MUSIC

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There exist certain parallels and dualities between the generation of a musical scale by the circle of fifths and the generation of a liturgical year by the analemma loop. The basic algorithm is the same:

	CIRCLE OF FIFTHS	ANALEMMA LOOP
1.	Select a base pitch	Select a base date
2.	Select the pitch factor	Select the date interval
3.	Generate the pitch sequence	Generate the date sequence
4.	Effect closure	Effect closure

Whereas the circle of fifths deals with pitch ratios, the analemma loop deals with time intervals. Thus the pitches in the circle of fifths form a geometric sequence, while the dates in the analemma loop form an arithmetic sequence.

The CIRCLE OF FIFTHS

Musical scales can be generated in many ways. One method is by iterated third harmonics, usually called the circle of fifths. The human ear tends to equate all even harmonics, hence discriminable notes must be derived from the odd harmonics. The third harmonic is the most important one. (The physicist says that the E` an octave above C is the third harmonic of C. The musician says that the E above C is the fifth of C,. where E an octave below E`.) For the third harmonic the pitch factor is 3. But to reduce to the base octave the pitch must be divided by 2. Hence the pitch factor in the circle of fifths is 3/2.

Useable scales will be obtained whenever the powers of 3/2 (the third harmonics) are approximately equal to some power of 2, since the sequence of pitches must close on an octave.

	Powers of 3/2		Powers of 2	Ratio
1	1.50	1	2.00	0.75
2	2.25			1.125
3	3.38	2	4.00	0.845
4	5.06			1.265
5	7.59	3	8.00	0.949*
6	11.39			1.424
7	17.09	4	16,00	1.068*
8	25.63			1.602
9	38.44	5	32.00	1.201
10	57.67			1.802
11	86.50	6	64.00	1.352
12	129.75	7	128.00	1.014*

We note that the ratios between a power of 3/2 and a power of 2 are close to unity in the three starred cases. The first starred case leads to a pentatonic scale. The second leads to a seven note scale and the last to a twelve note scale. This last was selected as the basic chromatic scale used in occidental music. However, the

12th root of 128 is 1.4983 not 1.5000, close but not exact. So in order to adjust for the non-exactness, the so-called equal temperament scale was devised in which the twelve notes were separated by a pitch factor of 1.05946 which is equal to the twelfth root of two. Adjustments are always required in order to fit a power of 3, 5, 7, ... to the octave, i.e to a power of 2.

Relating to the algorithm, we first choose a base pitch, say C = 286 hz. This pitch is then repeatedly multiplied by the pitch factor, 1.4983 and finally reduced to one octave, or it is simply multiplied by 1.05946 to generate the 12 pitches of the scale. Closure is effected either by adjusting the pitch factor or leaving some anomolous ratios. If the ratios are true third harmonics, the sequence will not close on an octave. If the scale is adjusted to close, the pitches are no longer true third harmonics. We have here an example of Godel's incompleteness theorem. The scale cannot be both perfect (true harmonics) and complete (close on an octave).

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THE ANALEMMA LOOP

The liturgical year may be generated by a fibonacci ratio of 8/13 or by the golden ratio (= 0.618034). These are values that effect near closure like the starred values in the circle of fifths example. The algorithm may be carried out in two ways. The first way requires the generation of numerical sequences directly from the ratios then translates these sequences into date intervals by multiplying by the length of the year. The second method first evaluates a basic date interval from the ratio and then generates the date sequence with this date interval.

TRANSFORMS

In the development of analysis several operations known as **transforms** were introduced. These operations had the property of altering the perspective on the objects being described. For example, a transform known as the Laplace transform

$$f(\alpha) = \int_{x=0}^{\infty} e^{-\alpha x} F(x)\, dx$$

has the property of converting derivatives and integrals into products and quotients or in general converting differential and integral equations into algebraic equations. Another operation known as the Fourier transform

$$f_s(n) = \int_{x=0}^{\pi} F(x) \sin(nx)\, dx$$

has the property of changing from a time perspective to a frequency perspective. Another way of looking at the Fourier transform is that it can analyze a continuous wave from and transform it into a spectrum of its harmonic contents.

An interesting example of this is the cochlea, the spiral shaped organ in the inner ear. The cochlea creates a spectrum of the sound wave received by the ear and sends the spectrum data on to the brain. The brain then establishes a fundamental frequency and separates its harmonics thus creating the sensation of pitch and timbre or tone color. In the outer world there is sound which is energy and information in wave form, while inside the brain there is a spectral analysis of the sonic information providing a fundamental and a set of harmonics each with an assigned relative intensity. The cochlea and brain have performed a fourier transform on the incoming energy-time information producing intensity-frequency information.

