 409 523 674

Counter Clockwise Σ
Clockwise Δ
Toward Center Δ

BELL TRIANGLE
ENCYCLOPEDIA of MATH p. 106
Eric Temple Bell

BELLK[illegible]3.WPD
4 - 9 - 24

BELL TRIANGLES

STEM CELL #1

INITIAL
$\overset{0\ 1}{1}$
Δ
→ 0
not → 2

0		1		1		1		1		1^{3}		1^{7}		1			
	1		$\boxed{0}$		0		0		0^{2}		0^{12}		0^{55}				
		1		$\boxed{0}$		0		0^{2}		0^{10}		0^{46}					
			1		$\boxed{0}$		0^{2}		0^{8}		0^{34}						
				1		$\boxed{0}^{2}$		0^{6}		0^{24}							
					1		$\boxed{0}^{4}$		0^{16}								
						1^{3}		$\boxed{0}^{12}$									
							1^{9}										
ORIGINAL BELL TRIANGLE																	
1		2		5		15		52		203		877		4140			
	1		3		10		37		151		674		3263				
		2		7		27		114		523		2589					
			5		20		87		409		2066						
				15		67		322		1657							
					52		255		1335								
						203		1080									
							877										

This triangle also uses the Δ theme

INITIAL
$\overset{1\ 2}{1}$
THIS IS B[illegible]'s
[illegible]INAL
ORIGINAL
BELL

This triangle the Δ theme and Σ theme give the same triangle

$\overset{0\ 1}{1}$ $\overset{1\ 0}{1}$ and $\overset{1\ 1}{0}$ can be converted into Σ-Δ at any point

The Free will is when a selection is +

Once → Σ-Δ, deterministic

So long as you choose 0
you will have choice

The CHOICE-NECESSITY
METAPHOR

choice regime until +
but choice regime is also deterministic
so long as it remains choice

To keep a potential for
choice always select 0
Selecting + launches a deterministic pattern

BELL TRIANGLES
"STEM CELLS"
△ only

STEM CELL #2

INITIAL
1 1
0

△
Triangle

Key
1 = 1 − 0
1 = 1 + 0

1		1		$\boxed{0}^{1}$		0		1		$\boxed{0}^{4}$		0		1		$\boxed{0}$		0	0
	0		1		$\boxed{0}^{2}$		1		1		$\boxed{0}$		1		1		$\boxed{0}$		0
		1		1		1		$\boxed{0}^{3}$		1		1		$\boxed{0}$		1		0	
			0		0		1		1		$\boxed{0}$		1		1		$\boxed{0}$		
				0		1		$\boxed{0}^{5}$		1		1		$\boxed{0}$		1			
					1		1		1		$\boxed{0}$		1		1				
						0		0		1		1		$\boxed{0}$					
							0		1		$\boxed{0}$		1						
								1		1		1							
									0		0								
										0									

Pattern

1 1
1 1
1 1 0 1 1
1 1
1 1

STEM CELL #3

INITIAL
1 0
1

1		0		1		1		$\boxed{0}^{3}$		1		1		$\boxed{0}^{9}$		1		1	0
	1		1		$\boxed{0}^{1}$		1		1		$\boxed{0}^{6}$		1		1		$\boxed{0}$		1
		0		1		1		$\boxed{0}^{4}$		1		1		$\boxed{0}$		1		1	
			1		$\boxed{0}^{2}$		1		1		$\boxed{0}^{8}$		1		1		$\boxed{0}$		
				1		1		$\boxed{0}^{5}$		1		1		$\boxed{0}$		1			
					0		1		1		$\boxed{0}$		1		1				
						1		$\boxed{0}^{7}$		1		1		$\boxed{0}$					
							1		1		$\boxed{0}$		1						
								0		1		1							
									1		$\boxed{0}$								
										1									

Pattern

1 1
1 1
1 1 0 1 1
1 1
1 1

Both use the
△ theme

The inner rows and diagonals in both triangles
have the form
.. 11 011011010 ..

EXCLUDING THE OUTER PRIMARY
LINES, THE INSIDES ARE IDENTICAL

PATTERNS FROM BELL

STEM CELL #1 p1

0		1		1		4		1		1		1		1
	1		$\boxed{0}$		0		0		0		0		0	
		1		$\boxed{0}$		0		0		0		0		
			1		$\boxed{0}$		0		0		0			
				1		$\boxed{0}$		0		0				
					1		$\boxed{0}$		0					
						1		$\boxed{0}$						
							1							

	0		1		3		9		31		121		523
		1		$\boxed{2}$		6		22		90		402	
			1		4		16		68		312		
I				3		12		52		244		1232	
					9		40		192		988		
						31		152		796			
							121		644				
								523					

0		1		1		3		11		43		185		873
	1		0		2		8		32		142		688	
		1		$\boxed{2}$		6		24		110		546		
			1		4		18		86		436			
	II			3		14		68		350				
					11		64		282					
						43		228						
							185		1058					
								873						

0		1		1		1		3		13		57		267		1361
	1		0		0		2		10		44		210		1094	
		1		0		2		8		34		166		884		
			1		$\boxed{2}$		6		26		132		718			
				1		4		20		106		586				
	III				3		16		86		480					
						13		70		394						
							57		324							
								267								

0		1		1		1		1		3		15		73		369		2015
	1		0		0		0		2		12		58		296		1646	
		1		0		0		2		10		46		238		1350		
			1		0		2		8		36		192		1012			
				1		$\boxed{2}$		6		28		156		920				
	IV				1		4		22		128		764					
						3		18		106		636						
							15		88		530							
								73		442								
									369									

STEM CELL #1 CONT p 2.

0		1		1		1		1		1		3		17		91		493
	1		0		0		0		0		2		14		74		402	
		1		0		0		0		2		12		60		328		
			1		0		0		2		10		48		268			
				1		0		2		8		38		220				
					1		$\boxed{2}$		6		30		182					
	V					1		4		24		152						
							3		20		128							
								17		108								
									91									

0		1		1		1		1		1		1		3		19		111		641
	1		0		0		0		0		0		2		16		92		530	
		1		0		0		0		0		2		14		76		438		
			1		0		0		0		2		12		62		362			
				1		0		0		2		10		50		300				
					1		0		2		8		40		250					
		VI				1		$\boxed{2}$		6		32		210						
							1		4		26		178							
								3		22		152								
									19		130									
										111										

0		1		1		1		1		1		1		1		3		21		133		815
	1		0		0		0		0		0		0		2		18		112		682	
		1		0		0		0		0		0		2		16		94		570		
			1		0		0		0		0		2		14		78		476			
				1		0		0		0		2		12		64		398				
					1		0		0		2		10		52		334					
						1		0		2		8		42		282						
		VII					1		$\boxed{2}$		6		34		240							
								1		4		28		206								
									3		24		178									
										21		154										
											133											

40 X 30

Summaries STEM CELL #1 p3

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
I	2	3	3	4	6	9	9	12	16	22	31	31	40	52	68	90	121												
II	2	2	3	3	4	6	8	11	11	14	18	24	32	43	43	54	68	86	110										
III	2	2	2	3	3	4	6	8	10	13	13	16	20	26	34	44	67	57	70	86	106								
IV	2	2	2	2	3	3	4	6	8	10	12	15	15	18	22	28	36	46	58	73	73	88	106						
~~V~~	~~2~~	~~2~~	~~2~~	~~2~~	~~2~~	~~3~~	~~3~~	~~5~~	~~7~~	~~9~~	~~11~~	~~13~~	~~15~~	~~18~~	~~18~~	~~21~~	~~26~~	~~33~~	~~42~~	~~53~~	~~66~~	~~81~~	~~99~~	~~99~~	~~117~~				
V	2	2	2	2	2	3	3	4	6	8	10	12	14	17	17	20	24	30	38	48	60	74	91	91	108				
VI	2	2	2	2	2	2	3	3	4	6	8	10	12	14	16	19	19	22	26	32	40	50	62	76	92	[illegible]	[illegible]		
VII	2	2	2	2	2	2	2	3	3	4	6	8	10	12	14	16	18	21	21	24	28	34	42	52	64	78	94	112	133

TOP ROWS

I	1	3	9	31	121	523
II	1	3	11	43	185	873
III	1	3	13	57	267	1361
IV	1	3	15	73	369	1015
V	1	3	17	91	493	
VI	1	3	19	171	641	
VII	1	3	21	133	815	

✓
✓
error somewhere

Δ20
Δ20

$\Delta=2$ $\Delta^2=2$ $\Delta^3=2$

$\Delta=2$

irregular

I	2	3	4	6	9	12	16	22	31	40	52	68	90	121															
II	2	3	4	6	8	11	14	18	24	32	43	54	68	86	110	142	185												
III	2	3	4	6	8	10	13	16	20	26	34	44	57	70	86	106	132	166	210	267									
IV	2	3	4	6	8	10	12	15	18	22	28	36	46	58	73	88	106	128	156	192	238	296	369						
V	2	3	4	6	8	10	12	14	17	20	24	30	38	48	60	74	91	108	128	152	182	220	268	328	402	493			
VI	2	3	4	6	8	10	12	14	16	19	22	26	32	40	50	62	76	92	111	130	152	178	210	250	300	362	438	530	641
VII	2	3	4	6	8	10	12	14	16	18	21	24	28	34	42	52	64	78	94	112	133	154	178	206	240	282	334	398	476
																											570	682	815

sequences of distinct numbers

BELL TRIANGLES

With an initial 0, ∃ a choice

Δ or Σ-Δ

The only
Δ triangles
are $\begin{matrix}0 & 1\\ & 1\end{matrix}$ $\begin{matrix}1 & 0\\ & 1\end{matrix}$ $\begin{matrix}1 & 1\\ & 0\end{matrix}$

all others are Σ-Δ's

Other Δ's
$\begin{matrix}0 & 0\\ & 1\end{matrix}$
$\begin{matrix}1 & 0\\ & 0\end{matrix}$
$\begin{matrix}0 & 1\\ & 0\end{matrix}$

choice

0 or 2

$\begin{matrix}1 & 1\\ & 0\end{matrix}$
Σ-Δ
Triangle

OUT OF STEM CELL #2 FIRST BRANCH

1		1		$\boxed{2}$		6		21		82		354					
	0		①		4		15		61		272						
		1		3		11		46		211							
			2		8		35		165								
				6		27		130									
					21		103										
						82											
OUT OF STEM CELL #3					FIRST		BRANCH										
1		0		1		3		10		39		169					
	1		1		$\boxed{2}$		7		29		130						
		0		1		5		22		101							
			1		4		17		79								
				3		13		62									
					10		49										
						39											

ONLY WHEN ∃ A ZERO IN THE INITIAL TRIAD ~~DOES~~ IS CHOICE POSSIBLE
Allowing Δ triangles. In all other cases Σ-Δ triangles

$\boxed{2}$ The branch point to specificity

STEM CELL #2

S	E	C	O	N	D		B	R	A	N	C	H		P	O	I	N	T
1		1		0		2		7		26		112		531				
	0		1		$\boxed{2}$		5		19		86		419					
		1		1		3		14		67		333						
			0		2		11		53		266							
				2		9		42		213								
					7		33		171									
						26		138										
							112											

T	H	I	R	D		B	R	A	N	C	H		P	O	I	N	T
1		1		0		0		1		4		16		75		376	
	0		1		0		1		3		12		59		301		
		1		1		1		$\boxed{2}$		9		47		242			
			0		0		1		7		38		195				
				0		1		6		31		157					
					1		5		25		126						
						4		20		111							
							16		91								
								75									

F	O	U	.R	TH		B	R	A	N	C	H		P	O	I	N	T
1		1		0		0		1		$\boxed{2}$		8		211		212	
	0		1		0		1		1		6		33		171		
		1		1		1		0		5		27		138			
			0		0		0		5		22		111				
				0		0		4		17		89					
					1		3		13		72						
						2		10		59							
							8		49								
								41									

Sol 1	2	3	4	6	8	11	15	21	27	35	46	61	82	103	130	165	211	272	353	
2	2	2	2	3	5	7	9	11	14	19	26	33	42	53	67	86	112	138	171	213
3	2	3	4	5	6	7	9	12	16	20	25	31	38	47	59	75	91	111	126	157
4	2	3	4	5	5	6	8	10	13	17	22	27	33	41	49	59	72	89	111	138

BELL STEM CELLS

PAGE 1

THE $\overset{10}{1}$ STEMCELL #3 SINGULAR POINTS IN RED

1		0		1		1		$\boxed{0}$		1		1		$\boxed{0}$				
	1		1		$\boxed{0}$		1		1		$\boxed{0}$		1					
		$\boxed{0}$		1		1		$\boxed{0}$		1		1						
			1		$\boxed{0}$		1		1		$\boxed{0}$							
				1		1		$\boxed{0}$		1								
					0		1		1									
						1		$\boxed{0}$										
							1											
F	I	R	S	T		S	I	N	G	U	L	A	R	→	2			
1		0		1		3		10		39		169		799				
	1		1		$\boxed{2}$		7		29		130		630					
		0		1		5		㉒		101		500						
			1		4		17		79		399							
				3		13		62		320								
					10		49		257									
						39		208										
							169											
SECOND				SINGULAR			POINT		TO	→	2							
1		0		1		1		4		17		73		342				
	1		1		$\boxed{0}$		3		13		56		269					
		0		1		3		10		43		213						
			1		$\boxed{2}$		7		33		170							
				1		5		26		137								
					4		㉑		111									
						17		90										
							73											

○ beginning of difference

Beginning ○ of difference

BELL STEM CELLS

BASED ON THE $^{10}_{1}$ STEMCELL PAGE 2

#3

TH	IRD	SIN	GULA	RITY	→	2													
1		0		1		1		$\boxed{2}$		7		31		150					
	1		1		$\boxed{0}$		1		5		24		119						
		0		1		1		4		19		95							
			1		$\boxed{0}$		3		15		76								
				1		3		12		61									
					2		⑨		49										
						7		40											
							31												

0
Beginning of difference

FO	UR	TH	SIN	GUL	ARI	TY	→	2											
1		0		1		1		$\boxed{0}$		3		13		58					
	1		1		$\boxed{0}$		1		3		10		45						
		0		1		1		$\boxed{2}$		7		35							
			1		$\boxed{0}$		1		5		28								
				1		1		4		23									
					0		3		19										
						3		⑯											
							13												

0
Beginning of difference

FI	F	TH		SIN	GUL	ARI	TY	→	2										
1		0		1		1		0		1		5		24		121			
	1		1		0		1		1		4		19		97				
		0		1		1		0		3		15		78					
			1		0		1		3		12		63						
				1		1		$\boxed{2}$		9		51							
					0		1		7		42								
						1		[illegible]		35									
							5	⑥	29										

BELL NUMBERS

First Selections

Stem Cell

First Branch Point

#1	2	3	4	6	9	12	16	22	31	40	52	68	90	121	152	192	244	312	402	523
#2	2	3	4	6	8	11	15	21	27	35	46	61	82	103	130	165	211	272	354	
#3	2	3	4	5	7	10	13	17	22	29	39	49	62	79	101	130	169	208	257	320
			Second Branch Point																	
#1	2	3	4	6	8	11	14	18	24	32	43	54	68	86	110	142	185	228	282	350
#2	2	2	2	3	5	7	9	11	14	19	26	33	42	53	67	86	112	138	171	213
#3	2	3	3	4	5	7	10	13	17	21	26	33	43	56	73	90	111	137	170	213
			THIRD BRANCH POINT																	
#1	2	3	4	6	8	10	13	16	20	26	34	44	57	70	86	106	132	166	210	267
#2	2	3	4	5	6	7	9	12	16	20	25	31	38	47	59	75	91	111	126	157
#3	2	3	3	4	5	7	9	12	15	19	24	31	40	49	61	76	95	119	150	
				FOURTH BRANCH POINT																
#1	2	3	4	6	8	10	12	15	18	22	28	36	46	58	73	88	106	128	156	192
#2	2	3	4	5	5	6	8	10	13	17	22	27	33	41	49	59	72	89	111	138
#3	2	3	3	4	5	6	7	9	12	15	19	24	29	35	42	51	63	78	97	121

INITIAL TRIADS

#1	#2	#3
0 1	1 1	1 0
1	0	1

After the initial few numbers all are unique also in NB DIMENSIONS

ARCREATE.WPD

DIMENAL5.WPD 2003-02-08 rev April 4, 2006 September 18, 2009

~~ARRAYS~~ ARRAYS

DIMENSIONAL MATRICES: INTRODUCTION

~~NETS~~

Dimensional matrices are an alternate approach to the relations that exist between the magnitudes of the fundamental constants of physics [initially c, G, and ħ] and the masses, sizes, and frequencies of material bodies ranging from sub-atomic particles to the universe itself. Traditionally the relations or linkages between physical bodies are organized around such concepts as force, action, energy, power, etc. Dimensional matrices show that in many cases relations may be viewed in different but equivalent ways. For example, equivalences between frequency resonance, energy conservation, and symmetries. The matrices also show that the richness of relations exceeds those commonly recognized or utilized.

The construction of a ~~matrix~~ NET starts with equation 1).

combine exponential ~~spaces~~ Grids

$$1)\quad M^a L^b T^e c^x G^y \hbar^z = M^u L^v T^w$$

input output

There are three sets of exponents each with three members:

The exponents **u, v, w,** in the right member are pre-assigned according to the dimensionality of the desired matrix. For example, to create a force-matrix, assign u=+1, v=+1, and w= –2; or to create a frequency matrix, assign u=0, v=0, and w = –1.

assume a A 3 dimensional matrix require the

The exponents **a, b, e,** in the left member are coordinate exponents that assign coordinates to M, L, ~~or M, T,~~ and or L,T depending on the designation of the dimensionality of the matrix.

But in practice we are restricted to 2 dimen i.e. we use only ML, MT or LT

The third set of exponents, **x, y, z,** are those derived for the fundamental constants. This set is a function of the input and coordinate exponents.

To determine the values of **x, y, z,** we rewrite equation 1) with the constants expressed in their dimensional form:

$$2)\quad \mathbf{M^a L^b T^e} \cdot \left\{\frac{\mathbf{L}}{\mathbf{T}}\right\}^{x} \cdot \left\{\frac{\mathbf{L^3}}{\mathbf{M \cdot T^2}}\right\}^{y} \cdot \left\{\frac{\mathbf{M \cdot L^2}}{\mathbf{T}}\right\}^{z} = \mathbf{M^u L^v T^w}$$

Arranging the exponents according to their parent parameter,

$$\begin{array}{ll} M: & a - y + z = u \\ L: & b + x + 3y + 2z = v \\ T: & e - x - 2y - z = w \end{array}$$

Solving for x, y, and z,

$$\begin{aligned} 2x &= u - 3v - 5w - a + 3b + 5e \\ 2y &= -u + v + w + a - b - e \\ 2z &= u + v + w - a - b - e \end{aligned}$$

General matrices
may be required to cover , all ~~to~~ possibilities. For example, a separate NET matrix, each covering the
numerical ranges of a and b, for different assigned value of e. In general, there can be six input arrangements:
a fixed, b and e variable   a and b fixed, e variable
b fixed, a and e variable   a and e fixed, b variable
e fixed, a and b variable   b and e fixed, a variable
Selecting one of these six options, three "pre-matrices" are to be generated: a matrix for x in terms of, (for example), a and b with fixed e, and similar matrices for y and for z. From these three matrices the basic matrix is constructed, whose elements each have the assigned dimensionality (eg force, MR/T2) with specified ranges for a and b, (the exponents of M and R respectively), and for a specified value of eFinally, from a basic matrix, several numerical matrices can be developed using specific values for M and R. For example, In a floating M,R matrix with input T-' , inserting mp for M and re for R to obtain all frequencies related to a proton 1n addition, several types of "restricted" basic matrices may be constructed. For example, matrices in which constraints are placed on c, G or 'i, such as a matrix that displays all forces in which planck's constant plays no role [z=0].
Examples: ARRAY
1: A Force M~~atri~~x
Rewriting equation 2) in logarithmic form,

1° Matrices with M and L
as the variables
~~[illegible]~~ " e = 0

M\L	-1	-0.5	0	0.5	+1	1.5	+2
+3	$\sqrt{G^5M^6/L^2hc^{11}}$		G^2M^3/hc^4		$\sqrt{G^3M^6L^2/h^3c^5}$		GM^3L^2/h^2c
+2.5		$\sqrt{G^4M^5/Lhc^9}$		$\sqrt{G^3M^5L/h^2c^6}$		$\sqrt{G^2M^5L^3/h^3c^3}$	
+2	G^2M^2/Lc^5		$\sqrt{G^3M^4/hc^7}$		GM^2L/hc^2		$\sqrt{GM^4L^4/h^3c}$
+1.5		$\sqrt{G^3M^3/Lc^8}$		$\sqrt{G^2M^3L/hc^5}$		$\sqrt{GM^3L^3/h^2c^2}$	
+1	$\sqrt{G^3M^2h/L^2c^9}$		GM/c^3		$\sqrt{GM^2L^2/hc^3}$		ML^2/h
+1/2		$\sqrt{G^2Mh/Lc^7}$		$\sqrt{GML/c^4}$		$\sqrt{ML^3/hc}$	
0	Gh/Lc^4		$\sqrt{Gh/c^5}$		L/c		$\sqrt{L^4 c/Gh}$
-1/2		$\sqrt{Gh^2/MLc^6}$		$\sqrt{Lh/Mc^3}$		$\sqrt{L^3/GM}$	
-1	$\sqrt{Gh^3/M^2L^2c^7}$		h/Mc^2		$\sqrt{L^2h/GM^2c}$		L^2c/GM
-3/2		$\sqrt{h^3/M^3Lc^5}$		$\sqrt{Lh^2/GM^3c^2}$		$\sqrt{L^3hc/G^2M^3}$	
-2	h^2/M^2Lc^3		$\sqrt{h^3/GM^4c^3}$		Lh/GM^2		$\sqrt{L^4hc^3/G^3M^4}$
-5/2		$\sqrt{h^4/GM^5Lc^4}$		$\sqrt{Lh^3/G^2M^5c}$		$\sqrt{L^3h^2c^2/G^3M^5}$	
-3	$\sqrt{h^5/GM^6L^2c^5}$		h^2/GM^3c		$\sqrt{L^2h^3c/G^3M^6}$		L^2hc^2/G^2M^3

= 0 $\frac{1}{c^3}\sqrt{\frac{(c^2L)^n}{(GM)^{n-2}}}$

G = 0 $\frac{1}{c}\sqrt{\left(\frac{Mc}{h}\right)^{n-2}L^n}$

ħ = 0 $c^{-3}\sqrt{\frac{L^n c^{2n}}{(GM)^{n-2}}}$

TIMATRX4.WPD

November 8, 2009 O = 0

TIME TABLE: T=T(G,M,L,ħ,c)

[T] = 1

$\frac{GM^4L^3}{\hbar^3}$

G = 0

M\L	0	0.5	+1	1.5	+2	+2.5	+3
+3	G^2M^3/hc^4		$\surd G^3M^6L^2/h^3c^5$		GM^3L^2/h^2c		$\surd GM^6L^6c/h^5$
+2.5		$\surd G^3M^5L/h^2c^6$		$\surd G^2M^5L^3/h^3c^3$		$\surd GM^5L^5/h^4$	
+2	$\surd G^3M^4/hc^7$		GM^2L/hc^2		$\surd GM^4L^4/h^3c$		6 M^2L^3c/h^2
+1.5		$\surd G^2M^3L/hc^5$		$\surd GM^3L^3/h^2c^2$		5 $\surd M^3L^5c/h^3$	
+1	T GM/c^3 0		$\surd GM^2L^2/hc^3$		4 ML^2/h		$\surd M^2L^6c^3/Gh^3$
+1/2		ψ $\surd GML/c^4$ 1		3 $\surd ML^3/hc$		$\surd ML^5c^2/Gh^2$	
0	$\surd Gh/c^5$	PLANCK	t L/c 2		$\surd L^4 c/Gh$		L^3c^2/Gh
-1/2		$\surd Lh/Mc^3$		τ $\surd L^3/GM$ 3		$\surd L^5c^3/G^2Mh$	
-1	K h/Mc^2		$\surd L^2h/GM^2c$		L^2c/GM 4		$\surd L^6c^5/G^3M^2h$
-3/2		$\surd Lh^2/GM^3c^2$		$\surd L^3hc/G^2M^3$		$\surd L^5c^4/G^3M^3$ 5	
-2	$\surd h^3/GM^4c^3$		Lh/GM^2		$\surd L^4hc^3/G^3M^4$		L^3c^3/G^2M^2 6
-5/2		$\surd Lh^3/G^2M^5c$		$\surd L^3h^2c^2/G^3M^5$		$\surd L^5hc^5/G^4M^5$	
-3	h^2/GM^3c		$\surd L^2h^3c/G^3M^6$		L^2hc^2/G^2M^3		$\surd L^6hc^7/G^5M^6$

ħ = 0

Notation: In the above table h is used for ħ, the Planck constant / 2π .

√ is for entire expression

The Geometric Mean Rule

for vertical, horizontal and ~~both~~ all diagonals

T ~~MATRIX~~ ARRAY.

6-25-10
office

A	B	C	D
T	ψ	t	τ
$\frac{GM}{C^3}$	$\sqrt{\frac{GML}{C^4}}$	$\frac{L}{C}$	$\sqrt{\frac{L^3}{GM}}$

Derivation of $T\tau^2 = t^3$
from A $\sqrt{AB}$ B

GEOME PROPERTY:

① 1) $B^2 = AC$
2) $C^2 = BD$

1)^3 $B^6 = A^3C^3$
1)×2) $B^3D = AC^3$
÷ $\frac{B^3}{D} = A^2$
$B^3 = A^2D$

2)^3 $C^6 = B^3D^3$
1)×2) $C^3A = B^3D$
÷ $\frac{C^3}{A} = D^2$

SPECIAL PROPERTY

② $AD^2 = C^3$
$DA^2 = B^3$

ARE THESE ② DERIVABLE FROM ①

How general are ② ?

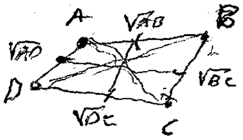
The $T\tau^2 = t^3$
and corresponding corrections
are genome properties

---

4 VERTICES COPLANAR TEST;

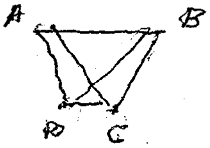
In general 4 vertices in 3 dim will for a tetrahedron. 4 Faces, 6 edges,

When will the 4 vertices be co-planer?



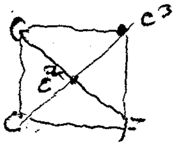
Evaluate the six geometric means

$\sqrt{AB}$ $\sqrt{BC}$
$\sqrt{AC}$ $\sqrt{BD}$
$\sqrt{AD}$ $\sqrt{CD}$



IF two geometric means are equal, the 4 points are co-planar

EXAMPLE

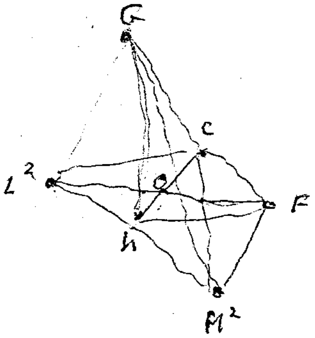


$\sqrt{GE} = c^2$ $\sqrt{Fc} = \sqrt{\frac{ML}{T3}}$
$\sqrt{Gc} = \sqrt{\frac{L^4}{MT^3}}$ $\sqrt{Fc^2} = \sqrt{\frac{ML^3}{T^4}}$
$\sqrt{Gc^3} = \sqrt{\frac{L^5}{MT^4}}$ $\sqrt{cc^3} = c^2$

$\sqrt{GF} = \sqrt{c^4}$

$F = \frac{c^4}{G}$

FORCES



$\sqrt{GM^2} = \sqrt{FL^2}$

$\sqrt{\hbar c} = \sqrt{FL^2}$

$\sqrt{GF} = \sqrt{c^4}$

$F = \frac{GM^3}{L^2}$ grav

$F = \frac{hc}{L^2} = \frac{Q^2}{L^2}$ elect

$F = \frac{c^4}{G}$ plank 49,082 ~~378~~ > $M = m_0$

$L = \sqrt{o}$

$F = M^2 \frac{c^3}{\hbar}$?, $= M^2 \times 58,407386$

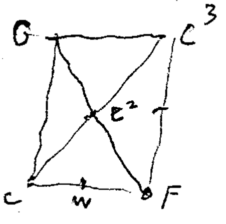
$F = \frac{W}{c}$? weak

$F = \frac{Mc}{T}$ mass x accel

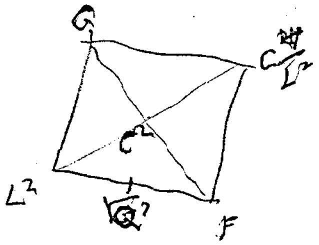
$F = \frac{Q^2}{L^2}$ electric

$\sqrt{\hbar F} = \sqrt{M^2c^3}$ $F = \frac{M^2c^3}{\hbar}$

SYMMETRIC
OR
NOT



PLANE



$FL^2 = Q^3$

$\sqrt{cF} = \sqrt{W}$

$F = \frac{W}{c} = \frac{\text{POWER}}{\text{Velocity}}$

$\sqrt{Fc^3} = \sqrt{\frac{M}{T}c^4}$

$F = \frac{Mc}{T}$ Mass x acceleration

FORCE ARRAY: $F=F(M,L,G,\hbar,c)$

$\hbar$

M\L	3	2	1	0	-1	-2	-3
-5							
-4							
-3	L^3c^{10}/G^4M^3		$Lc^7\hbar/G^3M^3$		$c^4\hbar^2/G^2M^3L$		$c\hbar^3/GM^3L^3$
-2		L^2c^8/G^3M^2		$c^5\hbar/G^2M^2$		$c^2\hbar^2/GM^2L^2$	
-1	$L^3c^9/G^3M\hbar$		Lc^6/G^2M		$c^3\hbar/GML$		$\hbar^2/ML^3$
0		$L^2c^7/G^2\hbar$		c^4/G		$c\hbar/L^2$	
1	$ML^3c^8/G^2\hbar^2$		MLc^2/Gh		Mc^2/L		GMh/L^3c 3
2		$M^2L^2c^6/G\hbar^2$		$M^2c^3/\hbar$		GM^2/L^2	
3	$M^3L^3c^7/G\hbar^3$		$M^3Lc^4/\hbar^2$		$GM^3c/L\hbar$		G^2M^3/L^3c^2
4		$M^4L^2c^5/\hbar^3$		$GM^4c^2/\hbar^2$		$G^2M^4/L^2c\hbar$	
5	$M^5L^3c^6/\hbar^4$		$GM^5Lc^3/\hbar^3$		$G^2M^5/L\hbar^2$		$G^3M^5/L^3c^3\hbar$
6							
7							

Ⓑ

P level
For all entries

$M = [illegible]_0 = -4.662$
$L = \ell_0 = -32.791$

$ML = \frac{\hbar}{c} = -37.453743$
$\frac{M}{L} = \frac{c^2}{G} = +28.128937$

$t_b^2 = -86.[illegible]6324$

FORCTRIX 2.WPD
~~TIMATRX0.WPD~~

F.

$$F \equiv \frac{c^4}{G} = 49.082578431$$

November 8, 2009

$M = m_p =$
$L = r_e =$

$m_p r_e = ML = \frac{\hbar}{c}\alpha M = -36.326670$
$\frac{m_p}{r_0} = \frac{M}{L} = \frac{1}{S}\frac{c^3}{G} = -11.226534$

B LEVEL

FORCE ~~TIME~~ TABLE: T=T(G,M,L,ħ,c)

[T] = 1

a M\L b	~~0.5~~ $-\frac{3}{2}$	-1 0	$-\frac{1}{2}$ ~~+0.5~~	+ ~~1~~ 0	~~+1.5~~ ½	~~+2~~ +1	~~+2.5~~ $+\frac{3}{2}$	2
+3						$\frac{c^4}{G} S^2 (\alpha M)^{-3}$		
+2.5					$\frac{c^4}{G} S^2 (\alpha M)^{-2}$		$\frac{c^4}{G} S (\alpha M)^{-3}$	
+2				$\frac{c^4}{G} S^2 (\alpha M)^{-1}$		$\frac{c^4}{G} S (\alpha M)^{-2}$		
+1.5			$\frac{c^4}{G} S^2$		$\frac{c^4}{G} S (\alpha M)^{-1}$		$\frac{c^4}{G} (\alpha M)^{-2}$	
+1				T $\frac{c^4}{G} S$ ✓		ξ $\frac{c^4}{G} (\alpha M)^{-1}$ ✓		
+0.5	η $\frac{c^4}{G} (\alpha M S)^2$				ψ $\frac{c^4}{G}$ ✓		$\frac{c^4}{G} S^{-1} (\alpha M)^{-1}$	
0		Z ✓ $\frac{c^4}{G} (\alpha M)^2 S$		~~t_0 $\frac{c^4}{G} \alpha M$ ✓~~		t $\frac{c^4}{G} S^{-1}$ ✓		
-0.5			γ $\frac{c^4}{G} (\alpha M)^2$ ✓				τ $\frac{c^4}{G} S^{-2}$ ✓	
-1			?	K ✓ $\frac{c^4}{G} S^{-1} (\alpha M)^2$				
-1.5								
-2						σ $\frac{c^4}{G} \frac{(\alpha M)^2}{S^3}$ ✓		
-2.5								
-3								

$\delta \frac{c^4}{G} \frac{(\alpha M)^3}{S^3}$

G = 1

$\frac{c^4}{G} (\alpha M)^{-3}$
G = ½

h = -1

φ $\frac{c^4}{G} \frac{1}{(\alpha M)^2 S}$ ✓

G = 0

h = -½

h = 0

h = ½

$\frac{c^4}{G} \frac{1}{(\alpha M)^2 S}$

$$F = \frac{c^4}{G} (\alpha\mu)^{x} S^{y}$$

where $x = 1 - (a+b)$
$y = a - b$

$G = 0 \Rightarrow S^{-1}$, $\hbar = 0 \Rightarrow (\alpha M)^0$
$G = \frac{1}{2} \Rightarrow S^{0}$, $\hbar = \frac{1}{2} \Rightarrow (\alpha M)^1$

$\hbar = 0$, $F = \frac{c^4}{G} S^{a-b}$
$h = 1$, $F = \frac{c^4}{G} S^{a-b} (\alpha M)^{-1}$
$G = 0$, F $\frac{c^4}{G} S^{-1} (\alpha M)^{1-(a+b)}$

For all $\hbar = 0$

$\frac{M}{L} = \frac{1}{S}\frac{c^2}{G}$ $M = m_p$
$L = r_e$

$G = 1$, $F = \frac{c^4}{G} S^0 (\alpha M)^{1-(a+b)}$

TIME TABLE: T=T(G,M,L,ħ,c)

[T] = 1

M\L	0	0.5	+1	1.5	+2	+2.5	+3
+3							
+2.5							
+2							
+1.5							
+1							
+1/2							
0							
-1/2							
-1							
-3/2							
-2							
-5/2							
-3							

Ⓤ

$M =$

$L =$

$ML = 80.$[illegible]$672 = S^3 \frac{\hbar}{c}$

$\frac{M}{L} = 24.747716 = \frac{1}{(\alpha\mu)^3} \frac{c^2}{G}$

$M_U^2 = \frac{\hbar c}{G} \left(\frac{S}{\alpha\mu}\right)^3$

$L_U^2 = \frac{G\hbar}{c^3} (\alpha\mu S)^3$

FORCTRIX 3.
~~TIMATRX0.WPD~~

November 8, 2009

UNIVERSE LEVEL FORCE ~~TIME~~ TABLE: T=T(G,M,L,ħ,c)

[T] = 1

2

M\L	-0.5	-0	-~~+~~0.5	~~+0~~ 0	+0.5	+1	+1.5
+3							
+2.5							
+2							
+1.5							
+1				T $\frac{c^4}{G}(\alpha\mu)^3$		$\frac{c^4}{G}\frac{1}{S^3}$ ξ	
+0.5				t_0 $c^4/G\ S^3$	ψ c^4/G		
0				~~$\frac{c^4}{G}(\alpha\mu)^3$~~		$\frac{c^4}{G}(\alpha\mu)^{-3}$ t	
-0.5							τ $\frac{c^4}{G}\frac{1}{(\alpha\mu)^6}$
-1							
-1.5							
-2							
-2.5							
-3							

Force

Cf $U \longleftrightarrow B$

$(\alpha\mu)^3 \longleftrightarrow S$

$(\alpha\mu)^{-3} \longleftrightarrow S^{-1}$

$\frac{c^4}{G}$ $S^{-3} \longleftrightarrow (\alpha\mu)^{-1}$

$S^{-9} \longleftrightarrow (\alpha\mu)^{-3}$

$S^3 \longleftrightarrow (\alpha\mu)$

TIME

$U \longleftrightarrow B$

$t_0 \left(\frac{S}{\alpha\mu}\right)^{+3/2} \leftrightarrow \left(\frac{S}{\alpha\mu}\right)^{-1/2} t_0$

TIMATRX0.WPD November 8, 2009

TIME TABLE: T=T(G,M,L,ħ,c)

[T] = 1

M\L	0	0.5	+1	1.5	+2	+2.5	+3
+3							
+2.5							
+2							
+1.5							
+1							
+1/2							
0	▨						
-1/2							
-1							
-3/2							
-2							
-5/2							
-3							

ARPRESS.WPD May 30, 2011

PRESSURE ARRAY P(M,L,cGh)

M\L	3	2	1	0	-1	-2	-3
-5							
-4							
-3			Lc^{10}/G^4M^3		$c^7\hbar/G^3M^3L$		$c^4\hbar^2/G^2M^3L^3$
-2		~~c^8/G^3M^2~~		c^8/G^3M^2		$c^5\hbar/G^2M^2L^2$	
-1			~~$Lc^9/G^3M\hbar$~~		c^6/G^2ML		$c^3\hbar/GML^3$
0				$c^7/G^2\hbar$		c^4/GL^2	
1					$Mc^5/LG\hbar$		c^2M/L^3
2						$c^3M^2/L^2\hbar$	
3							
4							
5							
6							
7							

Per $\frac{F}{L^2}$,

$Lc^9/G^3M\hbar$

per $F \cdot \frac{c^3}{G\hbar}$

THE PRESSURE ARRAY

$\frac{M}{Lt^2}$ $t = \frac{L}{c}$ $\frac{M}{L^3}c^2 = \rho c^2$

$= \frac{E}{V}$

$\tau^2 = \frac{L^3}{GM}$ $\frac{GM^2}{L^4} = \frac{GRAV}{L^2}$

$P = \frac{F}{L^2}$

$\psi^2 = \frac{GML}{c^4}$ $\frac{c^4}{G} \cdot \frac{1}{L^2}$ $\frac{P_F}{L^2}$ $ML = \frac{\hbar}{c}$ $\frac{G\hbar}{c^3}$ ✓

FORCE

PRESSURE

$T^2 = \frac{G^2M^2}{c^6}$ $\frac{c^6}{G^2LM}$ $\frac{P_F^2}{E \cdot L}$

$ML \times \frac{1}{T^2}$

$\frac{M}{T} \cdot \frac{1}{T^2}$

---

if $\frac{M}{L} = \frac{c^2}{G}$ $\frac{M}{Lt^2} = \frac{c^2}{Gt^2} = \frac{P_F}{L^2}$

$\frac{M}{Lt^2} = \frac{M^2c}{\hbar t^2}$ ✓ from $ML = \frac{\hbar}{c}$

from $\frac{M}{Lt^2} \cdot \frac{\hbar}{c} = \frac{M^2}{t^2}$ $\frac{cM^2}{\hbar t^2}$ $t^2 = \left(\frac{L}{c}\right)^2$ $\frac{c^3}{\hbar}\frac{M^2}{L^2}$

$\frac{F_P^2}{LV} = \left(\frac{M}{LT^2}\right)^2$

$\frac{M}{Lt^2}$ $\tau^2 = \frac{L^3}{GM}$ $\frac{cG}{\hbar}\frac{M^3}{L^3}$

$\psi^2 = \frac{GML}{c^4}$ $\frac{c^5}{G\hbar}\frac{M}{L}$

$\frac{F_P \Phi}{ML^2}$ ✓

$T^2 = \frac{G^2M^2}{c^6}$ $\frac{c^7}{G^2\hbar}$

$\frac{M}{L\,Time^2}$ ρc^2 $\frac{E}{V}$ $\frac{GRAV}{L^4}$ $\frac{F_P}{L^2}$ $\frac{F_P^2}{E \cdot L}$ $\frac{c^7}{G^2\hbar}$

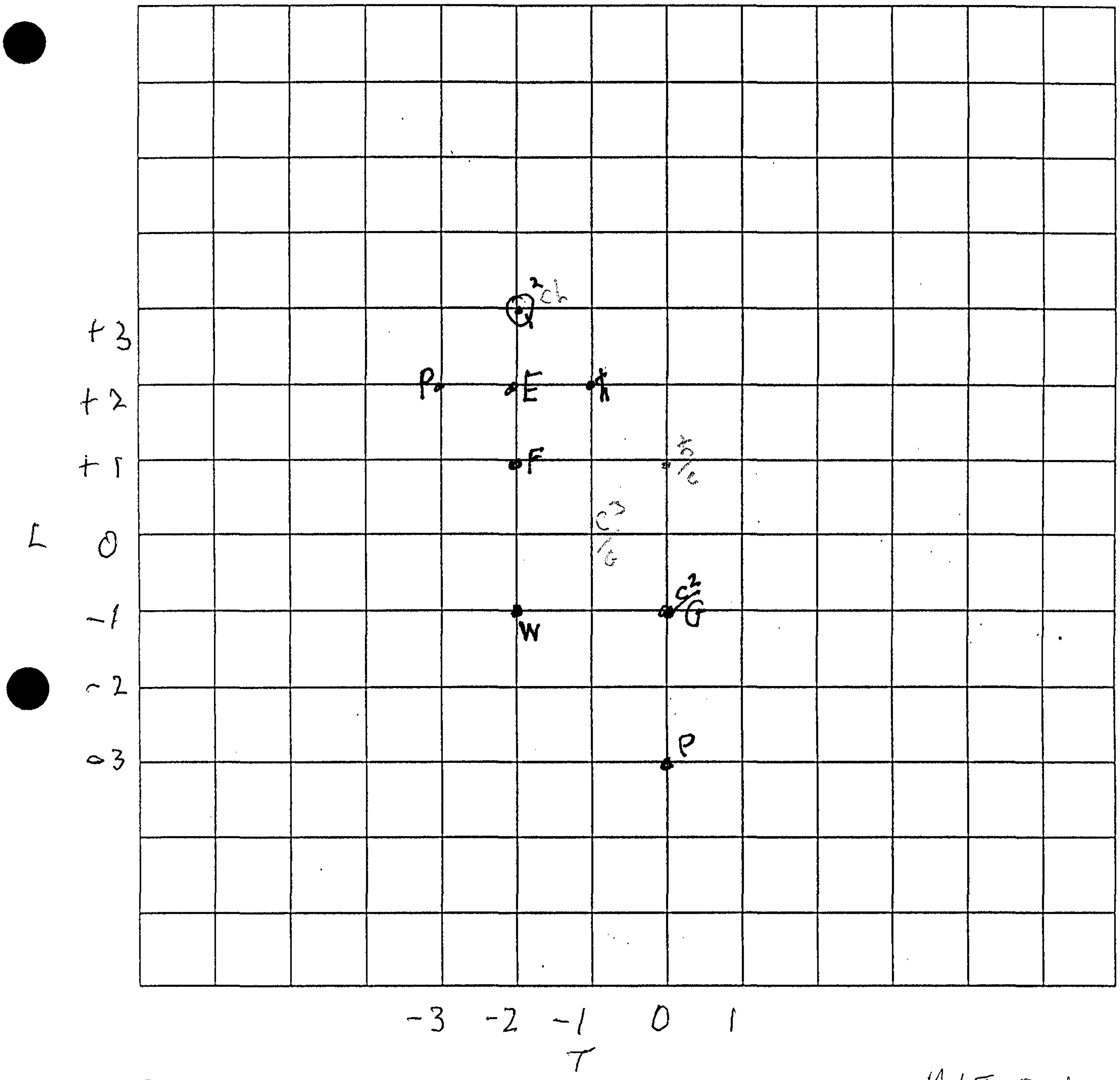
'' $\frac{c^4}{GL^2}$

ARMASS.WPD May 31, 2011

MASS ARRAY $M(L,T,c,G,h)$

T\L	3	2	1	0	-1	-2	-3
-5							
-4							
-3							
-2							
-1							
0							
1							
2							
3							
4							
5							
6							
7							

→ ARMASS



MLT Grid

Q^2 = charge^2

P = Power

E = Energy

ħ = Planck's constant

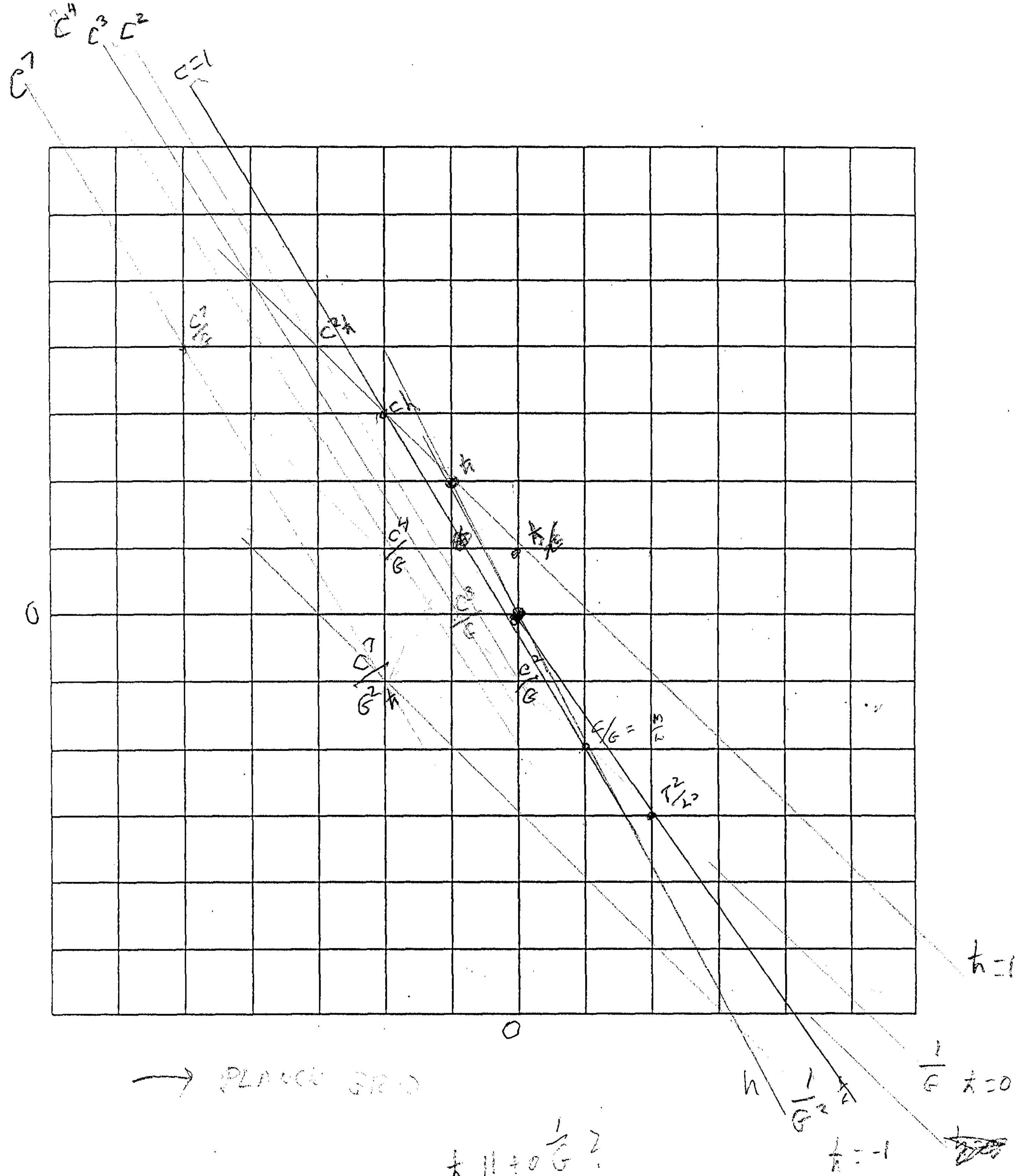
F = Force

W = Pressure $= \frac{E}{Volume} = \frac{F}{AREA} = \frac{C^7}{G^2 \hbar}$

PLANE

M = 1

$\nu = \sqrt{G\rho}$



$\hbar \parallel$ to $\frac{1}{G}$?

M=1

PLANE
M=1

$\frac{ML^2}{T}$ $\frac{MT^2}{L^3}$ $\frac{L^3}{T^2}$ w $\frac{L}{T}$

$\boxed{\frac{L^2}{T} \text{ w } \frac{L^3}{T^2}}$ $\frac{1}{G}$ $\frac{1}{G^2}$

$\hbar$

C~~ART~~IME.WPD
SPACE

Redo L instead of R

May 30, 2011

M\R	-1	-0.5	0	0.5	+1	1.5	+2
+3	$\surd G^5M^6/R^2hc^{11}$		G^2M^3/hc^4		$\surd G^3M^6R^2/h^3c^5$		GM^3R^2/h^2c
+2.5		$\surd G^4M^5/Rhc^9$		$\surd G^3M^5R/h^2c^6$		$\surd G^2M^5R^3/h^3c^3$	
+2	G^2M^2/Rc^5		$\surd G^3M^4/hc^7$		GM^2R/hc^2		$\surd GM^4R^4/h^3c$
+1.5		$\surd G^3M^3/Rc^8$		$\surd G^2M^3R/hc^5$		$\surd GM^3R^3/h^2c^2$	
+1	$\surd G^3M^2h/R^2c^9$		GM/c^3		$\surd GM^2R^2/hc^3$		MR^2/h
+1/2		$\surd G^2Mh/Rc^7$		$\surd GMR/c^4$	⊕×	$\surd MR^3/hc$	
0	Gh/Rc^4		$\surd Gh/c^5$		R/c		$\surd R^4c/Gh$
-1/2		$\surd Gh^2/MRc^6$		$\surd Rh/Mc^3$		$\surd R^3/GM$	
-1	$\surd Gh^3/M^2R^2c^7$		h/Mc^2		$\surd R^2h/GM^2c$		R^2c/GM
-3/2		$\surd h^3/M^3Rc^5$		$\surd Rh^2/GM^3c^2$		$\surd R^3hc/G^2M^3$	
-2	h^2/M^2Rc^3		$\surd h^3/GM^4c^3$		Rh/GM^2		$\surd R^4hc^3/G^3M^4$
-5/2		$\surd h^4/GM^5Rc^4$		$\surd Rh^3/G^2M^5c$		$\surd R^3h^2c^2/G^3M^5$	
-3	$\surd h^5/GM^6R^2c^5$		h^2/GM^3c		$\surd R^2h^3c/G^3M^6$		R^2hc^2/G^2M^3

$$k = \sqrt{\psi t} = \sqrt{T\tau} = \sqrt{t^3/\tau} = \sqrt{\psi^3/T}$$

$T\tau^2 = t^3$
$\tau T^2 = \psi^3$

$$\cancel{k} = \left(\frac{GML^3}{c^4}\right)^{1/4} = \left(Tt^3\right)^{1/4}$$

$\sqrt{\tau\psi^3}$?

$= \left(\psi^2 t^3\right)^{1/4}$ $\quad \sqrt{\psi t}$

$= \sqrt{T^2\tau^2}^{1/4}$ $\quad \sqrt{T\tau}$

FORCARAY.WPD November 30, 2009 June 29, 2010

FORCE ARRAY: $F=F(M,L,G,\hbar,c)$

$\hbar$

M\L	3	2	1	0	-1	-2	-3
-5							
-4							
-3	L^3c^{10}/G^4M^3		$Lc^7\,\hbar/G^3M^3$		$c^4\,\hbar^2/G^2M^3L$		$c\,\hbar^3/GM^3L^3$
-2		L^2c^8/G^3M^2		$c^5\,\hbar/G^2M^2$		$c^2\,\hbar^2/GM^2L^2$	
-1	$L^3c^9/G^3M\,\hbar$		Lc^6/G^2M		$c^3\,\hbar/GML$		$\hbar^2/ML^3$
0		$L^2c^7/G^2\,\hbar$		c^4/G	$\left(\frac{c^5\hbar}{GL^2}\right)^{1/2} *$	$c\hbar/L^2$	
1	$ML^3c^8/G^2\,\hbar^2$		$MLc^5/G\,\hbar$		Mc^2/L		$GM\,\hbar/L^3c$
2		$M^2L^2c^6/G\,\hbar^2$		$M^2c^3/\hbar$		GM^2/L^2	
3	$M^3L^3c^7/G\,\hbar^3$		$M^3Lc^4/\hbar^2$		$GM^3c/L\,\hbar$		G^2M^3/L^3c^2
4		$M^4L^2c^5/\hbar^3$		$GM^4c^2/\hbar^2$		$G^2M^4/L^2c\,\hbar$	
5	$M^5L^3c^6/\,\hbar^4$		$GM^5Lc^3/\hbar^3$		$G^2M^5/L\,\hbar^2$		$G^3M^5/L^3c^3\,\hbar$
6							
7							

$* = \frac{\hbar}{TL}$

~~$X = \left(\frac{GML^3}{c^6}\right)^{1/4} = \sqrt{4t} = \sqrt{T\tau}$~~

15-11

LENGTH ARRAY

CT $\quad cT^{2/3}t^{1/3}$ $\quad cT^{1/3}t^{2/3} = c(Tt^2)^{1/3}$

M \ T			0	1/3	2/3	3/3	4/3		2
1			$\frac{GM}{c^2}$						
				$\frac{t^{1/3}M^{2/3}G^{2/3}}{c}$					
1/3		$c\left(\frac{G\hbar}{c^5}\right)^{1/2}$			$t^{2/3}M^{1/3}G^{1/3}$				
0		$= ct_0$	$\left[G\hbar/c^3\right]^{1/2}$	$\left(\frac{G\hbar t}{c^2}\right)^{1/3}$	$(G\hbar c t^4)^{1/6}$	ct			
			$c(t_0^2)^{1/3}$	/ X	$c(t_0t^2)^{1/3}$ ✓		$\frac{c^2t^{4/3}}{G^{1/3}M^{1/3}}$		
				$c(t_0t)^{1/2}$				$\left(\frac{t^5c^9}{G^2M^2}\right)^{1/3}$	
−1			$\frac{\hbar}{Mc}$						$\frac{t^2c^4}{GM}$
		cT ct ct							

$c(\tau^2 t)^{1/3}$

$c(t^5T^{-2})^{1/3}$

$c\frac{t^2}{T}$

$R_s \quad R_c^2 = 8\pi^2 R^3$

$R = ct$ = metric radius

$R_s = \frac{GM}{c^2}$ = gravitational radius $= cT$?

R_c = local radius of curvature of space $= t^{2/3}M^{1/3}G^{1/3}V = \frac{L^{2/3}M^{1/3}G^{1/3}}{c^{2/3}}$

~~$R_c = c\,t^{1/3}t_0^{2/3}V$~~ $R_c = c^{1/2}\,t^{3/2}T^{-1/2}$

$\frac{C^6}{G^3} = 84.386\,812$

DENSITIES

$\frac{GM}{C^2} = R$

$\rho_1 = \frac{M}{L^3}$

$\rho_2 = \frac{M}{R^3} = \frac{C^6}{G^3 M^2}$

	$\bar{U}$	Q	-23.776 602 PROTON	-4.662404 PLANK	14.451796 D	33.565 995 ☆	52.680 194 U
ρ_1 $\frac{M}{L^3}$			+113,873 602	93.711618	52.~~199987~~ 102 000		-31.117244
ρ_2 $\frac{C^6}{G^3M^2}$			~~+113.873~~ 131.940 016	"	55,483120		-20.973 576
$\Delta \rho_2 \rho_1$			118.066 414 S^3	0	$(\alpha\mu)^3$		+10.143668 $(\alpha\mu)^9$
$\rho - \rho_1$			79.835 016 $S^2\alpha\mu$	0	$S(\alpha\mu)^2$		124.828 862 $S^3(\alpha\mu)^6$
$\rho - \rho_2$			-38.228 397 $\alpha\mu/S$	0	38.228 354 $S/\alpha\mu$		114.685194 $(S/\alpha\mu)^3$

TIMATRX2.WPD 2003-01-18

electric
m G
m M
? t

ARRAY

change to h

TIME TABLE: T=T(G,M,R,ħ,c)

[T] = 1

L

M \ R	0	0.5	+1	1.5	+2	+2.5	+3
+3	G^2M^3/hc^4		$\surd G^3M^6R^2/h^3c^5$		GM^3R^2/h^2c		$\surd GM^6R^6c/h^5$ ✓
+2.5		$\surd G^3M^5R/h^2c^6$		$\surd G^2M^5R^3/h^3c^3$		$\surd GM^5R^5/h^4$ ✓	
+2	$\surd G^3M^4/hc^7$		GM^2R/hc^2		$\surd GM^4R^4/h^3c$ ✓		M^2R^3c/h^2 ✓
+1.5		$\surd G^2M^3R/hc^5$		$\surd GM^3R^3/h^2c^2$ ✓		$\surd M^3R^5c/h^3$ ✓	
+1	T GM/c^3 ✓		$\surd GM^2R^2/hc^3$ ✓		MR^2/h ✓		$\surd M^2R^6c^3/Gh^3$ ✓
+1/2		$\surd GMR/c^4$ ✓ $a=0$		e^2 $\surd MR^3/hc$ ✓		$\surd MR^5c^2/Gh^2$ ✓	
0	$\surd Gh/c^5$ ✓		t R/c ✓		$\surd R^4c/Gh$ ✓		e^2 R^3c^2/Gh ✓
-1/2	[illegible] bound	$\surd Rh/Mc^3$ ✓		τ $\surd R^3/GM$ ✓ $a=2$		$\surd R^5c^3/G^2Mh$	
-1	h/Mc^2 ✓		$\surd R^2h/GM^2c$		R^2c/GM $a=B$		$\surd R^6c^5/G^3M^2h$
-3/2		$\surd Rh^2/GM^3c^2$		$\surd R^3hc/G^2M^3$		$\surd R^5c^4/G^3M^3$ 4	
-2	$\surd h^3/GM^4c^3$ ✓		e^2 Rh/GM^2 ?		$\surd R^4hc^3/G^3M^4$		R^3c^3/G^2M^2
-5/2		$\surd Rh^3/G^2M^5c$		$\surd R^3h^2c^2/G^3M^5$		$\surd R^5hc^5/G^4M^5$	
-3	h^2/GM^3c		$\surd R^2h^3c/G^3M^6$		R^2hc^2/G^2M^3		$\surd R^6hc^7/G^5M^6$

all $= \frac{c^4}{G}$ ✓

all $\frac{e^2}{L^2}$ r

all $\frac{GM^2}{L^2}$ ✓

$\frac{GM^2}{L^2}$

$- \frac{GM^2}{L^2}$

u h

all $= \frac{c^4}{G}$

[illegible]

Notation: In the above table h is used for ħ, the Planck constant / 2π .
√ is for entire expression

$\frac{\hbar}{G} = \frac{M^2}{c}$

$\hbar G = \frac{L^5}{T^3}$

$\frac{c^4}{G}\left(\frac{R}{L}\right)^a$

$T\tau^2 = t^3$

$$T = M^a \underset{L}{R}^b c^{\frac{3b-a-5}{2}} G^{\frac{a-b+1}{2}} \hbar^{\frac{1-a-b}{2}}$$

$ML = \frac{\hbar}{c}$

TIMATRX3.WPD 2003-01-18

TIME TABLE: T=T(G,M,R,ħ,c)

[T] = 1

M R	-3	-2.5	-2	-1.5	-1	-0.5	0
+3	$\surd G^7M^6h/R^6c^{17}$		G^3M^3/R^2c^7		$\surd G^5M^6/R^2hc^{11}$		G^2M^3/hc^4
+2.5		$\surd G^6M^5h/R^5c^{15}$		$\surd G^5M^5/R^3c^{12}$ -4		$\surd G^4M^5/Rhc^9$	
+2	G^3M^2h/R^3c^8		$\surd G^5M^4h/R^4c^{13}$		G^2M^2/Rc^5 -3		$\surd G^3M^4/hc^7$
+1.5		$\surd G^5M^3h^2/R^5c^{14}$		$\surd G^4M^3h/R^3c^{11}$		-2 $\surd G^3M^3/Rc^8$	
+1	$\surd G^5M^2h^3/R^6c^{15}$		G^2Mh/R^2c^6		$\surd G^3M^2h/R^2c^9$		T GM/c^3 -1
+1/2		$\surd G^4Mh^3/R^5c^{13}$		$\surd G^3Mh^2/R^3c^{10}$		$\surd G^2Mh/Rc^7$	
0	G^2h^2/R^3c^7		$\surd G^3h^3/R^4c^{11}$		Gh/Rc^4		$\surd Gh/c^5$
-1/2		$\surd G^3h^4/MR^5c^{12}$		$\surd G^2h^3/MR^3c^9$		$\surd Gh^2/MRc^6$	
-1	$\surd G^3h^5/M^2R^6c^{13}$		Gh^2/MR^2c^5		$\surd Gh^3/M^2R^2c^7$		h/Mc^2
-3/2		$\surd G^2h^5/M^3R^5c^{11}$		$\surd Gh^4/M^3R^3c^8$		$\surd h^3/M^3Rc^5$	
-2	$Gh^3/M^2R^3c^6$		$\surd Gh^5/M^4R^4c^9$		h^2/M^2Rc^3		$\surd h^3/GM^4c^3$
-5/2		$\surd Gh^6/M^5R^5c^{10}$		$\surd h^5/M^5R^3c^7$		$\surd h^4/GM^5Rc^4$	
-3	$\surd Gh^7/M^6R^6c^{11}$		$h^3/M^3R^2c^4$		$\surd h^5/GM^6R^2c^5$		h^2/GM^3c

Notation: In the above table h is used for ħ, the Planck constant / 2π .

√Is for entire expression

IF $\hbar = \frac{ML^2}{T}$

$G = \frac{L^3}{MT^2}$

rather than $\hbar \equiv -26976\ 924$

$G \equiv -7.175296$

→ →

	Ω	Q	B	P	D	☆	U
MASS			-23.776 602	-4.662403			52.680 194
LENGTH			-12.550068	-32.791 340			27.932478
TIME			-23.026889	-43.268 161			17.455 657
$\hbar$ ML^2/T			-25.849849	-26.976924			91.089 493
G L^3/MT^2			32.180176	-7.175296			-3.794074
$\hbar/G$			-58.030025	-19.801628			94.883 567
$G\hbar$			6.330327	-34.152220			87.295419
$\hbar c$			-15.373028	-16.500103			101.566 318
GM^2			-15.373028	-16.500102			101.566 314
ML/T^2			9.727 108	49.082579			45.701 358
GM^2/L^2			9.727 105	49.082 578			45.701 358
$\hbar^{\circ}/L^{\circ}$			9.727 108	49.082 577			45.701 358

$c^4 = 41.907\ 283$

$c^3 = 31.430\ 462$

$c^2 = 20.953641$

$c = 10.476\ 821$

$c^7 = 73.337\ 745$

2011

COTEMPREV.WPD

~~CONTEMPREV.WPD~~

TABLE Ia

[Measures in $\log_{10}$(cgs) units]

LEVEL	LENGTH	TIME =L/c	MASS	VOLUME	M/L	M ·L
units	centimeters	seconds	grams	centimeters^3	gr/cm	gr ·cm
DARK MTR.	–53.032 612	–63.509 434	14.451796	–159.097 836	67.484 408	–38.580 816
Planck c G ħ	$(G\hbar/c^3)^{1/2}$	$(G\hbar/c^5)^{1/2}$	$(c\hbar/G)^{1/2}$	$(G\hbar/c^3)^{3/2}$	c^2/G	$\hbar/c$
Planck numer	–32.791 340	–43.268 161	–4.662 403	–98.374 020	28.128 937	–37.453 745
BARYON	–12.550 068	–23.026 889	–23.776 602	–37.650 204	–11.226 534	–36.326 670
STAR	7.691 205	–2.785 617	33.565 995	23.073 614	25.874 790	41.257 200
UNIVERSE	27.932 478	17.455 657	52.680 194	83.797 432	24.747 716	80.612 672

TABLE Ib

[Measures in $\log_{10}$(cgs) units]

LEVEL	ENERGY Mc^2	POWER	FORCE	GRAVITY	M/V	M ·V
units	ergs	ergs/sec	dynes	dynes	gr/cm^3	gr ·cm^3
DARK MTR.	35.405 440	98.914 874	88.438 046	127.793 520	173.549 632	–144.646 040
Planck c G ħ	$(\hbar c^5/G)^{1/2}$	c^5/G	c^4/G	c^4/G	$c^5/\hbar G^2$	$\hbar^2/c^4G$
Planck numer	16.291 237	59.559 399	49.082 578	49.082 578	93.711 617	–103.036 423
BARYON	–2.822 960	20.203 929	9.727 108	10.854 182	13.873 602	–61.426 806
STAR	54.519 639	57.305 256	46.828 434	44.574 284	10.492 381	56.639 609
UNIVERSE	73.633 836	56.178 179	45.701 358	42.320 136	–31.117 238	6.477 626

10x16alt

$c \doteq 10.476821$
$G \doteq -7.175296$
$\hbar \doteq -26.976924$

			B	←	PLANCK	→	D	☆	U
MASS			-23.271602	$-\left(\frac{S}{\alpha M}\right)^{1/2}$	-4.662403	$\left(\frac{S}{\alpha M}\right)^{1/2}$	14.451796	33.565995	52.680194
LENGTH			-12.550068	$(\alpha M S)^{1/2}$	-32.791340	$(\alpha M S)^{1/2}$	-12.550068	7.691205	27.932478
~~FORCE~~ $\hbar$ ML²/T	—	—	—	—	~~-26.976924~~ ~~-43.268161~~	—	—	—	~~91.089493~~
L/c ~~W~~/T = L/c			-23.026889	$(\alpha M S)^{1/2}$	-43.268161	$(\alpha M S)^{-1/2}$	-23.026889	-2.785616	17.455657
G L³/MT²	—	—	—	—	-7.175296	—	—	—	—
Gh	—	—	—	—	-34.152220	—	—	—	—
Gh/c³	—	—	—	—	-65.582680	—	—	—	—
hc/G	—	—	—	—	-9, 324806	—	—	—	—
GMM			-15.373028	(αM)	-16.500102	S	22.855369	62.210840	101.566814
hc	—	—	—	—	-16.500102	—	—	—	—
FORCE ML/T²			9.727108	S^{-1}	49.082579	$(\alpha M)^{-1}$	47.955~~501~~504	46.828430	45.701358
GRAVITY GMM/L²			-29.628364	S^{-3}	49.082588	$(\alpha M)^{-3}$	46.828430	44.574282	42.320136
DENSITY			13.873602	$S^{-2}(\alpha M)^{-1}$	93.711617	$S^{-1}(\alpha M)^{-2}$	52.102000	10.492381	-31.117238
ENERGY			-2.822961	$S^{-1/2}(\alpha M)^{-1/2}$	16.291237	$S^{1/2}(\alpha M)^{1/2}$	35.405437	54.579632	73.633835
E/V			34.827243	$S^{-2}(\alpha M)^{-1}$	114.665257	$S^{-1}(\alpha M)^{-2}$	73.055638	31.446020	-10.163596

$\hbar \equiv -26.976924$

$c \equiv 10.476821$

$G \equiv -7.175296$

$hc = -16.500103$

B

$\frac{\frac{hc}{L^2}}{GM^2} = \frac{S}{\alpha\mu}$

10^x16

	Ω	Q	B	~	PLANCK	→	D	~~B~~ ☆	U
MASS	-62.004999	-42.890801	-23.776602	$\left(\frac{\alpha\mu}{S}\right)^{1/2}$	-4.662403	$\left(\frac{S}{\alpha\mu}\right)^{1/2}$	14.451796	33.565995	52.680194
LENGTH	27.932478	7	-12.550068	$(\alpha\mu S)^{1/2}$	-32.791340	$(\alpha\mu S)^{1/2}$	-12.550068	7.691205	27.932478
M/L	-90.437476	-50.582005	-11.226534	S^{-1}	28.128937	$(\alpha\mu)^{-1}$	27.001863	25.874790	24.747716
AREA			-25.100136	$\alpha\mu S$	-65.582680	$\alpha\mu S$	-25.100136	15.382410	55.864956
M/L^2			1.323534	$S^{-3/2}(\alpha\mu)^{-1/2}$	60.920277	$S^{-1/2}(\alpha\mu)^{-3/2}$	39.551932	18.183585	-3.184762
VOLUME			-37.650204	$(\alpha\mu S)^{+3/2}$	-98.374020	$(\alpha\mu S)^{3/2}$	-37.650204	+23.073612	83.797432
DENSITY	-145.802430	-65.964414	13.873602	$S^{-2}(\alpha\mu)^{-1}$	93.711617	$S^{-1}(\alpha\mu)^{-2}$	52.101000	10.492381	-31.117238
~~TEMS~~ ℏ/c/			8.600036	$(\alpha\mu S)^{-1}$	49.082579	$(\alpha\mu S)^{-1}$	8.600033	-31.882513	-72.365059
ENERGY	-41.051354	-21.937157	-2.822960	$S^{-1/2}(\alpha\mu)^{+1/2}$	16.291237	$S^{1/2}(\alpha\mu)^{1/2}$	35.405434	54.519632	73.633836
E/V	-124.858787	-45.013769	34.827244	$S^{-2}(\alpha\mu)^{-1}$	114.665257	$S^{-1}(\alpha\mu)^{-2}$	73.625638	31.446020	-10.163596
FORCE			9.727108	S^{-1}	49.082578	$(\alpha\mu)^{-1}$	47.955504	46.828430	45.701758
GRVITY			-29.628364	S^{-2}	49.082578	$(\alpha\mu)^{-2}$	46.828430	44.574282	42.320136
GM^2/L^4					114.665258	$S^{-1}(\alpha\mu)^{-3}$	71.928568	29.191878	+13.544812
POWER					59.559399	$(\alpha\mu)^{-1}$	58.432323		
TEMP									

$\frac{\hbar c}{L^2}$

$\frac{GM^2}{L^2}$

$\lambda \alpha\mu^3$

$\frac{M_U}{M_\Omega} = 114.685193$

$\frac{E_U}{E_\Omega} = 114.685190 = \left(\frac{S}{\alpha\mu}\right)^3$

$\frac{(E/V)_U}{(E/V)_\Omega} = 114.685195 = \left(\frac{S}{\alpha\mu}\right)^{3/2}$

$(E/V)_P = 114.665257$

$S(\alpha\mu)^2 = 41.609619$

$S^2(\alpha\mu) = 79.838016$

$\frac{\rho_U}{\rho_\Omega} = 114.685192$

$\left(\frac{S}{\alpha\mu}\right)^3$

$(\alpha\mu S)^{3/2} = 60.723817$

$S^{+1}(\alpha\mu)^3 = 42.736690$

$\frac{C^7}{G^2\hbar} = 114.665261 = \left(\frac{E}{V}\right)_P$

M/L 07.484410

Inverse Matter 14.451796
-53.032614 ~~55~~

-12.550068
-32.791341
20.241273

20.241273
32.791341
-53.032614

FORMULA	BARYON	←	PLANCK	→	DARK	STELLAR	UNIVERSE
M	-23.776602				14.451796	33.565995	52.680194
L	-12.550068				-12.550068	7.691205	27.932478
L ML	-36.326670	αM	-37.453744	S	1.901728	41.457199	80.612670
VOLUME							
~~DEL~~ M/L							
DENSITY							
~~ENERGY TIME~~							
~~LENT~~QCY							
~~ML~~E/V							
ML/T²							
GM²/L³							
CM²/L⁴							
hC/L³							

T=L/c

POWER

15 x 8

$\frac{c^4}{G} = 49.082578$

ENERGIES.WPD December 3, 2010 January 26, 2011

G:=-7.175296 c:=10.476821 h:=-26.976924

M:=52.680194 L:=27.932478 V = 83.797434

ENERGIES

$P \ \frac{E}{V}_{max} = \frac{c^7}{G^2 \hbar}$

114.665257 61

UNIVERSE:

M = 52,68019~~4~~ L = 27.932478 T = 17.455657 = L/c

E = – 44.432581 = ħ/T ✓

Δ = 114.685195 = $(S/\alpha\mu)^3$

E = 70.252614 = GM^2/L

Δ = 3.381222 = $(\alpha\mu)^3$

E = 73.633836 = Mc^2

Δ = 3.381222 = $(\alpha\mu)^3$

E = 77.015056 = c^4L/G

Δ = 121.447637 = $(\alpha\mu S)^3$

E = – 44.432581 = ħ/T

Δ = 118.066417 = S^3

E = 73.633836 = Mc^2

ET/V 128.230014

÷ ~~ħ~~ Ec

111.938776

-13.544820

~~2.746418~~ ~~1.485~~ ~~2104~~

29.836058

-10.163598

~~6.127640~~

-6.163598

128.230614

-10.163598

E planck = 16.291238 = ϵ

$h/T\epsilon = (\alpha\mu S)^{3/2}$

$GM^2/L\epsilon = S^{3/2} / (\alpha\mu)^{9/2}$

$Mc^2/\epsilon = (S/\alpha\mu)^{3/2}$

$c^4L/G\epsilon = (\alpha\mu S)^{3/2}$

	U	☆
$\frac{ML}{T^2}$	45.761758 ↓	46.828430 ↓
$\frac{GM^2}{L^2}$	42.320136 ↓	44.574282 ↓
$\frac{E}{V}$	-10.163596 ↓	31.446020 ↓
$\frac{c^4}{G}$	49.082578 ↑	49.082578 ↑
NET	↑	↓
L^2		
$\frac{GR}{L^3}$		
$\frac{c^4}{GL^2}$	-6.812278	33.700168

	sec	YEAR	BY	ANTILOG	
τ	19.146267	11.647164	2.647164	443.776 BY BY	
$\frac{L}{c}$ t	17.455657	9.956545	0.956545	9.047842 BY	4.524 BY ×3 = 13.571763 BY
ψ	15.765046	8.265934	MY 1.265434	184,261 MY	
$\frac{CM}{c_0}$ T	14.0744[illegible] 14.074435	6.575364	MY 0.575364	3.762 MY	
	12.383824	4.884712	TY 1.884712	76.685 TY	
	10.693213	3.194101	0.194101	1.564 TY	15.64 centuries
	9.002602	1.503490	Y "	31.878 Y	
$\frac{G^3M^3}{L^2c^7}$	7.311991		2.375428 days	237.371 days	φY = 225.7 139.5 365 2
	5.621380	0.684867 days		4.84 days	

86.400 sec in day

log = 4.936513

$$\frac{\tau}{t_0} = 62.414446 = S^{3/2}(\alpha\mu)^3$$

$$\frac{t}{t_0} = (\alpha\mu S)^{3/2} \qquad S^{3/2}(\alpha\mu)^{3/2}$$

$$\frac{\psi}{t_0} = S^{3/2} = 59.033307$$

$$\frac{T_v^2}{t_0^2} = 114.685192 \quad [0] \quad V = \left(\frac{S}{\alpha\mu}\right)^{3/2}$$

$$S^{3/2}(\alpha\mu)^{-3/2}$$

$\log_{10} = 2.059507$

log 7.499112 sec/yr

log 4.936514 sec/day

t_0 -43.268161

	log sec		/t_0
α	19.146268	~~4~~43.776 BY	$S^{3/2}(\alpha\mu)^3$
$\frac{L^2c}{GM}$	20.836879	21765 BY 22150	$S^{3/2}(\alpha\mu)^{9/2}$
$\sqrt{L^5c^9/GM^3}$	22.52749	1067,524 BY	$S^{3/2}(\alpha\mu)^6$
Life time of Brahma	21.690808	155,520 BY	$\frac{LB}{t_0} = 64.958969$ $\doteq S^{3/2}(\alpha\mu)^{21/4}$
Kalpa 1 day in life of Brahma	17.134596	4.320003 B.Y	0.2^{14}

cf. Stellar cycle 4.534 BY

1.12

$\frac{1}{t_0}$ 43.268366 sec

$\sqrt{}$ 21.634183

21.690808

cf. Life of Brahma ~~25~~

0.056625

LB 21.690808

43.381616

~~43.89~~

43.268761

0.113

LB 21.690808

$\sqrt{t_0}$ 21.634050

Δ = 0.057

	M_1	$M_2 = L\frac{c^2}{G}$	$M_3 = \frac{\hbar}{cL}$	L_1	$L_2 = \frac{G}{c^2}M$	$L_3 = \frac{\hbar}{cM}$
B	−23.776602	+15,578849	−24,903677	−12,550068	−51.905519	−13,677143
	$m_0\left(\frac{\alpha\mu}{S}\right)^{1/2}$	$M \cdot S$	$M/(\alpha\mu)$	$l_0(\alpha\mu S)^{-1/2}$	L/S	$L/(\alpha\mu)$
P	−4.662404	−4.662424	−4.662 424 ~~−9,324828~~	−32,791341	−32,791341	−32,791341
	m_0	m_0	m_0	l_0	l_0	l_0
D	14.451796	+15,578849	−24,903 677	−12,550068	−13,677121	−51 905541
	$m_0\left(\frac{S}{\alpha\mu}\right)^{1/2}$	$M \cdot (\alpha\mu)$	M/S	$l_0(\alpha\mu S)^{-1/2}$	$L/(\alpha\mu)$	L/S
✡	33,565995	35.820122	−45,144950	7,691 205	5.437078	−71,019730
	$m_0\frac{S}{\alpha\mu}$	$M \cdot (\alpha\mu)^2$	M/S^2	$l_0(\alpha\mu S)$	$L/(\alpha\mu)^2$	L/S^2
U	52.680194	56.061345	−65,386223	27.932478	24.551277	−90,133939
	$m_0\left(\frac{S}{\alpha\mu}\right)^{3/2}$	$M \cdot (\alpha\mu)^3$	M/S^3	$l_0(\alpha\mu S)^{3/2}$	$L/(\alpha\mu)^3$	L/S^3

$\frac{c^2}{G} = 28,128917 \quad \frac{\hbar}{c} = -37,453745$

$\frac{c^2}{G} \times \frac{\hbar}{c} = -9,324328 = m_0^2$

	M_1/M_2	M_1/M_3	M_2/M_3		L_1/L_2	L_1/L_3	L_2/L_3
B	-39.355449	1.127075	40.482526				
	s^{-1}	$(\alpha\mu)$	$\alpha\mu s$				
P							
D							
☆							
U	-3.381171	118.066417	121.447568		3.381201	118.066417	114.685206
	$-(\alpha\mu)^3$	s^3	$(\alpha\mu s)^3$		$(\alpha\mu)^3$	s^3	$(s/\alpha\mu)^3$

$$\frac{c^3}{G^2 k} = 114.665261$$

ML	M_1L_1	M_1L_2	M_1L_3	M_2L_1	M_2L_2	M_2L_3	M_3L_1	M_3L_2	M_3L_3
B									
P									
D									
☆									
U									

YANGHUI TRIANGLE

Also see

Pascal's Arithmetical Triangle
by A.W.F. Edward

YANGHUIS
GENERALIZATIONS OF PASCAL's TRIANGLE

EDGES

1 1 1 1 2 2 2 2

[1,1] [2,1] etc,

SYM ASYM

1234
1
2 etc.
3
4

primes, RANDOM

OPERATIONS

+, −, x, ^

TRUNCATIONS

1
1 1
1 2 1

□
3 5
3 8 5
3 11 13 5

□
□ □
6 6 1
6 12 7 1
6 18 19 8 1
6 24 37 27 9 1
6 30 61 64 36 10 1
6 36 91 125 100 46 11 1

cubes

□
□ □
□ □ □
a b c d

PASCAL

$$\frac{R!}{(R-D)!\,D!}\cdot 1^{R-D}$$

SHIFTS **FACTORS** EULER GENERALIZATION

$$\frac{R!}{(R-D)!\,D!}\,2^{R-D}$$

⋮

$$3^{R-D}$$

R = ROW
D = DIAGONAL

NOTATIONS

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}\ ;\quad \binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}\ ;\quad \binom{n}{k} = \binom{n}{n-k}$$

Applications of Yanghui's:
Venn Intersects — THE NUMBER OF INTERSECTS OF ORDER R FROM N INTERSECTING SETS

$$\# = \frac{N!}{R!\,(N-R)!}$$

Generalizations of Euler's Theorem $V - E + F = 2$
Prime Yanghui

NOTES: WOLFRAM's 256 RULES MAY BE CONSIDERED A GENERALIZATION OF PASCAL's TRIANGLE

SERIES: RECURSION, f(preceeding values)
Array: f(neighboring values)

W's rule or program is used recursively but the rule is explicit

Descartes: Number x, y, as position or address Analytic Geometry

2 tasks
1) Number as value ⇒ position, if code is known
2) position ⇒ value

Arrays, series, cellular or historic
recursive or explicit

HISTORY

LIU JU-HSIEH ?
CHIA HSIEN c. 1000
CHU SHIH-CHIEH 1303 "PRECIOUS MIRROR OF THE FOUR ELEMENTS"

EUROPE
PETRUS APIANUS 1527
BLAISE PASCAL 1654 [1623-1662]

REDISCOVERED?

The Obverse Pascal Triangle
refers to fractally to an exclusive set [on special rows]
If a template for evolution
exclusive set ~ homogenization (fewer parameters, less diversity)
could even be extinction
and fewer values

Sort of an obverse stem cell

Another direction for generalization of Yang hui's

Pascal $\text{Row}_R \sum \pm = 0^R$ $\text{row}_R \sum \frac{+}{+} = 2^R$

EULER CUBE $\text{Row}_R \sum \pm = 1^R$ $\text{row}_R \sum \frac{+}{+} = 3^R$

What Array would give

$\sum \pm = 2^R$ $\sum \frac{+}{+} = 4R$

~~$\sum \pm = M$ $\sum \frac{+}{+} = (M+2)^n$ $\frac{?}{1}$~~

Pascal Element (also Venn) $\frac{R!}{(R-D)!\,D!} 1^{R-D}$ R = row D = diagonal gives $\sum \frac{+}{+} = 2^R$ $\sum \pm = 0^R$

EULER ELEMENT $\frac{R!}{(R-D)!\,D!} 2^{R-D}$ give $\sum \frac{+}{+} = 3^R$, $\sum \pm = 1^R$

NEXT $\frac{R!}{(R-D)!\,D!} 3^{R-D}$ give $\sum \frac{+}{+}\ 4^R$, $\sum \pm = 2^R$

The Generalization appears to be:

$$\frac{R!}{(R-D)!\,D!} M^{R-D} \text{ gives } \sum \frac{+}{+}\ (M+1)^R, \quad \sum \pm = (M-1)^n$$

Generalize sum of all entries through row N

Another representation of the Pascal Yang hui is the number of intersects of order R, with N, primary element [see vol. 3]

The definition of YANG HUI may be an array whose elements are given by

$\binom{R}{D} M^{R-D}$

PASCAL M=1 $\sum \frac{+}{+} = 2^R$, $\sum \pm = 0^R$
EULER M=2 $\sum \frac{+}{+} = 3^R$, $\sum \pm = 1^R$
....
General M=M $\sum \frac{+}{+} = (M+1)^R$, $\sum \pm = (M-1)^R$

M=1
$\sum \frac{+}{+} = 2^R$
(a+b)

M=4
$\sum \pm = 2^R$
(a-b)

PASCAL	EULER	M = 3
1	1	1 1
1 1	2 1	3 1
1 2 1	4 4 1	9 6 1
1 3 3 1	8 12 6 1	27 27 9 1
1 4 6 4 1	16 32 24 8 1	81 108 54 12 1
1 5 10 10 5 1	32 80 80 40 10 1	243 405 270 90 15 1
1 6 15 20 15 6 1	64 192 240 160 60 12 1	729 1458 1215 540 135 18 1

M = 4

1

4 1

16 8 1

64 48 12 1

256 256 16 1

$\sum$ K rows

PASCAL	EULER	M = 3	M = P
$\sum_{+}^{+} \quad 2^R$	$\sum_{+}^{+} = 3^R$	$\sum_{+}^{+} = 4^R$	
$\sum_{R=0}^{K} 2^R = 2^{K+1} - 1$	$\sum_{0}^{K} 3^R = \frac{3^{K+1}-1}{2}$	$\sum_{0}^{K} 4^R = \frac{4^{K+1}-1}{3}$	$\sum_{0}^{K} P^R = \frac{(P+1)^{K+1}-1}{P-1}$

VENN DIAGRAMS are PASCAL YANGHUIS 4-9-19

N = # of elements, R = order of intersect

I = number of intersects of order R,

$$I(N,R) = \frac{N!}{R!(N-R)!}$$

Add Σ to Bell's Triangle

Σ

151
37 114
10 27 87 322
3 7 20 67 255
2 5 15 52 203
1 3 10 37 151
2 7 27 114
5 20 87
15 67
52

Δ

Bell's Triangle [Enc ~~of~~ Math p 106] E. T. BELL

1 2 5 15 52 203
1 3 10 37 151
2 7 27 114
5 20 87
15 67
52
203

Δ

Bell's initial values 1 2
1
2

can use any initial values

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$$

$$\frac{n}{k!(n-k)!} \quad \frac{n!}{(n-k)!}$$

$n - n+1$

ANOTHER PASCAL YANGHUI 04-09-19

IS THE VENN DIAGRAM

N elements and R order of intersect

$$\# = \frac{N!}{R!(N-R)!}$$ = Number of intersects of order R from N primary element

Number of intersects as F(N,R)

See Book ~~II~~ III

Cartesian Arrays vs Yanghuis

Cartesian Array = Analytic Geometry	Yanghuis
x, y, z	
Is continuous, utilizing real number	Discrete, Integers only
A number <u>pair</u> is used to locate a position in 2-space	A single number locates up to 2? positions*

THEOREM
AFTER INITIALS
$\Sigma H = a2^n$ for All Yanghuis of $[a, a]$ or $[a, b]$ species

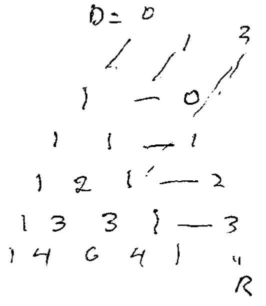
THEOREM:
THE SAME NUMBER CANNOT APPEAR MORE THAN
FOUR TIMES within A PASCAL TRIANGLE [excluding 1, of course]

Redo If R is odd $\hat{V}$ at ~ D/2

In the pascal triangle only 2 is unique in position
all other number occupy 3 or 4 positions

The numbers occupying 3 positions are
given by $\frac{(2n)!}{(n!)^2} = 6, 20, 70, 252, \ldots$

All others occupy 4 positions



Question: More than 4 positions?
i.e. can $\frac{R!}{D!(R-D)!}$ take on more than 4 values? R = row, D = diagonal or column

The largest numbers for a given R are $\frac{R!}{(R-D)!D!}$ when $R-D = D$

The 3's can't do more than 3

so if $\frac{(R+1)!}{(D+1)!(D+1)!} > \frac{R!}{D!D!}$, then $\nexists$ more than 3 or 4

$\frac{(R+1)!}{R!} > \frac{(D+1)!^2}{D!^2}$

D max
but $\hat{D} < \frac{R+1}{2}$ $\frac{5}{2}R$

i.e. if $R+1 > (D+1)^2$

proof

cf $\frac{(2n+1)!}{(n+1)!^2}$ vs $\frac{(2n)!}{(n!)^2}$ $(2n+2)(2n+1) > (n+1)^2$

i.e. $4n^2 + 6n + 2 > n^2 + 2n + 1$

$3n^2 + 4n + 1 > 0$ for all n

YANGUIS

YANGHUIS ARE GENERALIZATIONS OF PASCALS TRIANGLE.

THE "edges" in a Pascal Triangle are all ones.

1	Total	Classical Rows
1 1	for Triangle	$\Sigma_1 = 1$
1 2 1		$2 = 2$
1 3 3 1	$1+2+4+8+16 =$	$3 = 4$
1 4 6 4 1	$\Sigma_T = 2^R - 1$	$4 = 8$
. . . . .	$R = \#$ of rows	Σ_N 5 16 2^{N-1} $N = \#$ of row

IF WE REWRITE ~~FOR GENERALIZING,~~ THE TRADITIONAL TRIANGLE ~~IS WRITTEN~~ IN THE FORM:

1 1 1 1 1
1 2 3 4 5　　[1, 1]
1 3 6 10 15
1 4 10 20 35
1 5 15 35 70

The Generalized form allows the replacement of the ones by other values. e.g., symmetric, or even non-symmetric

3 3 3 3
3
3
3
[3, 3]

　3 3 3 3
5
5
5
5
[3, 5]

etc

~~The top row~~

The values in row G are given by

$$R_G = \frac{(N+G-2)!}{N!(G-1)!}[aN + b(G-1)]$$

in column G by

$$K_G = \frac{(N+G-2)!}{N!(G-1)!}[bN + a(G-1)]$$

Sums are the values in G+1

For symmetric Yanghuis $a = b$

$$R_G = K_G = \frac{a(N+G-1)!}{N!(G-1)!}$$

The rows and columns are diagonals in the original Triangle

OBVERSE

~~INVERSE~~ YANGHUIS

76 2^n property

5
0 5 5 — 2
5 0 5 × 2
10 5 5 5 5 — 4
×3 5 0 0 0 5 × 2
5 5 0 0 0 5 5
5 0 5 0 5 0 5
30 5 5 5 5 5 5 5 5 → 8

All symmetric
Yanghuis have
the "2, 4, 8, .. 2^n"
property of all ~~1~~'s 5's
whatever in the 2, 4, 8 .. 2^n row

In the case of A symmetric prime
number Yanghui, ~~th~~ the 2^n rows
are only primes

Do random
edges

Fibonacci

I

10 4
68 13

1
1 1
2 0 2
3 2 2 3 — 2^6
5 1 0 1 5
8 4 1 1 4 8
13 4 3 0 3 4 13
Powers 43 21 9 1 3 3 1 9 21 — 3^{n}_{0}
34 12 8 2 0 2 8 12 34
55 22 4 6 2 2 6 4 22 55
89 33 18 2 4 0 4 2 18 33 89
144 56 15 16 2 4 4 2 16 15 56 144 12

No result

1
2 2 — ×3
3 0 3
14 4 3 3 4 — odd, Σ 6
5 1 0 1 5 ×3
6 4 1 1 4 6
7 2 3 0 3 2 7
8 5 1 3 3 1 5 8 — odd Σ 18

SYMMETRIC YANGHUIS
ALL HAVE A "FRACTAL" LIKE
PROPERTY
OR
SOME ~~form~~ of homogeneity or exclusion
at the 2^n rows
[for laws of change?]

1
3 3
5 0 5
24 7 5 5 7 — odd Σ=10
9 2 0 2 9 × 3
11 7 2 2 7 11
13 4 5 0 5 4 13 Σ=30
60 15 9 1 5 5 1 9 15 — odd

Ask what is excluded
or what is common
to those included

2
4 4
6 0 6 ? Σ=12 14
8 6 6 8 —
10 2 0 2 10 × 3
12 8 2 2 8 12
14 4 6 0 6 4 14 — ? Σ=36 78
16 10 2 6 6 2 10 16

~~144 56 15 16 2 4 4 2 16 15 56 144 ⑫~~
~~233 200 71 31 18 6 8 6 18 31 71 200 233~~
~~377 433 271 102 49 24 14 14 24 49 102 171 433 377~~
~~610 810 704 273 151 73 38 28 38 73 151 273 704 810 610~~
11 446 ~~9472 987 1420 1514 977 424 224 111 66 66 111 224 424 977 1514 1420 987 16~~

nothing unusual

÷4
= 2368

YANG HUI S

PASCAL

$[a,b] = [1,1]$

			b = 0	1	2	3	4	5
	N = N		K_1	K_2	{1,1} K_3	K_4	K_5	K_6
a =	0	R_1		1	1	1	1	1
	1	R_2	1	2	3	4	5	6
{1,1}	2	R_3	1	3	6	10	15	21
	3	R_4	1	4	10	20	35	56
	N	R_5	1	5	15	35	70	126
	5	R_6	1	6	21	56	126	252

$[a,b] = [1,2]$

		N =	0	1	2	3	4	5
	N =		K_1	K_2	{2,1} K_3	K_4	K_5	K_6
	0	R_1		1	1	1	1	1
	1	R_2	2	3	4	5	6	7
{1,2}	2	R_3	2	5	9	14	20	27
	3	R_4	2	7	16	30	50	77
	4	R_5	2	9	25	55	105	182
	5	R_6	2	11	36	91	196	378

$[a,b] = [2,1]$

		N =	0	1	2	3	4	5
	N =		K_1	K_2	{1,2} K_3	K_4	K_5	K_6
	0	R_1		2	2	2	2	2
	1	R_2	1	3	5	7	9	11
{2,1}	2	R_3	1	4	9	16	25	36
	3	R_4	1	5	14	30	55	91
	4	R_5	1	6	20	50	105	196
	5	R_6	1	7	27	77	182	378

$[a,b] = [2,2]$

		N =	0	1	2	3	4	5
	N =		K_1	K_2	{2,2} K_3	K_4	K_5	K_6
	0	R_1		2	2	2	2	2
	1	R_2	2	4	6	8	10	12
{2,2}	2	R_3	2	6	12	20	30	42
	3	R_4	2	8	20	40	70	112
	4	R_5	2	10	30	70	120	252
	5	R_6	2	12	42	112	232	~~4144~~ 504

$[a,b] = [3,5]$

		N =	0	1	2	3	4	5
	N =		K_1	K_2	K_3	K_4	K_5	K_6
	0	R_1		3	3	3	3	3
	1	R_2	5	8	11	14	17	20
	2	R_3	5	13	24	38	55	75
	3	R_4	5	18	42	80	135	210
	4	R_5	5	23	65	145	280	490
	5	R_6	5	28	93	238	518	1008

$[a,b] = [1,6]$

		N =	0	1	2	3	4	5
	N =		K_1	K_2	K_3	K_4	K_5	K_6
	0	R_1		1	1	1	1	1
	1	R_2	6	7	8	9	10	11
	2	R_3	6	13	21	30	40	51
	3	R_4	6	19	40	70	110	161
	4	R_5	6	25	65	135	245	406
	5	R_6	6	31	96	231	476	882

GAUSS $\{\Delta . I\}$

VANGHUIS

SOME GENERALIZATIONS OF PASCAL

Σ
1 1
2 2 4
3 4 3 10
4 7 7 4 22
5 11 14 11 5 46
6 16 25 25 16 5

Fibonacci

1 Σ
1 1 2
2 2 2 6
3 4 4 3 14
5 7 8 7 5 32
8 12 15 15 12 8 70
13 20 27 30 27 20 13
21 33 47 57 57 47 33 21
34 54 80 104 114 104 80 54 34

2 4 8 15 27 47 80

□ 8X
3 5 8 1
3 8 5 16 2
3 11 13 5 32 4
3 14 24 18 5 64 8
3 17 38 42 23 5 128 16

The Obverse Fibonacci

Obverse:
~~at row 2^n~~ At row 2^n
The edges manifest themselves
e.g. p all ones
3 all 3's etc.
primes all prime (at 2, 4, 8, 16, ???

Fibonacci Obverse (Continue

55
89 33
144 56 15 16 2 4 4 2 16 15 56 144 —⑫ 10 4
233 88 41 1 14 2 0 2 14 1 41 88 233 68 26
377 145 47 40 13 12 2 2 12 13 40 47 145 377
610 232 98 7 27 1 10 0 10 1 27 7 98 232 610
987 378 134 91 20 26 9 10 10 9 26 20 91 134 378 987 —⑯ 3310 1336
244 43 7 6 17 1 0 1

20
$378 = 2 \cdot 17^2$ ✗
$17^2 = 289$

edge 1 1 2 3 5 8 13 21 34 55 89 144
1° diag 0 0 2 1 4 4 9 12 22 33 56 88 145
oscillate ±1

$\sum_n \text{edge}_n = \sum_{\text{even}\, n} 1° \text{ diag}$

33

2 6 1 3 18 4 18 15 41 47 98 134

15

Fibonacci $\boxed{\sum_{1}^{n} F_n = F_{n+2} - 1}$

Fn How many ways can we generalize? 04-05-31

ORDERS OF YANG-HUI TRIANGLES

1° ORDER PASQUAL

```
              1
            1   1
          1   2   1
        1   3   3   1
      1   4   6   4   1
    1   5  10  10   5   1
  1   6  15  20  15   6   1
1   7  21  35  35  21   7   1
     1°+2°
```

INVERSE [subtract]

```
              1                  Σ
            1   1                1
          1   0   1              2
        1   1   1   1            3
      1   0   0   0   1          4
    1   1   0   0   1   1        2
  1   0   1   0   1   0   1      4
1   1   1   1   1   1   1   1    2
                                 8
```

2° ORDER

PLUS

```
                    1            Σ
                  2   2          1
                3   4   3        4
              4   7   7   4      10
            5  11  14  11   5    22
          6  16  25  25  16   6  46
        7  22  41  50  41  22  7     94
      8  29  63  91  91  63  29  8
```

MINUS

```
                      1                      Σ
                    2   2                    1
                  3   0   3                  4
                4   3   3   4                6
              5   1   0   1   5              14
            6   4   1   1   4   6            12
          7   2   3   0   3   2   7          22
        8   5   1   3   3   1   5   8        24
      9   3   4   2   0   2   4   3   9      34
    10  6   1   2   2   2   2   1   6  10    36
  11  4   5   1   0   0   0   1   5   4  11
   7  1   4   1   0   0   1   4   1   7  12
      3   1   0   1   3   3   6   5
```

~~2n+2^n~~ 2n+2.12

3° ORDER

```
                    1                      Σ
                  3   3                    1     (2 +4)
                6   6   6                  6     (12 +6)
             10  12  12  10                18    (36 +8)
           15  22  24  22  15              44    (88 +10)
         21  37  46  46  37  21            98
       28  58  83  92  83  58  28          210
     36  86 141 175 175 141  86  36        330
                                           876
```

```
                    1                      Σ
                  3   3                    1
                6   0   6                  6
             10   6   6  10                12
           15   4   0   4  15              32
         21  11   4   4  11  21            38
       28  10   7   0   7  10  28          72
     36  18   3   7   7   3  18  36        90
       18  15   4   0   4  15  18          128
          3  11   4   4  11   3
            8   7   0   7   8
              1   7   7   1
                6   0   6
```

Add and subtract orders and inverse

FIBONACCI
YANGHUI

1
1 1
2 0 2
3 2 2 3
5 1 0 1 5
8 4 1 1 4 8
13 4 3 0 3 4 13
21 9 1 3 3 1 9 21
34 12 8 2 0 2 8 12 34
55 22 4 6 2 2 6 4 22 55
89 33 18 2 4 0 4 2 18 33 89
144 56 15 16 2 4 4 2 16 15 56 144
88 41 1 14 2 0 2 14 1 41 88
47 40 13 12 2 2 12 13 40 47
7 27 1 10 0 10 1 27 7
20 26 9 10 10 9 26 20
6 17 1 0 1 17 6
11 16 1 1 16 11
5 15 0 15 5
10 15 15 10
5 0 5
5 5
0

SQUARES
YANGHUI

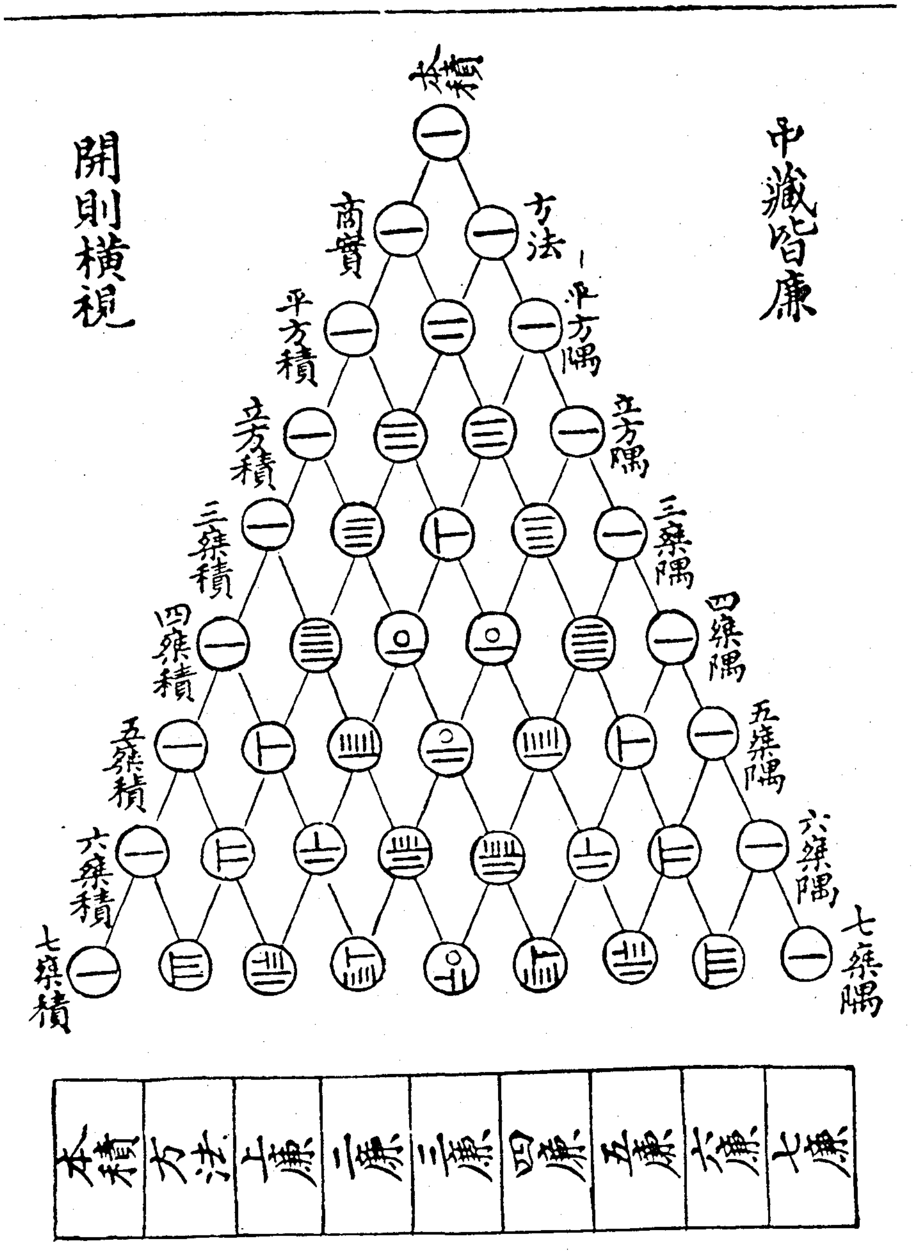
1
4 4
9 0 9
10 9 9 16
25 7 0 7 25
36 18 7 7 18 36
49 18 11 0 11 18 49
64 31 7 11 11 7 31 64
81 33 24 4 0 4 24 33 81

1
6 6
35 0 35
204 35 35 204
169 0 169

2
2 2
2 4 2
2 6 6 2
2 8 12 8 2

0
2 2
8 0 8
12 8 8 12
34 4 0 4 34
30 4 4 30

1
1 1
2 2 2
3 4 4 3
5 7 8 7 5
8 12 15 15 12 8
13 20 27 30 27 20 13
21 33 47 57 57 47 33 21
34 54 80 104 114 104 80 54
88 134 184 218



107 'Pascal's' Triangle was not invented by Blaise Pascal in 1654: it came from China. This diagram comes from Chu Shih-Chieh's *Precious Mirror of the Four Elements*, published in 1303. The caption refers to the triangle as the 'Old Method'; it had been expounded by the year 1100 by the mathematician Chia Hsien, who called it 'the tabulation system for unlocking binomial coefficients'.

of that date, now lost, entitled *Piling-up Powers and Unlocking Coefficients*, by Liu Ju-Hsieh.

The mathematician and poet Omar Khayyam discussed the Pascal Triangle somewhat indirectly about 1100. We do not know whether he got it from China or invented the elements of the system independently. But the first appearance of the Triangle in print in Europe was on the title page of the book on arithmetic of Petrus Apianus in 1527. Several succeeding mathematicians, such as Michael Stifel, considered it. And the Italian Nicolo Tartaglia, who was something of a scoundrel, claimed it as his own invention. But as far as we know, the inventor was indeed Liu Ju-Hsieh, 427 years before the appearance of the 'Pascal' Triangle in Europe.

Blaise Pascal (1623-62)

THE WORLD OF THE YANGHUI TRIANGLE

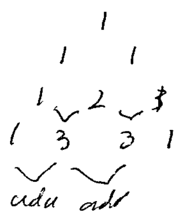
Mathematics is thought of as having two spouses–counting and measuring. These two were married and over time their progeny have included arithmetic, geometry, algebra, trigonometry, calculus, analysis, topology etc. And the mathematical dynasty is still very powerful having great influence in the kingdoms of physics, chemistry, economics, and many others. The mating of two members of the dynasty, number and position, resulted in a very gifted child known as A.G. {analytic geometry}, who was reared by one Rene Descartes (1596–1650). A.G. put number and position together in a linear and orthogonal manner that was so powerful that it eclipsed alternate ways of joining them. But A.G. had an older sibling, Chu, who had joined number and position in a more subtle and complex way. Chu would remind us that there are many ways besides that of A.G. to organize number and position, quantity and location, accumulation and direction. What is his story?

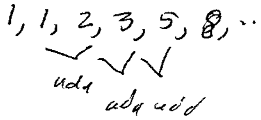
Sometime in the early 14th century the Chinese mathematician Chu Shih-Chieh published his epic *Precious Mirror of the Four Elements* which contained a numerical triangle rich in inter-related properties. This triangle, also known as the *Yanghui Triangle,* was rediscovered three centuries later by the French mathematician and physicist, Blaise Pascal (1623-1662). Today in the West it is commonly known as Pascal's Triangle. The simplest algorithm for its construction requires that the elements of each successive row be equal to the sum of the two elements located symmetrically above. This is illustrated here in the first six rows of the triangle:

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

But the triangle has many other mathematical properties. Note that the sum of the numbers in each row is a power of two. And note that each number is equal to the sum of all the numbers that are above it in the diagonal immediately above. [true for both the right-down diagonal and the left-down diagonal]. Further the numbers in each row are the coefficients in the expansion of the binomial $(a + b)^R$ where R is the number of the row, [The top single 1 is taken as row zero] The numbers in the second diagonal, 1,3,6,10,15,... are the triangular figurate numbers. [The diagonal consisting of all 1's is the zero order diagonal] The numbers in the third diagonal 1,4,10, 20, 35,... are the tetrahedral figurate numbers. The numbers in each of the subsequent diagonals are hyper tetrahedral figurate numbers corresponding to 4th, 5th, 6th, ... dimensional hyper-tetrahedra. The sums in the over-three-down-one diagonals are the Fibonacci numbers, 1,1,2,3,5,8,13,21,.... Each entry in the triangle , row R, diagonal D, has the value, R!/D!(R-D)!, which is the number of combinations of R distinct items take D at a time. The sum of the numbers in diagonal D down to and including row R is (R+1)!/(D+1)!(R-D)! . Concluding this partial list of properties, note that the sum of all the entries in the triangle down to and including row R is equal to $2^{R+1}-1$.

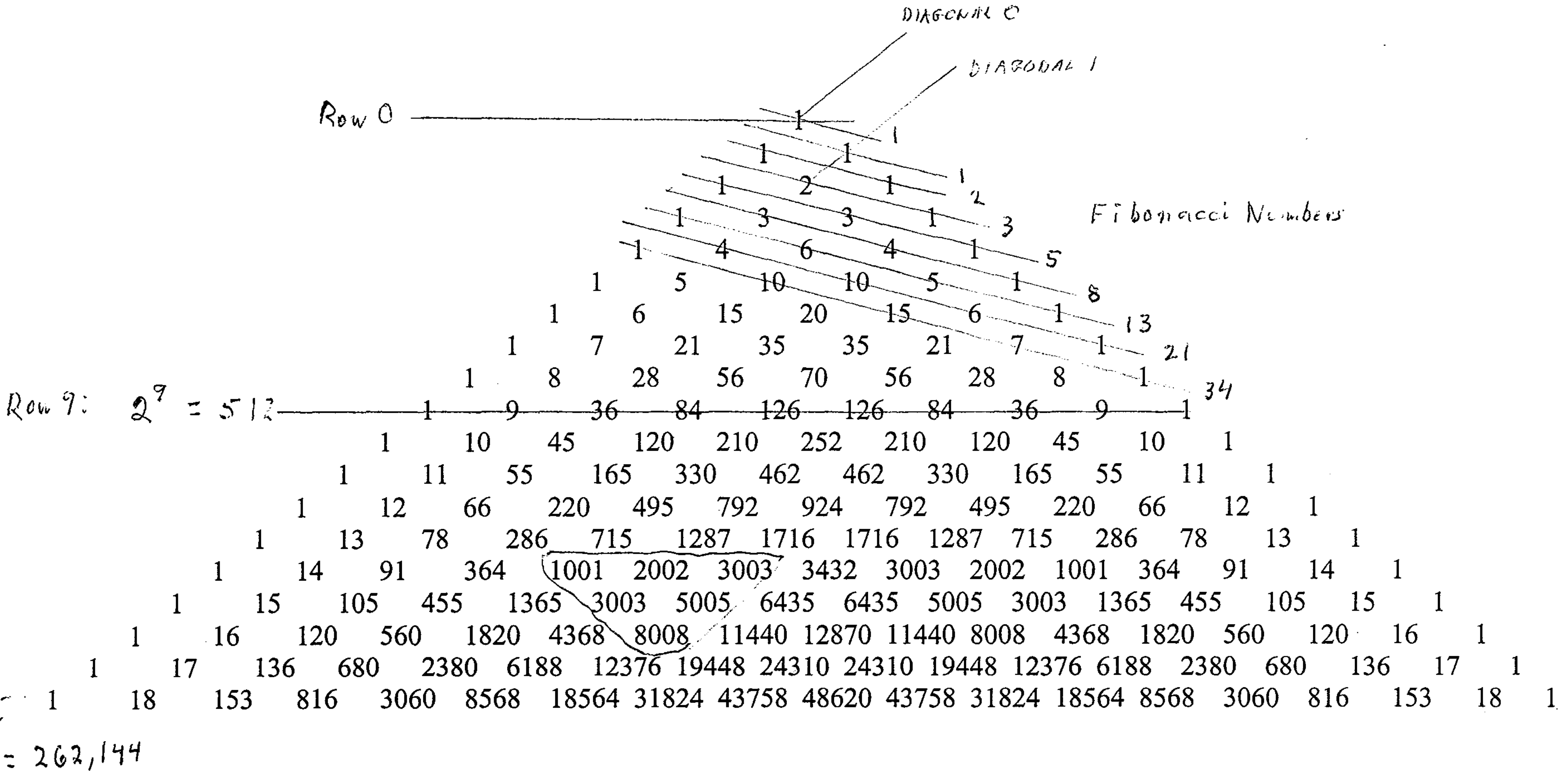
The Pascal Triangle is ~ numerically to the Fibonacci Sequence





1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1

Counting the apex 1 as row zero, the horizontal rows are designated by R. The ////diagonals are designated by D beginning with the left set of 1's as D = 0. The left–3, down–1 diagonals are designated by F. Each entry in the triangle W[R,D] = R!/D!(R-D)! The sum of the entries along diagonal D to row R is (R+1)!/(D+1)!(R-D)! The entries in a given row, R, are the coefficients of the binomial expansion, $(a + b)^{R.}$ The sum of the numbers in each row is $2^{R.}$ The total value of all the numbers in the first H rows is $2^{H} - 1$ The sums of the entries in F diagonals are the Fibonacci numbers. Symmetry allows \\\\ diagonals to be used as //// diagonals. Same with F.



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1

$$W[R,D] = \frac{R!}{D!\,(R-D)!}$$

$$\Sigma\ D \text{ to } R = \frac{(R+1)!}{(D+1)!\,(R-D)!}$$

Σ of top R rows $= 2^{R+1} - 1$; Σ for 19 rows $\underset{0 \text{ to } 18}{}$ $2^{19} - 1 = 524,287$

PASCAL'S TRIANGLE

Fibonacci 1 1 2 3 5 8 13

D=4 D=4

$W[8,4] = \frac{8!}{4!(8-4)!} = 70$

$\Sigma[8,4] = \frac{9!}{5!(9-5)!} = 126$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
$256 = 2^8 =$ R=8 ~~1 8 28 56 70 56 28 8 1~~
1 9 36 84 $\boxed{126}$ 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 362 362 330 165 55 11 1
1 12 66 220 495 692 724 692 495 220 66 12 1
1 13 78 286 715 1187 1416 1416 1187 715 286 78 13 1
1 14 91 364 1001 1902 2603 2832 2603 1902 1001 364 91 14 1
1 15 105 455 1365 2903 4505 5435 5435 4505 2903 1365 455 105 15 1
1 16 120 560 1820 4268 7408 9940 10870 9940 7408 4268 1820 560 120 16 1
1 17 136 680 2380 6088 11676 17348 20810 20810 17348 11676 6088 2380 680 136 17 1
1 18 153 816 3060 8468 17764 29024 38158 41620 38158 29024 17764 8468 3060 816 153 18 1
19 171 969 3876 11528 26232 46788 67182 79778 79778 67182 46788 26232 11528 3876 969 171 19

Total in top 8 rows $= 2^9 - 1 = 511$

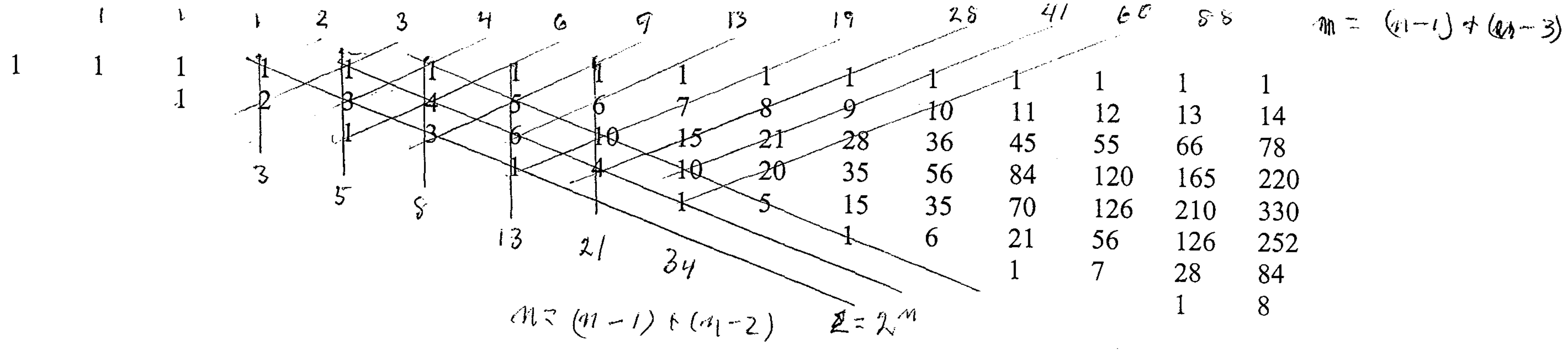
PASTRI1.WPD

																1
															1	1
														1	2	1
													1	3	3	1
												1	4	6	4	1
											1	5	10	10	5	1
										1	6	15	20	15	6	1
									1	7	21	35	35	21	7	1
								1	8	28	56	70	56	28	8	1
							1	9	36	84	126	126	84	36	9	1
						1	10	45	120	210	252	210	120	45	10	1
					1	11	55	165	330	462	462	330	165	55	11	1
				1	12	66	220	495	792	924	792	495	220	66	12	1
			1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	1
		1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	14	1
	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	105	15	1
1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	560	120	16	1

PASTRI2.WPD

				2^4	2^5						11	12				
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		1	3	6	10	15	21	28	36	45	55	66	78	91	105	120
			1	4	10	20	35	56	84	120	165	220	286	364	355	560
				1	5	15	35	70	126	210	330	495	715	1001	1365	1820
					1	6	21	56	126	252	462	792	1287	2002	3003	4368
						1	7	28	84	210	462	924	1716	3003	5005	8008
							1	8	36	120	330	792	1716	3432	6435	11440
								1	9	45	165	495	1287	3003	6435	12870
									1	10	55	220	715	2002	5005	11440
										1	11	66	286	1001	3003	8008
											1	12	78	364	1365	4368
												1	13	91	455	1820
													1	14	105	560
														1	15	120
															1	16
																1

PASTRI3.WPD



	1	1	1	2	3	4	6	9	13	19	28	41	60	88	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
		1	2	3	4	5	6	7	8	9	10	11	12	13	14
				1	3	6	10	15	21	28	36	45	55	66	78
						1	4	10	20	35	56	84	120	165	220
								1	5	15	35	70	126	210	330
										1	6	21	56	126	252
												1	7	28	84
														1	8
			3	5	8	13	21	34							

m = (n−1) + (n−3)

m = (n−1) + (n−2)

Σ = 2^n

Columns = Fibonacci Numbers

BLUE ~ 2^n
GREEN ~ Fibonacci
RED ~ Fibonacci

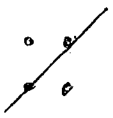
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
1	3	6	10	15	21	28	36	45	55	66	78	91	105	120		
1	4	10	20	35	56	84	120	165	220	286	364	355	560			
1	5	15	35	70	126	210	330	495	715	1001	1365	1820				
1	6	21	56	126	252	462	792	1287	2002	3003	4368					
1	7	28	84	210	462	924	1716	3003	5005	8008						
1	8	36	120	330	792	1716	3432	6435	11440							
1	9	45	165	495	1287	3003	6435	12870								
1	10	55	220	715	2002	5005	11440									
1	11	66	286	1001	3003	8008										
1	12	78	364	1365	4368											
1	13	91	455	1820	$\frac{(n+4)!}{5!(n-1)!}$											
1	14	105	560	$\frac{(n+3)!}{4!(n-1)!}$												
1	15	120	$\frac{(n+2)!}{3!(n-1)!}$													
1	16	$\frac{(n+1)!}{2!(n-1)!}$														
1	$\frac{n!}{1!(n-1)!}$															

values: $\frac{(n-1)!}{0!(n-1)!}$

in general $\frac{(n+u)!}{(u+1)!(n-1)!}$

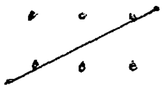
SUMS OF COLUMNS OR ROWS

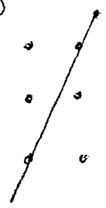
Formula to immediate right of column



$V = (V-1) + (V-1)$
$= 2(V-1)$

$V = (V-1) + (V-2)$





$V = (V-1) + (V-2)$

?

THE YANGHUI TRIANGLES

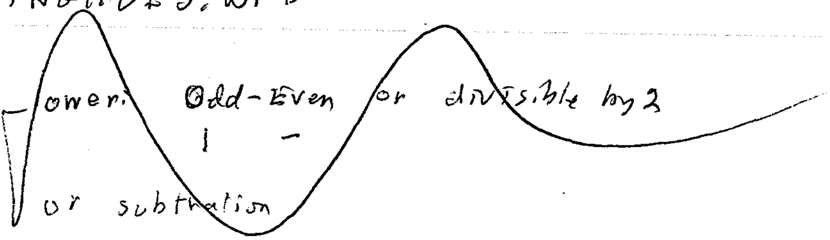
1
1 1
1 2 1
1 3 3 1 - 4
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1 - 8
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 209 252 209 120 45 10 1 X
1 11 55 165 330 462 452 330 165 55 11 1=12
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1=16
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1
1 1
1 0 1
1 1 1 1 4
1 0 0 0 1
1 1 0 0 1 1
1 0 1 0 1 0 1
1 1 1 1 1 1 1 1 8
1 0 0 0 0 0 0 0 1
1 1 0 0 0 0 0 0 1 1
1 0 1 0 0 0 0 0 1 0 1
1 1 1 1 0 0 0 0 1 1 1 1
1 0 0 0 1 0 0 0 1 0 0 0 1
1 1 0 0 1 1 0 0 1 1 0 0 1 1
1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 - 16
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1
1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 1
1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1
1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1
1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1
1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1
1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1
1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 32

YANGHUI2.WPD

THE YANGHUI TRIANGLES

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 209 252 209 120 45 10 1
1 11 55 165 330 462 452 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1
1 14 91 364 1001 2002 3003 3432 3003 2002 1001 364 91 14 1
1 15 105 455 1365 3003 5005 6435 6435 5005 3003 1365 455 105 15 1
1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1
1 17 136 680 2380 6188 12376 19448 24310 24310 19448 12376 6188 2380 680 136 17 1
1 18 153 816 3060 8568 18564 31824 43758 48620 43758 31824 18564 8568 3060 816 153 18 1
1
1 1
1 - 1
1 1 1 1
1 - - - 1
1 1 - - 1 1
1 - 1 - 1 - 1
1 1 1 1 1 1 1 1
1 - - - - - - - 1
1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 - - - - - - - - - - - - - - - 1
1 1 - - - - - - - - - - - - - - 1 1
1 - 1 - - - - - - - - - - - - - 1 - 1
1 1 1 1 - - - - - - - - - - - - 1 1 1 1
1 - - - 1 - - - - - - - - - - - 1 - - - 1
1 1 - - 1 1 - - - - - - - - - - 1 1 - - 1 1
1 - 1 - 1 - 1 - - - - - - - - - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 - - - - - - - - 1 1 1 1 1 1 1 1
1 - - - - - - - 1 - - - - - - - 1 - - - - - - - 1
1 1 - - - - - - 1 1 - - - - - - 1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1 - - - - - 1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

YANGHUI3.WPD



ower: Odd-Even or divisible by 2

| –

or subtration

from "chaos + fractals"

Odd - Even
black white

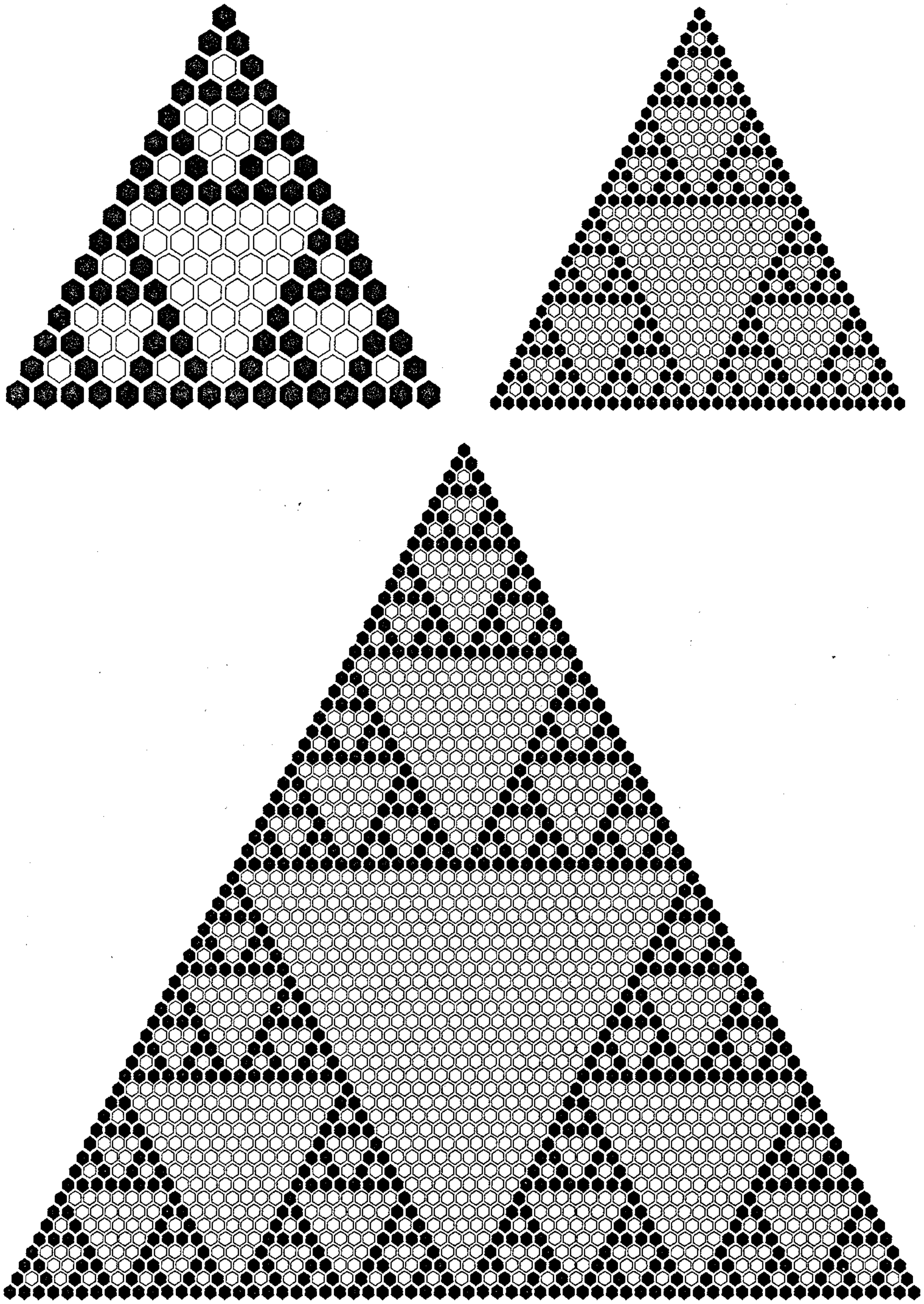


Figure 2.26 : Color coding of even and odd entries in the Pascal triangle with 16, 32, and 64 rows.

÷ by 2 Δ

WHITE SEQUENCE

$W_0=0$, $W_1=1$

$W_2=6$ $\quad W_n = 4W_{n-1} + 2^{n-1}$

$W_3=28$

$W_4=120$

BLACK

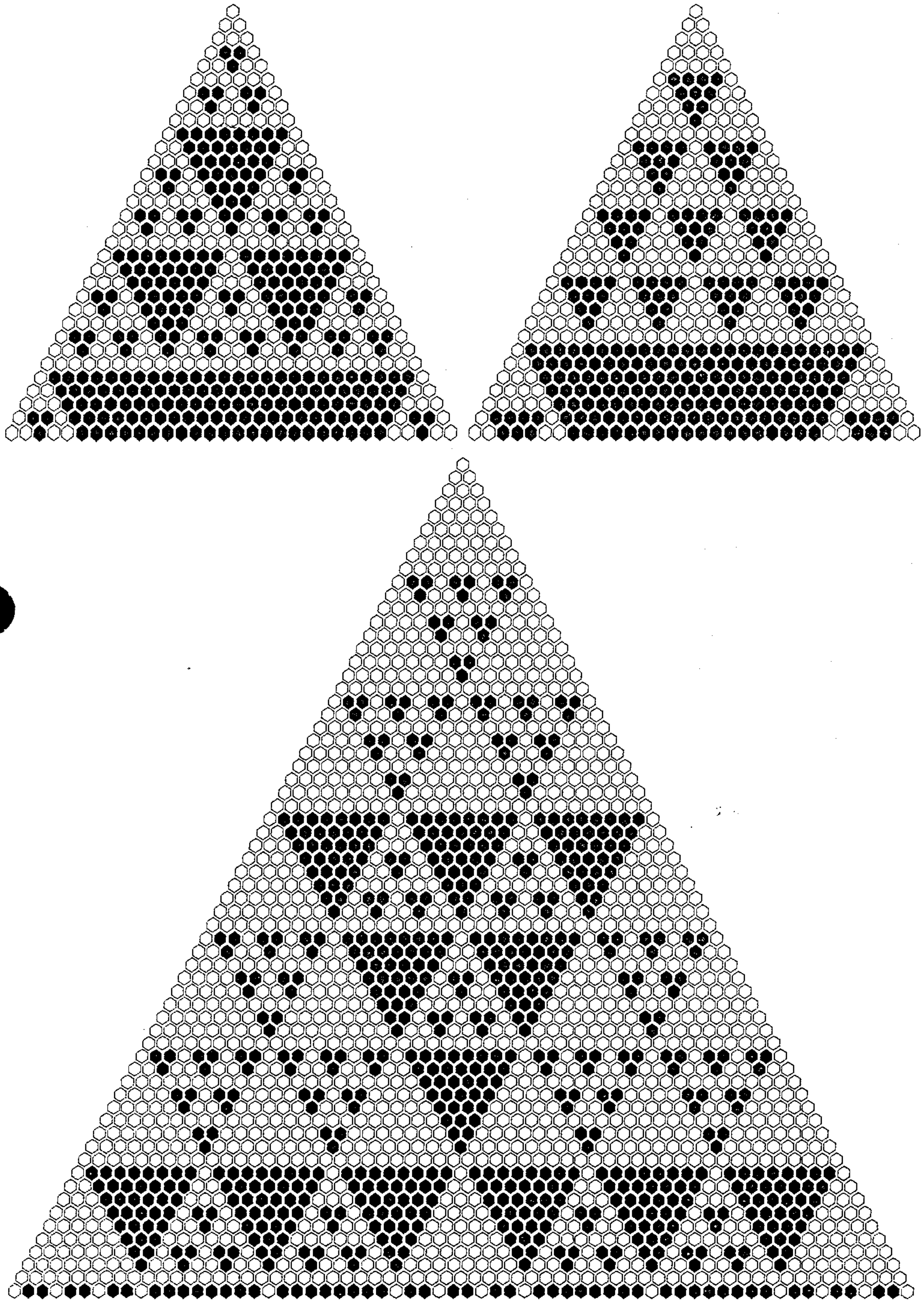
$B_1=9$ $\quad B_n = 3^{n+1}$

$B_2=27$

$B_3=81$

Species of Fractals

3 5 9



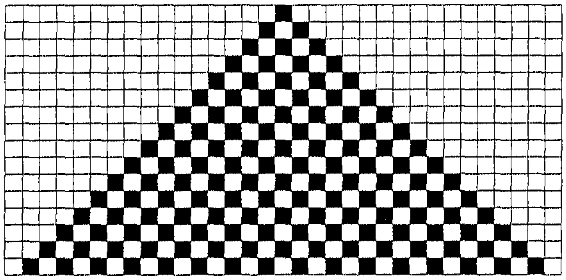
igure 2.27 : Color coding the Pascal triangle. Black cells denote divisibility by 3 (top left), by 5 (top right) nd [illegible] (bottom). 9

$\div 3 \quad \Delta$

$B_1 = 3 \qquad W_1 = 45 - 9 = 36 = 3^2 + 3^3$

$B_2 = 36$

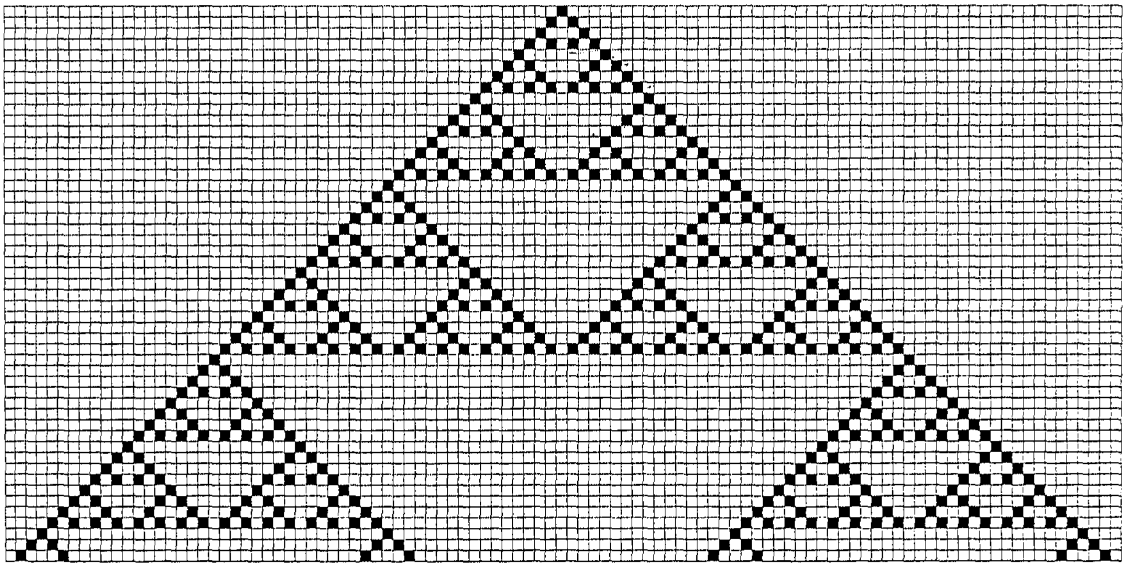
WOLFRAM NEW SCIENCE



A cellular automaton with a slightly different rule. The rule makes a particular cell black if either of its neighbors was black on the step before, and makes the cell white if both its neighbors were white. Starting from a single black cell, this rule leads to a checkerboard pattern. In the numbering scheme of Chapter 3, this is cellular automaton rule 250.

This pattern is however again fairly simple. And we might assume that at least with the type of cellular automata that we are considering, any rule we might choose would always give a pattern that is quite simple. But now we are in for our first surprise.

The picture below shows the pattern produced by a cellular automaton of the same type as before, but with a slightly different rule.



A cellular automaton that produces an intricate nested pattern. The rule in this case is that a cell should be black whenever one or the other, but not both, of its neighbors were black on the step before. Even though the rule is very simple, the picture shows that the overall pattern obtained over the course of 50 steps starting from a single black cell is not so simple. The particular rule used here can be described by the formula $a_i' = Mod[a_{i-1} + a_{i+1}, 2]$. In the numbering scheme of Chapter 3, it is cellular automaton rule 90.

Fractal Dimension $= \frac{\log 9}{\log 16} = 0,7924812$ ✓

$= \frac{\log \text{black}}{\log \text{total}}$ $\frac{\log 27}{\log 64} = 0,7924812$ ✓

EULER
YANGHUI

A GENERALIZATION OF EULER'S FORMULA
For "Cubes"

04 -09 -15

Pascal #'s
$\Sigma \pm = 0$ $\Sigma + = 2^n$

		V	E	F	S								$\Sigma\pm$	$\Sigma +$
	DIM	$+D_0$	$-D_1$	$+D_2$	$-D_3$	$+D_4$	$-D_5$	$+D_6$	$-D_7$	$+D_8$	...	D_p		
	0	1											1	1
	1	2	1										1	3
square	2	4	4	1									1	3^2
cube	3	8	12	6	1								1	3^3
tesseract	4	16	32	24	8	1							1	3^4
	5	32	80	80	40	10	1						1	3^5
	6	64	192	240	160	60	12	1					1	3^6
	7	128	448	672	560	280	84	14	1				1	3^7
	8	256	1024	1792	1792	1120	448	112	16	1			1	3^8
p=column n=row q=diagonal														
	n	2^n	$n2^{n-1}$	$\frac{n!\,2^{n-2}}{(n-2)!\,2!}$	$\frac{n!\,2^{n-3}}{(n-3)!\,3!}$	$\frac{n!\,2^{n-4}}{(n-4)!\,4!}$	$\frac{n!\,2^{n-5}}{(n-5)!\,5!}$	$\frac{n!\,2^4}{(n-4)!\,4!}$	$\frac{n!\,2^3}{(n-3)!\,3!}$	$2n(n-1)$ ✓	$2n$ ✓	1		Q_2 $\frac{n!\;2^2}{(n-2)!\;2!}$

Q_5 $\frac{n!\,2^5}{(n-5)!\,5!}$ 4   8 3   3   2   1   0

diagonal formulae

values $D_0 = 2^n$

$D_1 = \frac{n!}{(n-1)!} 2^{n-1}$

collumn formulae $D_2 = \frac{n!}{(n-2)!\,2!} 2^{n-2}$

$D_3 = \frac{n!}{(n-3)!\,3!} 2^{n-3}$

- - - - - -

General Column or diagonal $D_p = \frac{n!}{(n-p)!\,p!} 2^{n-p}$

above D's ~ Pascal D's

Euler $V - E + F = 2$

$\Sigma = D_0 - D_1 + D_2 - D_3 + D_4 - D_5 + \dots = 1$

General Horizontal Formulae

$$(2-1)^n = 2^n - n2^{n-1} + \frac{n!\;2^{n-2}}{(n-2)!\,2!} - \frac{n!}{(n-3)!\,3!} 2^{n-3} + \dots = 1$$

cf $(a+b)^n = a^n - na^{n-1}b + \frac{n!\,a^{n-2}}{(n-2)!\,2!} b^2 \dots$

q = diagonal numb

General Diagonal $Q_p = \frac{n!\,2^q}{(n-q)!\,q!}$

Diagonal q

$D_p = Q_p$ when $n - p = p$, $p = \frac{n}{2}$, $n = 2p$

We are delving into a world we cannot visualize

YANGHUIS

Euler [CUBE] ~~W~~ PASCAL

column = p
row = n

$E(n,p) = \frac{n!}{(n-p)!\,p!} 2^{n-p}$

column = K
row = R

$P(R,K) = \frac{R!}{(R-K)!\,K!}$ i^{n-p}

Standardize
Define Row Horizontal
Column vertical
Diagonal

i.e. $P(R,K) * 2^{R-K} = E(R,K)$

++ Σ rows $= 3^n$
+− Σ rows $= 1$
sum of all entries through row N
$S_2 = \frac{3^{N+1}-1}{2}$ OK
Sum check ✓

++ Σ rows $= 2^n$
+− Σ rows $= 0$
Sum of all entries through Row N
$S_1 = 2^{N+1} - 1$

$3S - S = \varsigma$
s
$S = 1 + 3 + 3^2 + \cdots + 3^G$
$3S = 3 + 3^2 + \cdots + 3^{G+1}$
$3S - S = 3^{G+1} - 1$
$S = \frac{3^{G+1}-1}{3-1}$

$S = 1 + 2 + 2^2 + \cdots + 2^a$
$2S = 2 + 2^2 + \cdots + 2^{a+1}$
$2S - S) = \underline{2^{a+1} - 1}$

THE SEARCH
FOR PATTERNS
involves what?

TETRAHEDRON

		V	E	F	S		+− Σ	++ Σ	S
	0	1					1	1	
	1	2	1				1	3	2
Triangle	2	3	3	1			1	7	4
Tetrahed	3	4	6	4	1		1	15	8
	4	8	16	14	6	1	1	45	20

OCTAHEDRON

		V	E	F	S		$\Sigma \pm$	++ $\Sigma \pm$	8
	0	1					1	1	
	1	2	1				1	3	2
Tri	2	3	3	1			1	7	4
									20
Oct	3	6	12	8	1		1	27	64
	4	12	30	28	10	1	1	81	
	5						1 1	243	

DODECAHEDRON

							++ Σ
	0	1				1	1
	1	2	1			1	3
Pentagon	2	5	5	1		1	11
	3	20	30	12	1	1	63

ICOSAHEDRON

0	1			
1	2	1		
2	3	3	1	
3			20	1

PASCAL [M=1]

$$V = \frac{R!}{(R-K)!\,K!}\,1^{R-K}$$

e.g. Row 5, R=5, K = 0 1 2 3 4 5

V = 1 5 10 10 5 1 $\Sigma_-^+ = 0$ $\Sigma_+^+ = 2^5$

EULER [M=2]

$$V = \frac{R!}{(R-K)!\,K!}\,2^{R-K}$$

Row 5, R=5 K = 0 1 2 3 4 5

V = 32 80 80 40 10 1 $\Sigma_-^+ = 1^5$ $\Sigma_+^+ = 3^5$

GENERALIZED [M=3]

$$V = \frac{R!}{(R-K)!\,K!}\,3^{R-K}$$ $\Sigma_-^+ = 2^5$ $\Sigma_+^+ = 4^5$

K = 0, 1, 2, 3, 4, 5

V = 243, 405, 270, 90, 15, 1

. . . . . .

[M = M] $\Sigma_-^+ = [M-1]^5$, $\Sigma_+^+ = [M+1]^5$

or for Row R $\Sigma_-^+ = [M-1]^R$, $\Sigma_+^+ = [M+1]^R$

TOTALS FOR G ROWS

PASCAL [M=1] $T = 2^{G+1} - 1$

EULER [M=2] $T = \dfrac{3^{G+1} - 1}{3-1}$

[M=3] $T = \dfrac{4^{G+1} - 1}{4-1}$

[M=M] $T = \dfrac{(M+1)^{G+1} - 1}{M}$

We can visualize a tessarack
and the all orthogonal "cube"
generalizations - fit with our
ideas of higher dimensions,
BUT, the continuation of the
non-orthogonal may
infer the existence of several
varieties of higher dimensional space

Non-orthogonal higher dimensionality
too difficult to visualize

RHOMBOIDS

RAMRHOMB.WPD

RAMAJUNAN NUMBERS RHOMBOID

January 8, 2004

σ9										33255424									
σ8									4870144		28385280								
σ7								713216		4156928		24228352							
σ6							104448		608768		3548160		20680192						
σ5						15296		89152		519616		3028544		17651648					
σ4					2240		13056		76096		443520		2585024		15066624				
σ3				328		1912		11144		64952		378568		2206456		12860168			
σ2			48		280		1632		9512		55440		323128		1883328		10976840		
σ1		7		41		239		1393		8119		47321		275807		1607521		9369319	
B	1		6		35		204		1189		6930		40391		235416		1372105		7997214
Δ1		5		29		169		985		5741		33461		195025		1136689		6625109	
Δ2			24		140		816		4756		27720		161564		941664		5488420		
Δ3				11		676		3940		22964		133844		780100		4546756			
Δ4					560		3264		19024		110880		646256		3766656				
Δ5						2704		15760		91856		535376		3120400					
Δ6							13056		76096		443520		2585024						
Δ7								63040		367424		2141504							
Δ8									304384		1774080								
Δ9										1469696									

RAMRHOMB.WPD

RHOMBUS OF RAMAJUNAN NUMBERS

January 8, 2004

Σ9	33255424									
Σ8	4870144	28385280								
Σ7	713216	4156928	24228352							
Σ6	104448	608768	3548160	20680192						
Σ5	15296	89152	519616	3028544	17651648					
Σ4	2240	13056	$760\overset{9}{4}6$	443520	2585024	15066624				
Σ3	328	1912	11144	64952	378568	2206456	12860168			
Σ2	48	280	1632	9512	55440	323128	1883328	10976840		
Σ1	7	41	239	1393	8119	47321	275807	1607521	9369319	
B	1	6	35	204	1189	6930	40391	235416	1372105	7997214
Δ1	5	29	169	985	5741	33461	195025	1136689	6625109	
Δ2	24	140	816	4756	27720	161564	941664	5488420		
Δ3	116	676	3940	22964	133844	780100	4546756			
Δ4	560	3264	19024	110880	646256	3766656				
Δ5	2704	15760	91856	535376	3120400					
Δ6	13056	76096	443520	2585024						
Δ7	63040	367424	2141504							
Δ8	304384	1774080								
Δ9	1469696									

26^2

$\Sigma 1_n + \Delta 1_n = \Sigma 1_{n+1} - \Delta 1_{n+1} = 2 B_{n+1}$

1912
+676
2588
−2472
116

1912
−676
1236
2
2472

$\Sigma 6 = 2 \Delta 8$

$\Delta 6 = \Sigma 4$

$\Sigma 4_n = \Delta 6_n$

RAMRHOMB.WPD RHOMBUS OF RAMAJUNAN NUMBERS January 8, 2004

Σ9										33255424									
Σ8									4870144		28385280								
Σ7								713216		4156928		24228352							
Σ6							104448		608768		3548160		20680192						
Σ5						15296		89152		519616		3028544		17651648					
Σ4					2240		13056		760$\overset{9}{4}$6		443520		2585024		15066624				
Σ3				328		1912		11144		64952		378568		2206456		12860168			
Σ2			48		280		1632		9512		55440		323128		1883328		10976840		
Σ1		7		41		239		1393		8119		47321		275807		1607521		9369319	
B	1		6		35		204		1189		6930		40391		235416		1372105		7997214
Δ1		5		29		169		985		5741		33461		195025		1136689		6625109	
Δ2			24		140		816		4756		27720		161564		941664		5488420		
Δ3				116		676		3940		22964		133844		780100		4546756			
Δ4					560		3264		19024		110880		646256		3766656				
Δ5						2704		15760		91856		535376		3120400					
Δ6							13056		76096		443520		2585024						
Δ7								63040		367424		2141504							
Δ8									304384		1774080								
Δ9										1469696									

RAMAJUNAN NUMBERS RHOMBOID

January 8, 2004

Σ9										33255424									
Σ8									4870144		28385280								
Σ7								713216		4156928		24228352							
Σ6							104448		608768		3548160		20680192						
Σ5						15296		89152		519616		3028544		17651648					
Σ4					2240		13056		760$\overset{9}{4}$6		443520		2585024		15066624				
Σ3				328		1912		11144		64952		378568		2206456		12860168			
Σ2			48		280		1632		9512		55440		323128		1883328		10976840		
Σ1		7		41		239		1393		8119		47321		275807		1607521		9369319	
B	1		6		35		204		1189		6930		40391		235416		1372105		7997214
Δ1		5		29		169		985		5741		33461		195025		1136689		6625109	
Δ2			24		140		816		4756		27720		161564		941664		5488420		
Δ3				116		676		3940		22964		133844		780100		4546756			
Δ4					560		3264		19024		110880		646256		3766656				
Δ5						2704		15760		91856		535376		3120400					
Δ6							13056		76096		443520		2585024						
Δ7								63040		367424		2141504							
Δ8									304384		1774080								
Δ9										1469696									

THE FIBONACCI RHOMBOID

Σ_8	987	1597													
Σ_7	377	610	987	1597											
Σ_6	144	233 -	377	610	987										
Σ_5	55	89 -	144	233	377	610									
Σ_4	21	34 -	55	89	144	233	377	610				2584			
Σ_3	8	13 -	21	34	55	89	144	233	377	610	987	1597			
Σ_2	3	5 -	8	13	21	34	55	89	144	233	377	610	987		
Σ_1	1	2 -	3	5	8	13	21	34	55	89	144	233	377	610	
F	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377
Δ_1	1	0	1	1	2	3	5	8	13	21	34	55	89	144	
Δ_2	1	1	0	1	1	2	3	5	8	13	21	34	55		
Δ_3	0	1	1	0	1	1	2	3	5	8	13	21			
Δ_4	1	0	1	1	0	1	1	2	3	5	8				
Δ_5	1	1	0	1	1	0	1	1	2	3					
Δ_6	0	1	1	0	1	1	0	1	1						
Δ_7	1	0	1	1	0	1	1	0							
Δ_8	1	1	0	1	1	0	1								

$\sum_{1}^{n} F_i = F_{n+2} - 1$ $\qquad$ $F_{n+2} = F_{n+1} + F_n = \sum_{1}^{n} F_i + 1$

$\Sigma_1 + \Delta_1 = \Sigma_1 - \Delta_1$ (shifted 1) ; $\Sigma_2 - \Delta_2 = 2[\Sigma_1 + \Delta_1]$

$$A_{n+2} = 10A_{n+1} - 10A_n$$

TWO SEQUENCES

RHOMBOIDS

A

[0, 1]

from Explicit

$\Sigma_4 A$ 1224 10881

$\Sigma_3 A$ 123 1101 9780

$\Sigma_2 A$ 12 a 111 c 990 8790

$\Sigma_1 A$ 1 11 100 890 7900

A 0 1 b 10 d 90 800 7100 63000 559000

$\Delta_1 A$ 1 9 80 710 6300 55900

$\Delta_2 A$ 8 71 630 5590 49600

$\Delta_3 A$ 63 559 4960 44010

$\Delta_4 A$ 496 4401 39050

$\Delta_5 A$ 3905 34649

For all rhomboid

a + b = c − d

$$\boxed{\Sigma_{A}^{n}+\Delta_{A}^{n} = \Sigma_1^{n+1} - \Delta_1^{n+1}}$$

B

[1, 2]

$\Sigma_3 B$ 117 972 8550

$\Sigma_2 B$ 15 102 870 7680

$\Sigma_1 B$ 3 12 90 780 6900

B 1 2 10 80 700 6200 55000 488000

$\Delta_1 B$ 1 8 70 620 5500

$\Delta_2 B$ 7 62 550 4880

$\Delta_3 B$ 56 488 4330

$\Delta_4 B$ 433 3842

A + B = 1, 3, 20, 170, 1500, 13300, 118000,

A − B −1 −1 0 10 100 900 8000 71000

= 10 A

$\Delta_1 B + \Sigma_1 B$ = 4, 20, 160, 1400, 12,400

$\Sigma_1 B - \Delta_1 B$ = 2, 4, 20, 160, 1400

$\Sigma_1 A + \Delta_1 A$ = 2, 20, 180, 1600, 14,200

$\Sigma_1 A - \Delta_1 A$ = 0, 2, 20, 180, 1600

FACTORI TRIANGLE

$F(n) = n \cdot F(n-1)$

RHOMBOID

n! ~~n(n+1) = n+2~~

Σ ↑

11743

1631 10112

261 1370 8742

49 212 1158 7584 57720

11 38 174 984 6600 61120

3 8 30 144 840 5760 45360

~~(n+1) - n(n-1)~~ N! 1 2 6 24 120 720 5040 40320 362880 3628800

1 4 18 96 600 4320 35280 322560 3265920

↓ 3 14 78 504 3720 30960 287280 2943360

Δ 11 64 426 3216 27240 256320 2656080

53 362 2790 24024 229080 2399760

309 2428 21234 204840 2170680

2119 18806

16687

n = 1 2 3 4 5

$\Sigma_1 - \Delta_1 = 2 \quad 4 \quad 12 \quad 48 \quad 240$

2 3 4 5

$(\Sigma_1 - \Delta_1)_n = n(\Sigma_1 - \Delta_1)_{n-1}$

6486

$$\begin{array}{r} 2790 \\ 8742 \\ \hline 11532 \end{array}$$

5046

$$\begin{array}{r} 8742 \\ 2790 \\ \hline 12 \end{array}$$

504046

50446

984

24 + 720 = 744

504

$\frac{1488}{2} = 744$

126

$$\begin{array}{r} 174 \\ 78 \\ \hline 252 \end{array}$$

$\frac{252}{2} = 126$

① + $\boxed{-1}$ = 2F+

Tautologies

$(F_n + F_{n+1}) + (F_{n+1} - F_n) = 2F_{n+1}$

$(F_n + F_{n+1}) - (F_{n+1} - F_n) = 2F_n$

```
/
?
C
D   \   •   :   A   K   L   M   N   R   S   ã   ä   å   ñ           &
'   +   ,   -   .   ,   ƒ   £
¤   |   §   Ä   Ç   È   É   ç   è   í   ð   ñ   ò           0   4
5   8   9   :   w   y   z   ½   á   ä   æ   ç   Œ       Â   Ã   Äú÷ñëñëñ
ëñëñ÷÷è÷ ÷âññññññññññññâñññññë ññññ ñññññññ ñññññññññññññññññññññññññññññññññ
ññññ ñ ñ ñ
 @          P


        Y                                   >           r   Ì   Í   Ñ   ü
      !
  &   H   ™       š   >
  œ               ¬   Ì   Í   Î   Ï   Ð   Þ   þ   ÿ                   %
  8   9
'
   L   M   N       Î   Â   ,   -
  Å                    â       ç               Ã   Å   Sûûôôîîàâèèàèàèè
                                   Îûûôèûa                 èèèèèèûèèûèûîà   5
     $
```

$n+1 = n(n+1) \approx 2^n$

1, 2, 2, 4, 8

EULER NUMBERS (p 581)

~~TRIANGLE~~

RHOMBOID

$a + b = 2770 = c - d$

Σ

↑ 1584

72 1612 53352

+6 66 $\overset{a}{1446}$ $\overset{c}{51906}$

E 1 5 61 1385 50,521 2,702,765 199,360,981

4 56 $\overset{b}{1324}$ $\overset{d}{49136}$

↓ 52 1268 47812

Δ 1216 46644

$$\begin{array}{r} 1446 \\ \underline{1324} \\ 2770 \end{array}$$

$$\begin{array}{r} 1512 \\ \underline{1268} \\ 2780 \end{array}$$

$$\begin{array}{r} 1584 \\ \underline{1216} \\ 2800 \end{array}$$

Tautologies =

$$\frac{(+1) - (-1)}{2} = E-$$

$$\frac{(+1) + (-1)}{2} = E+$$

Only tautologies

04-12-01

I Golden Section
II Stem Cell
ARRAYS AND MAPPING
GROWTH
IN THE NATURAL
ORDER

RHOMBOIDS

~~ARRAYS~~ WITH EQUAL Δ's
repetitive Δ's

A SERIES WHOSE Δ's and Σ's are all the same
alternate skips

34 55
13 21 34 55
Σ_2 5 8 13 21 34
Σ_1 2 3 5 8 13 21 34 55 89 144
Fibonacci Series F 1 1 2 3 5 8 13 21 34 55 89 144
Δ_1 0 1 1 2 3 5 8 13 21
Δ_2 0 1 1 2 3 5 8
0 1 1 2 3

$$\boxed{\Sigma_N = F_{N+2} - 1}$$

~~Scien~~
Science
Whitehead
Repetition
Fibonacci
Nature

2^n Rhomboid

$$\boxed{\Sigma_N = 2^{n+1} - 1}$$

$2 \cdot 3^n$
$2^2 \cdot 3^n$
$2^3 \cdot 3^n$

$3^4 \cdot 2^n$ Σ_4 162
$3^3 \cdot 2^n$ Σ_3 54 108 216 432
$9 \cdot 2^n$ Σ_2 18 36 72 144 288
$3 \cdot 2^n$ Σ_1 6 12 24 48 96 192
related to Pascal horizontal rows 2^n 2 4 8 16 32 64 128 256
Δ_1 2 4 8 16 32 64 128
Δ_2 2 4 8 16 32 64
2 4
2

see also the ~~bo~~ $(1+b)^n$ $(1-b)^n$ etc

PASCAL
1 20
1 1 2
1 2 1 2^2
1 3 3 1 2^3
1 4 6 4 1 2^4

---

18 36 72 144 288
Δ_1 18 36 72 144
Δ_2 18 36 72

The Δ's of Each Σ in the above are equal

For 2^n and all its Σ's, their Δ's are equal but their Σ's change

cf. stem cell
choose ↓ unchanged
choose ↑ new

$(b+1)^n$ $b(b+1)^n$ $b^2(b+1)^n$ $b^3(b+1)^n$

$(b+1)^5$

$(b+1)^4$ $b(b+1)^4$

$(b+1)^3$ $b(b+1)^3$ $b^2(b+1)^3$

$(b+1)^2$ $b(b+1)^2$ $b^2(b+1)^2$ $b^3(b+1)^2$

$(b+1)$ $b(b+1)$ $b^2(b+1)$ $b^3(b+1$ $b^4(b+1)$

1 b b^2 b^3 b^4 b^5

$(b-1)$ $b(b-1)$ $b^2(b-1)$ $b^3(b-1)$ $b^4(b-1)$

$(b-1)^2$ $b(b-1)^2$ $b^2(b-1)^2$ $b^3(b-1)^2$ $b^3(b-1)^n$

$(b-1)^3$ $b(b-1)^3$ $b^2(b-1)^3$ $b^2(b-1)^n$

$(b-1)^4$ $b(b-1)^4$ $b(b-1)^n$

$(b-1)^5$ $(b-1)^n$

$(b+1)^5 b^n$
$(b+1)^4 b^n$
$(b+1)^3 b^n$
$(b+1)^2 b^n$
$(b+1) b^n$
b^n
$(b-1) b^n$
$(b-1)^2 b^n$
$(b-1)^3 b^n$
$(b-1)^4 b^n$
$(b-1)^5 b^n$

FULCRUM NUMBERS

B

Iterated
DIFFERENCE TRIANGLES

→ ~~Add~~ Sums

B
1 6 35 204 1189 6930 40391 235416 ↓ diff
5 29 169 985 5741 33461 195025
24 140 816 4756 27720 161564
116 676 3940 22964 133844
560 3264 19024 110880
2704 15760 91856
13056 76096
63040

← diff

a sum is not the opposite of a diff

diff

reversing direction does not +⟷−

ΔB
4 19 92 444 2144 10352 49984
15 73 352 1700 8208 39632
58 279 1348 6508 31424
221 1069 5160 24916
848 4091 19756
3243 15665
12422

diff

$\Delta_2 B$
11 43 163 627 2395 9179
32 120 464 1768 6784
88 344 1304 5016
256 960 3712
704 2752
2048

diff

goes into "oscillation
Negative values !!

$\Delta_3 B$
21 56 168 448 1344
35 112 280 896
77 168 616
91 448
357

diff

14 42 14 266
28 −28 252
$\Delta_4 B$
0 280
280

14 −28 280
−42 308
$\Delta_5 B$
350

−56 392
448

```
OTECT=AUTO
          @PJL SET ECONOMODE=OFF
                                @PJL SET RESOLUTION=1200
                                                        @PJL SET BITSPERPIXEL=1



0F╡ó4@se:)↓↔-Bvi;*→▲.é┘ßP§☻♦#å·µR▬♥♦$Æ ╠Z◄☺♠ ù ╤]↕☻!G╕ñ5&                /
0I╝⌐6'
      1)↑∟,>ob8(↑∟+=na8¶☻♥#˜≡█M¶☻♥"|ϵ┘L►☺♣▼Ä ╟W►☺♣▼î —V&
```

PRIME PRE-RHOMBOIDS

$\exists$ several "paths" classical: $\Sigma_1\Sigma_2\Sigma_3$ and $\Delta_1\Delta_2\Delta_3$
but try $\Sigma\Delta$ combinations
$\Delta\Sigma$
some reversible, some ~~irreversible~~

Σ_3 ㉑ 33 50 72 96 120 144 172 206 230 274 308
$\Delta_1\Sigma_3$ 12 17 22 24 24 24 28 34 24 44 34
Σ_2 ⑧ ⑬ 20 30 42 54 66 78 94 112 128 146 162
$\Delta_1\Sigma_2$ ⑤ 7 10 12 12 12 12 16 18 16 18 16

6·2 6·3 6·4 6·5 6·6 6·7 6·10 6·13 6·14
Σ_1 ③ ⑤ ⑧ 12 18 24 30 36 42 52 60 68 78 84
PRIMES ① 2 3 5 7 11 13 17 19 23 29 31 37 41 43
Δ_1 1

~~$\Delta_1\Sigma_1$ 2 3 4 6 6 6 6 8 10 8 8 10 6~~
$\Delta_1\Sigma_1$ 2 3 4 6 6 6 6 6 10 8 8 10 6

$\boxed{\Sigma_1\Delta_1\Sigma_1 \quad 5 \quad 7 \quad 10 \quad 12 \quad 12 \quad 12 \quad 12 \quad 16 \quad 18 \quad 16 \quad 18 \quad 16}$

$$\Sigma_1\Delta_1\Sigma_1 = \Delta_1\Sigma_2$$

$\Delta_2\Sigma_2$ 2 3 2 0 0 0 4 4 -2 2 -2

THE VERTICAL SLICES

If single member sub-groups are included, the total number of sub-groups of all sizes, T, in a group of N members will be given by the sums of the vertical columns in the above table.

N=	1	2	3	4	5	6	7	8	9	10
T=	1	3	7	15	31	63	127	255	511	1023

where the values of T are given by the formula:

$$3) \qquad \mathbf{T = 2^N - 1}$$

Note that T is equal to the total number of **networks** of all sizes possible in a group of size N.

The sum of all the numbers in the matrix can be calculated by adding the numbers in the above T sequence. The sums of the values of T to the Nth column are given by,

$$4) \qquad \mathbf{\Sigma T_N = 2^{N+1} - (N+2) = 2T_N - N}$$

THE DIAGONAL UP-TO-THE-RIGHT SLICES [////]

The first row in the next table gives the sums, Σ[//], of the numbers in the up-to-the-right slices of the first matrix. The second row, ΔΣ[//], gives their differences.

Σ[//] = ΣF	1	2	4	7	12	20	33	54	88	143
ΔΣ[//] = F		1	2	3	5	8	13	21	34	55

Note that the numbers in the ΔΣ[//] row are the **Fibonacci Numbers**, F.
The second matrix will consist of a row of Fibonacci numbers, F, together with other rows giving sums and differences.

N	1	2	3	4	5	6	7	8	9	10	11
Σ_3F	1	4	11	25	51	97	176	309	530	894	1490
Σ_2F	1	3	7	14	26	46	79	133	221	364	596
ΣF	1	2	4	7	12	20	33	54	88	143	232
F	1	1	2	3	5	8	13	21	34	55	89
ΔF	0	1	1	2	3	5	8	13	21	34	55
Δ_2F	1	0	1	1	2	3	5	8	13	21	34
Δ_3F	-1	1	0	1	1	2	3	5	8	13	21
Δ_4F	2	-1	1	0	1	1	2	3	5	8	13
Δ_5F	-3	2	-1	1	0	1	1	2	3	5	8
Δ_6F	5	-3	2	-1	1	0	1	1	2	3	5
Δ_7F	-8	5	-3	2	-1	1	0	1	1	2	3

FRACTALS

3 5

FRACTDIM.WP6
[91/06/10;96/01/13;96/04/04;97/09/26;98/08/21;98/09/29]

FRACTAL DIMENSION

The modern concept of what we call a *fractal* probably began with the discovery by Galileo of the moons of Jupiter. Through subsequent centuries seeing the same form on two different scales — Copernicus' planets revolving about the sun and Galileos moons revolving about Jupiter — intrigued the imaginations of philosophers, scientists, and mathematicians. Emmanuel Swedenborg (1734) noted, " Nature is always the same and identical with hereself", while Jonathan Swift (1733) captured the idea in verse,

So, Naturalists observe, a Flea
Hath smaller Fleas that on him prey,
And these have smaller Fleas to bite 'em,
And so proceed ad infinitum.

Lewis Fry Richardson (1922) repeated this motif ,

Big whorls have little whorls,
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscosity.

The concept of fractal also emerged in attempts to explain why the sky is dark, the so-called Cheseau-Olbers Paradox. Speculators in this area included Immanuel Kant (1755), Johann Lambert (1761), John Herschel (1848), Edward Fournier d'Albe (1907) and Carl Charlier (1922). Mathematicians pursued like concepts through their interest in self-similar sets, Georg Cantor (1915), and "monster" curves, Felix Hausdorf (1914). But the ultimate sealing of the fractal concept both by generalizing it and naming it was the work of the mathematician, Benoit B. Mandelbrot (1977). And today fractals are everywhere.

It has been a matter of much amazement on the part of philosophers from the Greeks to Einstein that the structures of pure thought we call mathematics appear to have an isomorphic relation to the physical world. That mathematical constructs can be successfully used to explain and predict physical phenomena is itself a phenomenon that up to the present has eluded explanation. However, there are hiati in the successful representations of the world by mathematics. In particular several difficulties arise when treating the infinitely large and the infinitesimally small. While the geometry of Euclid, for example, has been most useful in the solution of myriads of problems, its sizeless points, diameterless lines, and thickless planes frequently lead to singularities and non-sensical physical conclusions. When mathematical thinking turned to the paradoxes implicit in the infinitely large and small, it opened new regions to the successful mathematical representation of the physical world.

The sizeless points of Euclid vs. the finite atoms of nature are but one example of the general dichotomy of continuum vs discretum. There is the continuousness of geometry vs. the discreteness of arithmetic; the continuous real numbers vs the discrete natural numbers; in technology, the analogue vs. the digital; in space, extension vs. separation; and in time, duration vs. interval. There appear to be two distinct worlds, or is it perhaps only two world descriptions, that need to be reconciled — the classical world of continuity and the quantized world of Max Planck.

There have been many mathematical approaches to the resulting paradoxes. Some, which should be mentioned, are Cantor's studies of transfinite sets, Hausdorf and Besicovitch's dimension, Lesbegue's theory of measure, and Mandelbrot's fractal dimension. Also related to this area are the finite difference calculus and some of the work of Buckminster Fuller. All are concerned with bridging the gap between the sizeless elements of abstract thought and the finite elements of physical experience.

The development of the concept of fractal, pioneered by Mandelbrot, has led to new isomorphisms between the formulae of mathematics and the laws and patterns of nature. Complex patterns in nature, such as shore lines and mountain ridge contours, always considered too complicated to be mathematically treated, have suddenly been made accessible through relatively simple expressions. At the present time not only are unexpected new isomorphisms being generated, but reexamination of classical models in such areas as geology and astronomy has led, through the fractal approach, to new and deeper insights.

SPACES OF FRACTIONAL DIMENSION

In enquiring into what ways the sizeless species of thought may be rendered useful representations of the finite elements of physical experience, one device is the concept of fractal or fractional dimension. The idea of fractal dimension requires abandoment of the view of homogeneity of space. Traditionally, conceptual spaces from Euclid to Riemann have been uniform or homogeneous spaces. However, to conform to physical space our conceptual spaces must be allowed to contain *gaps* or regions of "under density" and *fills* or regions of "over density". Only those spaces devoid of gaps and fills, having uniform density, turn out to have the integral dimensions, one, two, three,... of the spaces of mathematical thought. Thus to render our concepts of space more compatible with physical space, the concept of variable density, gaps and fills, turns out to be useful.

One approach to spaces with fractional or fractal dimension can be formulated as follows:
First consider spaces consisting only of two values of density, elements possessing extension and gaps possessing separation.
Let **E** represent an *element* possessing extension. An element can be a line segment, square, cube, etc. and let **u** be a unit of length, area, volume, etc.
The *extension* of E is measured in units **u**. (for example E = 5u, 8u,...eu, etc)

Let **G** represent a gap or *no-element*, whose *separation* is also measured in units **u.** (G=5u, 8u,...gu, etc). Next construct a module out of elements (E's) and gaps (G's). Let **M** represent a *module* composed of R elements and gaps together. Let A be the number of elements in **M**. The extension of **M** will be A **E** = Aeu, and the separation contained within **M** will be (R-A)**G** = (R-A)gu, giving the size of **M** = AE + (R-A)**G**. If elements and no-elements are of the same size, **E**=**G** then the size of **M** will be = RE.

With A = the number of elements in **M** and R the total of elements and gaps, fractal dimension d is defined by $A = R^d$, or d = log(A)/log(R).

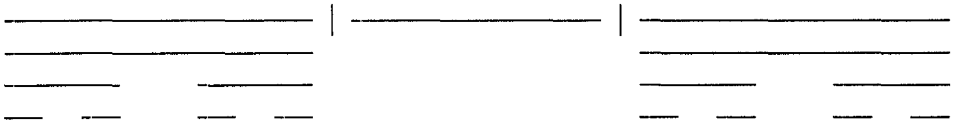
If we note that extension is manifested as appearance and separation as emptiness, then this so-called Hausdorf fractal dimension is the ratio of the logarithms of the number of appearance segments in a module to the number of appearance plus emptiness segments in the module. Or d is the ratio of the logarithms of the manifested to the total manifested and unmanifested.

In order that fractal dimension be consistent with classical notions of dimension, the fractal dimension must reduce to ordinary dimension when all segments are manifest, no gaps. That is whenever a line, area, or volume is filled in completely, the dimension should be an integer.

Examples:

I The Cantor Set

Take as the element a line segment of length 3 units = ___.
E = ___
Let R = 3, then **M** = 3 **E** = _________ = 9 units
Remove the central **E**, ___ ___ leaving A = 2
The fractal dimension of the Cantor set is then,
d = log(2)/log(3) = 0.631
The Cantor set continues this operation with the resulting
d = log(manifest)/log(total) = 0.631



II A straight line

Take u, **E**, and **M** as before
R again = 3 **M** = 3 **E** = _________ = 9 units
If the line is left solid, A then is = 3 and
the fractal dimension d = log(3)/log(3) = 1, which is the proper dimension for a line.

INVWANG.DBF. May 30, 2004

```
                               1
                              1 1
                             1 0 1
                            1 1 1 1
                           1 0 0 0 1
                          1 1 0 0 11
                         1 0 1 0 1 0 1
                        1 1 1 1 1 11 1
                       1 0 0 0 0 0 0 01
                      1 1 0 0 0 0 0 0 11
                     1 0 1 0 0 0 0 01 0 1
                    1 1 1 1 0 0 0 0 1 1 1 1
                   1 0 0 0 1 0 0 0 1 0 0 0 1
                  1 1 0 0 11 0 0 1 1 0 0 1 1
                 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1
                1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
               1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
              1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
             1 01 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
            1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
           1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
          1 1 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 1 0 0 1 1
         1 0 1 0 1 0 1 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 1
        11 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1
       1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 01
      1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 0 1 1
     1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1 0 0 0 0 0 1 0 1
    1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 1
   1 0 0 0 1 0 0 01 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1
  11 0 0 1 1 0 0 1 1 0 0 1 1 0 0 1 1 0 0 11 0 0 1 1 0 0 1 1
 101 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 01 0 1 0 1 0 1 0 1
1 111 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 11 1 1 1 1 1 1 1 1 1
```

INVWANG2.DBF. May 30, 2004

THE INVERSE PASCAL TRIANGLE
SUBTRACTION INSTEAD OF ADDITION

```
                                                              1
                                                            1   1
                                                          1   0   1
                                                        1   1   1   1
                                                      1   0   0   0   1
                                                    1   1   0   0   1   1
                                                  1   0   1   0   1   0   1
                                                1   1   1   1   1   1   1   1
                                              1   0   0   0   0   0   0   01
                                            1   1   0   0   0   0   0   0   1   1
                                          1   0   1   0   0   0   0   0   1   0   1
                                        1   1   1   1   0   0   0   0   1   1   1   1
                                      1   0   0   0   1   0   0   0   1   0   0   0   1
                                    1   1   0   0   1   1   0   0   1   1   0   01  1
                                  1   0   1   0   1   0   1   0   1   0   1   0   1   0   1
                                1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
                              1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
                            1   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   1
                          1   0   1   0   0   0   0   0   0   0   0   0   0   0   0   01  0   1
                        1   1   1   1   0   0   0   0   0   0   0   0   0   0   0   0   1   1   1   1
                      1   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   1   0   0   01
                    1   1   0   0   1   1   0   0   0   0   0   0   0   0   0   0   1   1   0   0   1   1
                  1   0   1   0   1   0   1   0   0   0   0   0   0   0   0   0   1   0   1   0   1   0   1
                1   1   1   1   1   1   1   1   0   0   0   0   0   0   0   0   1   1   1   1   1   1   1   1
              1   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   1
            1   1   0   0   0   0   0   0   1   1   0   0   0   0   0   0   1   1   0   0   0   0   0   0   1   1
          1   0   1   0   0   0   0   0   1   0    1   0   0   0   0   0   1   0   1   0   0   0   0   0   1   0   1
        1   1   1   1   0    0   0   0   1   1   1   1   0   0   0   0   1   1   1   1   0   0   0   0   1   1   1   1
      1   0   0   0   1   0   0   0   1   0   0   0   1   0   0   0   1   0   0   0   1   0   0   0   1   0   0   0   1
    1   1   0   0   1   1   0   0   1   1   0   0   1   1   0   0   1   1   0   0   1   1   0   0   1   1   0   0   1   1
  1   0   1   0   1   0   1   0   1   0   1   0   1   0   1   0   1   0   1   0   1   01  0   1   0   1   0   1   0   1
1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
```

INVWANG2.WPD May 30, 2004

THE INVERSE PASCAL TRIANGLE
SUBTRACTION INSTEAD OF ADDITION

```
                                                              1
                                                            1   1
                                                          1   -   1
                                                        1   1   1   1
                                                      1   -   -   -   1
                                                    1   1   -   -   1   1
                                                  1   -   1   -   1   -   1
                                                1   1   1   1   1   1   1   1
                                              1   -   -   -   -   -   -      -1
                                            1   1   -   -   -   -   -   -   1   1
                                          1   -   1   -   -   -   -   -   1   -   1
                                        1   1   1   1   -   -   -   -   1   1   1   1
                                      1   -   -   -   1   -   -   -   1   -   -   -   1
                                    1   1   -   -   1   1   -   -   1   1   -      -1   1
                                  1   -   1   -   1   -   1   -   1   -   1   -   1   -   1
                                1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
                              1   -   -   -   -   -   -   -   -   -   -   -   -   -   -   -   1
                            1   1   -   -   -   -   -   -   -   -   -   -   -   -   -   -   1   1
                          1   -   1   -   -   -   -   -   -   -   -   -   -   -      -1   -   1
                        1   1   1   1   -   -   -   -   -   -   -   -   -   -   -   -   1   1   1   1
                      1   -   -   -   1   -   -   -   -   -   -   -   -   -   -   -   1   -   -      -1
                    1   1   -   -   1   1   -   -   -   -   -   -   -   -   -   -   1   1   -   -   1   1
                  1   -   1   -   1   -   1   -   -   -   -   -   -   -   -   -   1   -   1   -   1   -   1
                1   1   1   1   1   1   1   1   -   -   -   -   -   -   -   -   1   1   1   1   1   1   1   1
              1   -   -   -   -   -   -   -   1   -   -   -   -   -   -   -   1   -   -   -   -   -   -   -   1
            1   1   -   -   -   -   -   -   1   1   -   -   -   -   -   -   1   1   -   -   -   -   -   -   1   1
          1   -   1   -   -   -   -   -   1   -   1   -   -   -   -   -   1   -   1   -   -   -   -   -   1   -   1
        1   1   1   1   -   -   -   -   1   1   1   1   -   -   -   -   1   1   1   1   -   -   -   -   1   1   1   1
      1   -   -   -   1   -   -   -   1   -   -   -   1   -   -   -   1   -   -   -   1   -   -   -   1   -   -   -   1
    1   1   -   -   1   1   -   -   1   1   -   -   1   1   -   -   1   1   -   -   1   1   -   -   1   1   -   -   1   1
  1   -   1   -   1   -   1   -   1   -   1   -   1   -   1   -   1   -   1   -   1      -1   -   1   -   1   -   1   -   1
1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1   1
```

FRACTAL3.WPD

```
1
1 1
1 - 1
1 1 1 1
1 - - - 1
1 1 - - 1 1
1 - 1 - 1 - 1
1 1 1 1 1 1 1 1
1 - - - - - - - 1
1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 - - - - - - - - - - - - - - - 1
1 1 - - - - - - - - - - - - - - 1 1
1 - 1 - - - - - - - - - - - - - 1 - 1
1 1 1 1 - - - - - - - - - - - - 1 1 1 1
1 - - - 1 - - - - - - - - - - - 1 - - - 1
1 1 - - 1 1 - - - - - - - - - - 1 1 - - 1 1
1 - 1 - 1 - 1 - - - - - - - - - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 - - - - - - - - 1 1 1 1 1 1 1 1
1 - - - - - - - 1 - - - - - - - 1 - - - - - - - 1
1 1 - - - - - - 1 1 - - - - - - 1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1 - - - - - 1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

∠30°

FRACTAL3.WPD

```
1
1 1
1 - 1
1 1 1 1
1 - - - 1
1 1 - - 1 1
1 - 1 - 1 - 1
1 1 1 1 1 1 1 1
1 - - - - - - - 1
1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
1 - - - - - - - - - - - - - - - 1
1 1 - - - - - - - - - - - - - - 1 1
1 - 1 - - - - - - - - - - - - - 1 - 1
1 1 1 1 - - - - - - - - - - - - 1 1 1 1
1 - - - 1 - - - - - - - - - - - 1 - - - 1
1 1 - - 1 1 - - - - - - - - - - 1 1 - - 1 1
1 - 1 - 1 - 1 - - - - - - - - - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 - - - - - - - - 1 1 1 1 1 1 1 1
1 - - - - - - - 1 - - - - - - - 1 - - - - - - - 1
1 1 - - - - - - 1 1 - - - - - - 1 1 - - - - - - 1 1
1 - 1 - - - - - 1 - 1 - - - - - 1 - 1 - - - - - 1 - 1
1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1
1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1
1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

INVYANG.DVD May 31, 2004

INVERSE OF THE YANGHUI TRIANGLE

- = 0

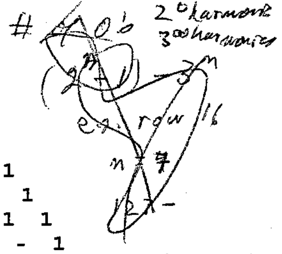
```
                               1
                              1 1
                             1 - 1
                            1 1 1 1
                           1 - - - 1
                          1 1 - - 1 1
                         1 - 1 - 1 - 1
                        1 1 1 1 1 1 1 1
                       1 - - - - - - - 1
                      1 1 - - - - - - 1 1
                     1 - 1 - - - - - 1 - 1
                    1 1 1 1 - - - - 1 1 1 1
                   1 - - - 1 - - - 1 - - - 1
                  1 1 - - 1 1 - - 1 1 - - 1 1
                 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
                1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
               1 - - - - - - - - - - - - - - - 1
              1 1 - - - - - - - - - - - - - - 1 1
             1 - 1 - - - - - - - - - - - - - 1 - 1
            1 1 1 1 - - - - - - - - - - - - 1 1 1 1
           1 - - - 1 - - - - - - - - - - - 1 - - - 1
          1 1 - - 1 1 - - - - - - - - - - 1 1 - - 1 1
         1 - 1 - 1 - 1 - - - - - - - - - 1 - 1 - 1 - 1
        1 1 1 1 1 1 1 1 - - - - - - - - 1 1 1 1 1 1 1 1
       1 - - - - - - - 1 - - - - - - - 1 - - - - - - - 1
      1 1 - - - - - - 1 1 - - - - - - 1 1 - - - - - - 1 1
     1 - 1 - - - - - 1 - 1 - - - - - 1 - 1 - - - - - 1 - 1
    1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1
   1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
  1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1
 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Total symbols through row n $= \frac{n(n+1)}{2}$

Σ of 1's above and in row n

Row and above	Σ 1's		#0	$\frac{n(n+1)}{2}$
1	1	+	0	1
2	3	+	0	3 ✓
4	9	+	~~8~~1	10 ✓
8	27	+	9	36 ✓
16	81	~~n=4~~	55	136 ✓
when 2^n	3^n			✗

of circle of 5ths and Pythagorean Scales !!



Pythagorean Scale i.e. Music ~ Fractal

$\frac{3}{2}$

$\frac{9}{8}$

$\frac{27}{16}$

$\frac{81}{64}$

⋮

$Z_0 = 1$

The module M = 1
T_0 1 1
1 - 1
1 1 1 1

The module Z_1 = - - - = 6 zeros
- -
-

The module Z_2 = - - - - - - - = 28 zeros.
- - - - - -
- - - - -
- - - -
- - -
- -
-

$T_0 = M = 10$ $5 \cdot 4/2$
$T_1 = \cancel{Z_1 + 3M} = 36 = 3T_0 + Z_1$ $9 \cdot 8/2$
$T_2 = 3T_1 + Z_2 = 136$ $17 \cdot 16/2$
$T_3 = 3T_2 + Z_3 = 528$ $33 \cdot 32/2$

The M-Modules and the Z-Modules

The module Z_3 = 120 zeros

The module $Z_n = 2^{2n+1} - 2^n$ zeros

$Z_n = 3 \cdot Z_{n-1} + T_{n-2}$

$T_n = 4 \cdot T_{n-1} - 2^{n+1}$

$T_n = 4(T_{n-1} - 2^{n-1})$

$Z_n = 4Z_{n-1} + 2^n$

YANGHUI

m	n	R	M	T	O	H
2	1	4	9	10	1	0.954242
3	2	8	27	36	9	0.919721
4	3	16	81	136	55	0.894516
5	4	32	243	528	285	0.876213

$$\lim_{m \to \infty} H \mapsto \frac{\log 3}{\log 4} = 0.792481$$

$R = 2^{n-1}$ $\quad \frac{R(R+1)}{2}$

[illegible] $\overset{m\to\infty}{\searrow}$ 0.792481

Same as Wolfram

$M = 3^{n+1}$

$$T = \frac{2^{n-1}(2^{n-1}+1)}{2}$$

or

$R = 2^m$

$M = 3^m$

$$T = \frac{2^m(2^m+1)}{2}$$

$2^{m-1}(2^m+1)$

$2^{2m-1} + 2^{m-1}$

$$\frac{\log M}{\log T} = \frac{m \log 3}{(m-1)\log 2 + \underset{\substack{\downarrow \\ m \log 2}}{\log(2^m+1)}}$$

$$\frac{m \log 3}{(2m-1)\log 2}$$

$m \log 2 + \log(2^m+1) - \log 2$

$(m-1)\log 2 + \overset{\text{as } m}{m \log 2}$

$$\lim_{m \to \infty} \frac{\log 3}{(2-\frac{1}{m})\log 2} = \frac{\log 3}{\log 4} = 0.792481$$

INVYANG.DVD May 31, 2004

INVERSE OF THE YANGHUI TRIANGLE

- = 0

```
                               1
                              1 1
                             1 - 1
                            1 1 1 1
                           1 - - - 1
                          1 1 - - 1 1
                         1 - 1 - 1 - 1
                        1 1 1 1 1 1 1 1
                       1 - - - - - - - 1
                      1 1 - - - - - - 1 1
                     1 - 1 - - - - - 1 - 1
                    1 1 1 1 - - - - 1 1 1 1
                   1 - - - 1 - - - 1 - - - 1
                  1 1 - - 1 1 - - 1 1 - - 1 1
                 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
                1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
               1 - - - - - - - - - - - - - - - 1
              1 1 - - - - - - - - - - - - - - 1 1
             1 - 1 - - - - - - - - - - - - - 1 - 1
            1 1 1 1 - - - - - - - - - - - - 1 1 1 1
           1 - - - 1 - - - - - - - - - - - 1 - - - 1
          1 1 - - 1 1 - - - - - - - - - - 1 1 - - 1 1
         1 - 1 - 1 - 1 - - - - - - - - - 1 - 1 - 1 - 1
        1 1 1 1 1 1 1 1 - - - - - - - - 1 1 1 1 1 1 1 1
       1 - - - - - - - 1 - - - - - - - 1 - - - - - - - 1
      1 1 - - - - - - 1 1 - - - - - - 1 1 - - - - - - 1 1
     1 - 1 - - - - - 1 - 1 - - - - - 1 - 1 - - - - - 1 - 1
    1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1 - - - - 1 1 1 1
   1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
  1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1 - - 1 1
 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
```

Total symbols thrugh row n $= \frac{n(n+1)}{2}$

$\frac{B}{T}$ $\frac{9}{10}$ $\frac{27}{36}$ $\frac{81}{136}$

0.954242

0.91972

0.894

Σ of 1's above and incl row n

Row and above	Σ 1's		# 0	$\frac{n(n+1)}{2}$
1	1	+	0	1
2	3	+	0	3 ✓
4	9	+	~~8~~1	10 ✓
8	27	+	9	36 ✓
16	81	~~n=4~~	55	136 ✓
when 2^n	3^n			

cf circle of 5ths and Pythagorean Scales!!

~~of~~ 0's 2^n harmonic 3^n harmonics

$(2^n(+1)) - 3^n$

e.g. row 16

n = 4

Pythagoras Scale i.e.

$\frac{3}{2}$

$\frac{9}{8}$

$\frac{27}{16}$

$\frac{81}{64}$

⋮

Music ~ Fractal

$Z_0 = 1$

The module M =

```
        1
  T0   1 1
      1 - 1
     1 1 1 1
```

The module Z_1 =

```
- - -
 - -
  -
```

= 6 zeros

The module Z_2 =

```
- - - - - - -
 - - - - - -
  - - - - -
   - - - -
    - - -
     - -
      -
```

= 28 zeros.

$T_0 = M = 10$ 5·4/2

$T_1 = \cancel{Z_1 + 3M} = 36 = 3T_0 + Z_1$ 9·8/2

$T_2 = 3\,T_1 + Z_2 = 136$ 17·16/2

$T_3 = 3\,T_2 + Z_3 = 528$ 33·32/2

The module Z_3 = 120 zeros

The module $Z_n = 2^{2n+1} - 2^n$ zeros

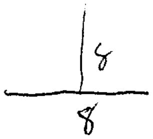
$Z_n = 3 \cdot Z_{n-1} + T_{n-2}$

$T_n = 4 \cdot T_{n-1} - 2^{n+1}$

$T_n = 4(T_{n-1} - 2^{n-1})$

$Z_n = 4Z_{n-1} + 2^n$

Alternate view



$8 \times 8 = \frac{64}{2} = 32$ ~~w~~ $T = 36$

~~w~~ $T = 136$

$n = 3$

$\frac{27}{32}$ $\frac{81}{128}$

0.9509 .9056

nyet

$\frac{8(9)}{2}$

$$\begin{array}{r} 8 \cdot 17 \\ 8 \\ \hline 136 \end{array}$$

$$\begin{array}{r} 243 \\ 3 \\ \hline 729 \end{array}$$

$$\begin{array}{r} 729 \\ 3 \\ \hline 2187 \\ 3 \\ \hline 6561 \end{array}$$

$$\begin{array}{r} 33 \\ 16 \\ \hline 198 \\ 33 \\ \hline 528 \\ 243 \\ 285 \end{array}$$

$$\begin{array}{r} 65 \\ 32 \\ \hline 130 \\ 195 \\ \hline 2080 \\ 729 \\ \hline 1351 \end{array}$$

$$\begin{array}{r} 129 \\ 64 \\ \hline \end{array}$$

$$\begin{array}{r} 257 \\ 128 \end{array}$$

$$\begin{array}{r} 8256 \\ 2187 \\ \hline 6069 \end{array}$$

$$\begin{array}{r} 32,896 \\ 6\ 561 \\ \hline 26\ 335 \end{array}$$

$$\lim_{n \to \infty} \frac{T}{E} \quad \frac{2^{n-1}(2^n+1)}{3^n}$$

$$= \left(\frac{2}{3}\right)^n \frac{(2^n+1)}{2}$$

$$\underset{0}{\downarrow} \qquad \to \infty$$

$$\frac{2^{2n-1} + 2^{n-1}}{3^n} = \frac{1}{2} \quad \frac{2^{2n} + 2^n}{3^n}$$

$$= \frac{1}{2}\left[\underset{\infty}{\underset{\downarrow}{\left(\frac{4}{3}\right)^n}} + \underset{0}{\underset{\downarrow}{\left(\frac{2}{3}\right)^n}}\right]$$

$$\lim \frac{T}{E} = \infty \qquad \lim \frac{E}{T} = 0$$

$$\lim \frac{Z}{E} = \frac{T-E}{E} = \frac{T}{E} - 1$$

$$\lim \frac{T}{Z} = \frac{T}{T-E} \quad \frac{\infty}{\infty}$$

$$= \frac{2^{n-1}(2^n+1)}{2^{n-1}(2^n+1) - 3^n}$$

$$\lim \frac{Z}{E} = \infty - 1 = \infty$$

i.e. → Mostly ZERO

$$\frac{\frac{T}{E}}{\frac{T}{E} - 1} \qquad \frac{1}{1 - \frac{E}{T}} \qquad = 1$$

$$\boxed{n \to \infty \quad T = Z}$$

05-02-06

A Saltatory Limit [Zeno returns]

COSMIC CURIOSITIES
2004 #36

SOME CURIOUS LIMITS

The symmetrical Δ + → − hidden 2

Row #,	# of 1's in row,	# of 1's in and above row E	Z T − E # of zeros above row	T $\frac{n(n+1)}{2}$ Total Δ
1	1	1	0	
2	2	3	0	
$4=2^2$	= 4	$9=3^2$	1	10
$8=2^3$	= 8	$17=3^3$	9	36
$16=2^4$	= 16	$81=3^4$	55	136
$32=2^5$	= 32	$243=3^5$	285	528
$64=2^6$	= 64	$729=3^6$	1351	2080

17
8
136
81
55

16
33
48
48
28

528
243
285

65
32
130
195
2080
729
1351

$$\lim_{n\to\infty}\frac{T-E}{E} = 1+\lim\left(\frac{T}{E}\right) = 1+\lim\frac{n(n+1)}{2\,3^n}$$

i.e. lim of # of zeros as n →
$\simeq 1$

$$\frac{1}{2}\left(\frac{n^2+n}{3^n}\right)\to 0$$

$$\frac{n^2}{3^n}+\frac{n}{3^n}$$

$$\lim\frac{T}{E}=0 \ni$$

$$\frac{\log n}{n\log 3}\to 0$$

$T-E \simeq 7$

$$\lim\frac{Z}{E}=1 \ni \#Z=\#1$$

but n does not go Newton wise to ∞ by by leaps of 2^n

$$\frac{T}{E}=\frac{1}{2}\frac{n^2+n}{3^n}\to 0 \text{ as } n\to\infty$$

i.e. # of 1s →} Total # of symbols

part > whole

both #1 and #0 > whole

Continuous limits vs ~~Eigen~~ limits Saltatory

n	SIZE OF Z ZERO PATTERNS	T P TOTAL 0's, 1's in A	U Total 1's	E Total zeros $P_n - U_n$
0	0	3	3	0
1	1	10	9	1
2	6	36	27	9
3	28	136	81	55
4	120	~~136~~ 528	243	285
5	496	~~536~~ 2080	729	1351

E – Z

0
0
3
27
135
855

$Z_n = 2^{n-1}(2^n-1)$ $P_n = 2^n(2^{n+1}+1)$ $U_n = 3^{n+1}$

~~$P_n = 2^{n+1}(2^{n}+1)$~~

$\frac{\log B}{\log T}$ not constant

for Wolfram is constant

9	27	81
10	36	136

0.9542425
0.919...
0894
⋮

$P_{n+1} = 3P_n + Z_n$

n	E
0	0
1	1
2	$6 + 3 = 9$
3	$28 + 3\cdot 6 + 3^2\cdot 1 = 55$
4	$120 + 3\cdot 28 + 3^2\cdot 6 + 3^3\cdot 1 = 285$
5	$496 + 3\cdot 120 + 3^2\cdot 28 + 3^3\cdot 6 + 3^4\cdot 1 = 1351$

combine

$$2^{n-1}(2^n-1) + 3\cdot 2^{n-2}(2^{n-1}-1) + 3^2 2^{n-3}(2^{n-2}-1) + \cdots + 3^{n-1}\cdot 1 = 2^n(2^{n+1}+1) - 3^{n+1}$$

$$T_n = \frac{2^n(2^n+1)}{2} = 2^{n-1}(2^n+1)$$

SIZE OF "0" Module

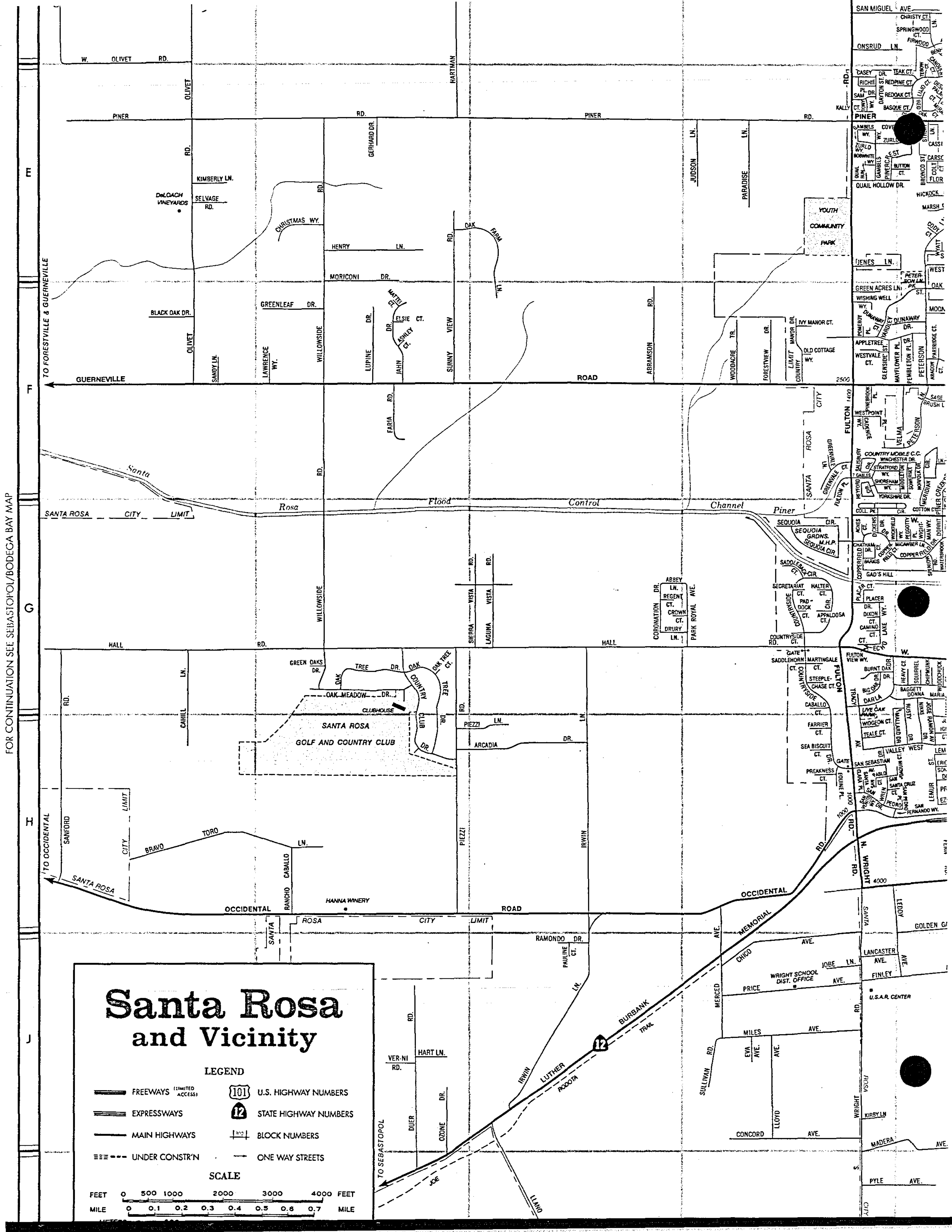
$2^{n-1}(2^n-1)$ Z $2^{n-1}(2^n-1)$

n	0
0	0
1	1
2	6
3	28
	120

$\frac{T}{Z} = \frac{2^n+1}{2^n-1} \to 1$

$\frac{0's}{Total}$ 0.1, 0.25, 0.404.., 0.541.. → 1

$\frac{1's}{Total}$ 0.9 0.75 0.596.., 0.460.. → 0


Santa Rosa
and Vicinity
LEGEND
FREEWAYS (LIMITED ACCESS)
101 U.S. HIGHWAY NUMBERS
EXPRESSWAYS
12 STATE HIGHWAY NUMBERS
MAIN HIGHWAYS
202 BLOCK NUMBERS
UNDER CONSTR'N
ONE WAY STREETS
SCALE
FEET 0 500 1000 2000 3000 4000 FEET
MILE 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 MILE
FOR CONTINUATION SEE SEBASTOPOL/BODEGA BAY MAP
E
F
G
H
J
TO FORESTVILLE & GUERNEVILLE
TO OCCIDENTAL
TO SEBASTOPOL
W. OLIVET RD.
OLIVET RD.
HARTMAN
PINER RD.
GERHARD DR.
JUDSON LN.
PARADISE LN.
KIMBERLY LN.
DeLOACH VINEYARDS
SELVAGE RD.
CHRISTMAS WY.
HENRY LN.
OAK FARM LN.
MORICONI DR.
BLACK OAK DR.
GREENLEAF DR.
MATTEI CT.
ELSIE CT.
ASHLEY CT.
LUPINE DR.
JAHN DR.
SUNNY VIEW RD.
ABRAMSON RD.
WOODACRE TR.
FORESTVIEW DR.
COUNTRY MANOR DR.
IVY MANOR CT.
OLD COTTAGE WY.
LIMIT
YOUTH COMMUNITY PARK
SANDY LN.
LAWRENCE WY.
WILLOWSIDE RD.
GUERNEVILLE ROAD
2500
FARIA RD.
Santa Rosa Flood Control Channel
Piner
SANTA ROSA CITY LIMIT
SANTA ROSA CITY
FULTON 1400
FULTON PL.
GREENVALE LN.
GREENVALE CT.
SEQUOIA CIR.
SEQUOIA GRDNS. M.H.P.
SEQUOIA CIR.
SADDLEBACK CIR.
SECRETARIAT CT.
PAD-DOCK CT.
HALTER CT.
CIR.
APPALOOSA CT.
COUNTRYSIDE DR.
ABBEY LN.
REGENT CT.
CROWN CT.
DRURY LN.
CORONATION DR.
PARK ROYAL AVE.
SIERRA VISTA RD.
LAGUNA VISTA RD.
HALL RD.
COUNTRYSIDE CT.
GATE
SADDLEHORN CT.
MARTINGALE CT.
STEEPLE-CHASE CT.
CABALLO CT.
FARRIER CT.
SEA BISCUIT CT.
PREAKNESS CT.
EQUINE PL.
GREEN OAKS DR.
OAK TREE DR.
OAK TREE CT.
OAK MEADOW DR.
COUNTRY CLUB DR.
CLUBHOUSE
SANTA ROSA GOLF AND COUNTRY CLUB
PIEZZI LN.
ARCADIA DR.
PIEZZI RD.
IRWIN LN.
CAHILL LN.
SANFORD RD.
CITY LIMIT
BRAVO TORO LN.
RANCHO CABALLO
SANTA ROSA
OCCIDENTAL ROAD
HANNA WINERY
SANTA ROSA CITY LIMIT
RAMONDO DR.
PAULINE CT.
MERCED AVE.
MEMORIAL
CHICO AVE.
WRIGHT SCHOOL DIST. OFFICE
PRICE AVE.
JOBE LN.
MILES AVE.
EVA AVE.
SULLIVAN RD.
LLOYD
CONCORD AVE.
LUTHER BURBANK
12
RODOTA TRAIL
JOE
LLANO
IRWIN
VER-NI RD.
HART LN.
DUER RD.
OZONE DR.
1000
N. WRIGHT RD.
4000
SANTA ROSA
LEDDY AVE.
GOLDEN GA
LANCASTER AVE.
FINLEY
U.S.A.R. CENTER
WRIGHT RD.
KIRBY LN
MADERA AVE.
S.
PYLE AVE.
CITY
SAN MIGUEL AVE.
CHRISTY CT.
SPRINGWOOD CT.
FIRWOOD
ONSRUD LN.
CASEY DR.
TEAK CT.
RICHIE
REDPINE CT.
SAM DR.
DAYTON ST.
REDOAK CT.
BASQUE CT.
KALLY CT.
PINER
GAMBELS WY.
COVE
ZURLO
ZURLO WY.
BOBWHITE LN.
PINECREST
BUTTON CT.
QUAIL RUN
QUAIL HOLLOW DR.
BRONCO ST.
CASSI
CARSO
COLT CT.
FLOR
HICKOCK
MARSH
CODY
WYATT
JENES LN.
PETER-SON LN.
GREEN ACRES LN.
OAK
WISHING WELL WY.
WEST
DUNAWAY DR.
MOON
POMEROY PL.
YARDLEY
APPLETREE
WESTVALE CT.
GLENSIDE ST.
MAYFLOWER PL.
PEMBLETON PL.
PETERSON
ARAGON
PARTRIDGE CT.
SAGE BRUSH L
PINEBROOK PL.
WESTPOINT
CADENCE WY.
VELMA
PETERSON
COUNTRY MOBILE C.C.
WINCHESTER DR.
SALISBURY
STRATFORD WY.
SHOREHAM WY.
YORKSHIRE DR.
MIDDLETON
SOMERSET
NORFOLK DR.
MERIDIAN CIR.
PINER CREEK
COTTON
COLL. PK.
SEQUOIA
DICKENS
COPPERFIELD DR.
MICAWBER LN.
COPPER FIELD DR.
WATERBROOK
DORRIT
GAD'S HILL
PLACER CT.
PLACER DR.
DIXON CT.
CAMINO CT.
LAKE WY.
ECHO
FULTON VIEW WY.
BURNT OAK DR.
BIG OAK DR.
SQUIRREL
CHIPMUNK
WOODCHUCK
BAGGETT
DONNA
MARIA
DARLA
TRACY AV.
LIVE OAK PARK
WIDGEON CT.
TEALE CT.
MALLARD DR.
RUSTY
NIKKI
JOSE RAMON AV.
VALLEY WEST
LEMUR
SAN SEBASTIAN
SANTA CRUZ
SAN PEDRO
SAN FERNANDO WY.
FULTON

Also classify
by x 2
x 3
etc.

+1

```
       |
      | |
     | | |
    | | | |
   | | | | |
  | | | | | |
 | | | | | | |
| | | | | | | |
```

+2

```
              |
            | | |
          | | | | |
        | | | | | | |
      | | | | | | | | |
    | | | | | | | | | | |
  | | | | | | | | | | | | |
| | | | | | | | | | | | | | |
```

+3

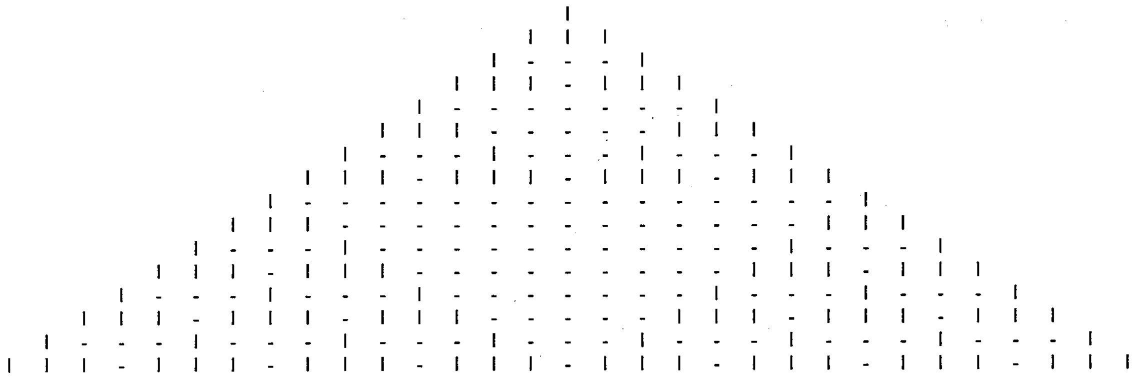
```
                   |
                | | | |
            | | | | | | |
         | | | | | | | | | |
      | | | | | | | | | | | | |
  | | | | | | | | | | | | | | | |
 | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | |
```

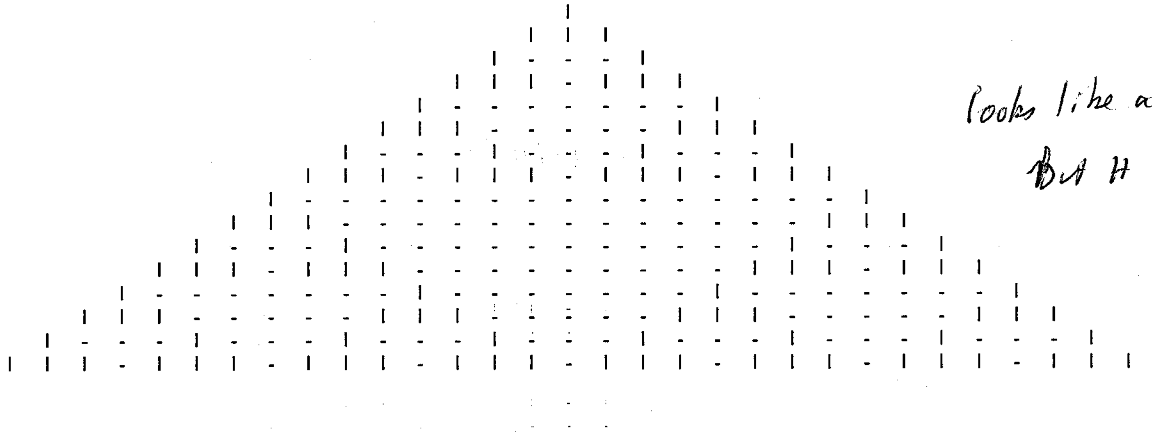
x short 1
x -1

+4

```
                            |
                        | | | | |
                    | | | | | | | | |
                | | | | | | | | | | | | |
            | | | | | | | | | | | | | | | | |
        | | | | | | | | | | | | | | | | | | | | |
    | | | | | | | | | | | | | | | | | | | | | | | | |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
```

PLUS TWO FRACTALS





Looks like a fractal
But H varie

M	n	R	M	T	T − M	H
$4 \cdot 3$	2	4	12	16	4	0.89624
$4 \cdot 3^2$	3	8	36	64	28	0.86165
$4 \cdot 3^3$	4	16	108	256	148	0.844361
$4 \cdot 3^4$	5	32	324	1024	700	0.833985

Tim H same
as Yonghui
and Wolfram

$M = 4 \cdot 3^{n-1}$

$2^n \; 3(M-1) \; R^2$

$H \to ?$

$R = 2^n$

$T = R^2 = 4^n$

$M = 4 \cdot 3^{m-1}$

$R = 2^m, \; T = 2^{2m} = 4^m$

$m \to \infty$

$$\frac{M}{T} = \frac{4 \cdot 3^{n-1}}{4^n}$$

$$\frac{M}{T} = \frac{4 \cdot 3^{m-1}}{4^m} = \frac{3^{n-1}}{4^{m-1}}$$

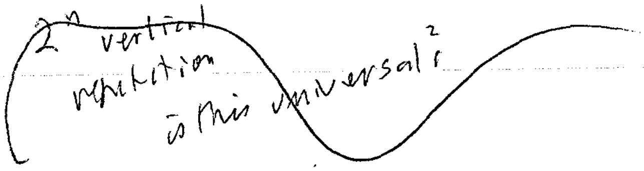
$$\frac{\log M}{\log T} = \frac{\log 3}{\log 4} = 0.792481$$

$$\frac{\log M}{\log T} = \frac{\log 4 + (n-1)\log 3}{n \log 4} = \frac{1}{n} + \frac{n-1}{n}\frac{\log 3}{\log 4}$$

$$n \to \infty \quad \to \frac{\log 3}{\log 4}$$

$$\frac{\log T}{\log M} = 1.2618595$$

cf



2^n vertical repetition is this universal?

The Wolfram Fractal

R = row
M = # black (manifest)
T = total #
U = T − M
$H = \frac{\log M}{\log T}$

$H^{-1} = 1.2618595$

n	R	M	T	U	H
1	4	9	16	7	0,7294812
2	8	27	64	37	"
3	16	81	256	175	" CONSTANT
4	32	243	1024	781	"

$R = 2^{n+1}, M = 3^{n+1}, T = R^2 \qquad \frac{M}{T} = \frac{3^{n+1}}{4^{n+1}} \to 0$ as $n \to \infty$

but logs constant at $\frac{3}{4}$

THE YANGHUI FRACTAL

n	R	M	T	U	H
1	4	9	10	1	0,954242
2	8	27	36	9	0,919721
3	16	81	136	55	0.894516
4	32	243	528	285	0.826213

$$\frac{\log M}{\log T} = \frac{(n+1)\log 3 + \log 2}{n+1 \log 2 + \underset{\substack{\downarrow \\ n+1 \log 2}}{\log(2^{n+1}+1)}}$$

$$\xrightarrow[n\to\infty]{} \frac{\log 3}{\log 4} = 0.7294812$$

$$R = 2^{n+1}, \quad M = 3^{n+1}, \quad T = \frac{R(R+1)}{2} = \frac{2^{n+1}(2^{n+1}+1)}{2}, \quad \frac{M}{T} = \frac{2 \cdot 3^{n+1}}{2^{n+1}(2^{n+1}+1)} \approx \frac{3^{n+1}}{4^{n+1}} \xrightarrow[n\to\infty]{} 0$$

THE "+2" FRACTAL

n	R	M	T	U	H
1	4	12	16	4	0,89624
2	8	36	64	28	0.86165
3	16	108	256	148	0.84436
4	32	324	1024	700	0.83398

$$R = 2^{n+1}, \quad M = 4 \cdot 3^n, \quad T = R^2 = 4^{n+1}$$

$$\frac{M}{T} = \frac{4 \cdot 3^n}{4^{n+1}} = \left(\frac{3}{4}\right)^n \xrightarrow[n\to\infty]{} 0 \qquad \frac{\log M}{\log T} = \frac{n \log 3 + \log 4}{(n+1)\log 4} = \frac{n \log 3 + \log 4}{n \log 4 + \log 4}$$

$$= \frac{\log 3 + \frac{\log 4}{n}}{\log 4 + \frac{\log 4}{n}} \xrightarrow[n\to\infty]{} \frac{\log 3}{\log 4} = 0.7294812$$

For All $\frac{M}{T} \to 0$, $\frac{\log M}{\log T} \to \frac{\log 3}{\log 4}$, or $\frac{\log M}{\log T} \equiv \frac{\log 3}{\log 4}$

FRACTAL2.WPD

February 7, 2005

```
                              1
                            1 1 1
                          1 - - - 1
                        1 1 1 - 1 1 1
                      1 - - - 1 - - - 1
                    1 1 1 - 1 1 1 - 1 1 1
                  1 - - - 1 - - - 1 - - - 1
                1 1 1 - 1 1 1 - 1 1 1 - 1 1 1
              1 - - - 1 - - - 1 - - - 1 - - - 1
            1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1
          1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
        1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1
      1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
    1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1
  1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1 - - - 1
1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1 - 1 1 1
```

Hausdorf Dimension as $n \to \infty = 1$

An area with
Uniformity
has the H.D. of 1

~ line

Perhaps
1 should be
added to H.D
in all cases
fractal here

THIS IS NOT A FRACTAL
IT is A "SOLID"
With M/[illegible] = → 1

alternating modules
of size 4

YANGHUI successive rows +1

Usual Fractals " " +2 as above

what about +3, +4, ... ?

THIS HAS TWO MODULES
4 1's and 4 -'s

$$H = \frac{\log M}{\log T}$$

Ratio

$\frac{1}{3}, \frac{3}{6}, \frac{6}{10}, \frac{10}{15}$

n	R	M	T	H	$\overset{V}{T-M}$
1	2	4	4	1	0
2	4	12	16	0.896	4
3	6	24	36		
4	8	40	64	0.886	24
~~5~~	~~10~~	~~220~~	~~100~~		

$M \doteq R$

4 12 24 40

$4 \cdot \frac{R(R-1)}{2}$

$M = 2R(R+1)$

$R = 2^n \quad T = R^2$

$M = ?$

n	R	M	T
1	2	4.1	4
2	4	4.3	16
3	6	4.6	36
4	8	4.10	64
5	10	4.15	

$$\frac{\log M}{\log T} = \frac{\log 2 + \log n + \log(n+1)}{\log 4 + 2\log n}$$

$$= \frac{1 + \frac{\log 2}{\log n} + \frac{\log(n+1)}{\log n}}{2 + \frac{\log 4}{\log n}}$$

$$\to \frac{1}{2} + \frac{1}{2}\lim \frac{\log n+1}{\log n} = 1$$

$= 1$

$\therefore 1$

$2n \qquad R^2 = 4n^2$

$$M = 4 \cdot \frac{(n)(n+1)}{2} = 2n(n+1)$$

At low resolving power
= uniform

$$\frac{M}{T} = \frac{2n(n+1)}{4n^2}$$

$$= \frac{2}{4} \quad \frac{n^2+n}{n^2}$$

$$= \frac{1}{2}\left(1 + \frac{1}{n}\right)$$

$$\to \frac{1}{2} \text{ as } n \to \infty$$

i.e. equal white
and black modules

~ uniformity

WOLFRAM'S FRACTAL

R	M	T	U	H
4	9	16	7	0.7924812
8	27	64	37	0.7924812
16	81	256	175	0.7924812
32	243	1024	781	0.7924812

H constat

A True Fractal has a constant Hausdorf dimension

WOLFRAM's FRACTAL

WOLFRAM

$\frac{\log B}{\log T} = \text{const} = 0.792 \ldots$

Each successive row adds 2

Hence 1, 3, 5, 7, ...

Row	B	T
4	9	16
8		
16		

The number in row	is	TOTAL NUMBER in row R and above
1	1	1
2	3	4
3	5	9
4	7	16
.....		
R	2R-1	R^2

large SIZE OF WHITE PATTERNS ~~NUMBER~~ In and above row	row
4	4
8	16
16	
32	

	# in row	n	IN + ABOVE Row #	TOTAL	large pattn ZERO's	Total ZEROS	~~Total ones~~	TOTAL 1's
2^3-1	7	1	4	$16 = 4^2 = 2^4$	$4 = 2^3$	7		9
2^4-1	15	2	8	$64 = 8^2 = 2^6$	$16 = 2^4$	22+16 = 38		26
2^5-1	31	3	16	$256 = 16^2 = 2^8$	$64 = 2^6$			
2^6-1	63	4	32	2^{10}	2^8			
		5	64					
		n	2^{n+1}					

Large Z pattern only above rows 4, 8, 16, 32, ...

R#	# in row	TOTAL IN + ABOVE	TOTAL BLACK	TOTAL WHITE in + above
1	1	1	1	0
2	3	4	3	1
3	5	9	5	4
4	7	16	9	7
5	9	25	11	14
	2R-1	R^2		

Key rows
4
8
16
etc

$\frac{W}{T} = 0, \frac{1}{4}, \frac{4}{9}, \frac{7}{16}, \frac{14}{25}$ ≅ 0, 0.25, 0.4, 0.4375, 0.56 → 1

→ 0

$\frac{B}{T} = 1, \frac{3}{4}, \frac{5}{9}, \frac{9}{16}, \frac{11}{25}$

Fractal Dimension?

$\frac{\log \text{Black}}{\log \text{Total}}$

Summation of odd numbers

n	1	2	3	4	5	6	7	8
N	1	3	5	7	9	11	13	15

$n = \frac{N+1}{2}$

$$\sum_1^N = \left(\frac{N+1}{2}\right)^2 = n^2$$