It is not clear whether the spiral shape of the cochlea is for any purpose other than economy of space. A straight tube of diminishing diameter with nerve sensors located linearly in the same way as they are in the cochlea would seem to perform the same function, all else being the same. However, spirals possess other important properties that may play a role in effecting the transform.

Another interesting example of the human transformation of information from the time-energy patterns of nature into an alternate information form is in the Weber-Fechner Law which states that inner information is proportional to the logarithm of the sensation received. This is true for optical information (cf the astronomers logarithmic scale of stellar magnitudes) and aural information (the logarithmic decibel scale for intensity of sound). Humans interact with the world by creating a transformed inner world which samples from the cosmos that which its sensors and processors can extract.

MUSENT.WPC 1995

INTRODUCTION TO MUSICAL STRUCTURE

For a complete discussion of the organization of any body of knowledge or praxis, two complementary approaches are required: 1) The histoorical approach--describing the actual path by which the present state of knowledge or praxis was ~~arrived at~~ reached and 2) The morphological approach--describing all of the <u>possibilites</u> that may be seen from the vantage point, ~~and~~ or disdvantage point,of the present. The path of development tells us about process--how we arrive at our structures and products. The second or morphological approach, in putting together as complete a structure as possible, best shows us where we may go in the future. Both of these approaches will be used in describing the origins of musical scales.

A further word about process vs. product or recipe vs. blueprint. A given structure may be made by more than one process, but a given process leads to but one structure. or a given place may be reached by many paths but a given path (branches being counted as separate paths) leads to but one place. This basic asymmetry between process and product, path and place, link and node, relation and entity infers the necessity of at least two non-interchangeable, non-dual elementals in the universe. Thus our basic theories must be founded on <u>dichotomous</u> sets. We shall in the present instance see that a qiven scale may be derived in several ways ~~but any~~ but any given method of derivation leads to but one scale. This asymmetry is of importance in relating the historical approach to the morphological approach. We could have ended up at the same place that we find ourselves today, even though we had followed other paths of evolution. The number of possible species (of scale, for example) may be quite limited even though the number of possible evolutionary paths is large.

All of this is contained in the relation between the number of nodes and the minimum number of paths linking them. If N is the number of nodes in a network, then the minimum number of <u>essential</u> paths connecting them is $N(N-1)/2$. It follows that $N < N(N-1)/2$ whenever $N > 3$.

Human creativity is constrained by the basic properties of the natural world, the properties of materials and substances, the laws of chemistry and physics, and the nature of our own beings. Yet within these natural bounds frequently our option space remains too large for our human information processing capacities to cope with. In this event we further restrict ourselves arbitrarily by introducing our own constraints--both,conscious and unconscious. These constraints may be cultural, social, legal, psychological whatever. They are agreed upon either tacitly or by conscious subscription. Artistic creativity usually takes the form of exploration of an arbitrarily restricted option space. Musical creativity, like other artistic creativity, consists of the intuitive and systematic exploration of an arbitrarily delimited option space. Its essence is the search for the aesthetic possibilities allowable within the constraints--the variatians on a theme. In particular in music we employ various arrangements of tonal elements that are permitted us by certain restrictive agreed upon rules. These rules derive both from the nature of sound and from our own physiological and psychological natures. These rules usually take the form of an organizational framework about which we structure the substance with which we wish to work. In the case of music the organizational frameworks are the musical scales which determine the set of permitted and disallowed sonic elements and their relations. The substances with which we are working in music are sound and time although it may be somewhat redundant to speak of both since sound is not abailable to us except through the its fluctuations in time.

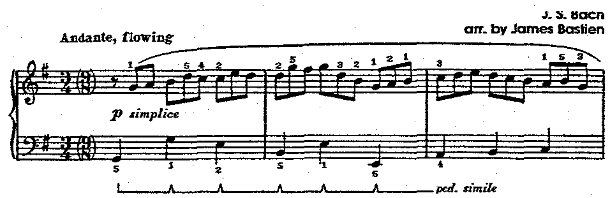
We shall thus take as our point of departure the processes and products through which we organize sound. These musical scales tell us what tones we may use and what tones are excluded and what are the relationships between the tones. Music may thus be defined as the ordered arrangement of sound according to certain agreed upon rules.

But on some deeper level what we call music is in no sense arbitrary but is a para-language by which we describe ourselves and the world in which we have our being. In some fundamental way the tones and their arrangements map with sound the basic essence of the universe. The physical world is certainly harmonically organized--the frequencies of nature--not only those of sound waves--bear certain definite ratios to one another. When expressed in pure number, these ratios, whether in music or physics, contain the secret of what may and what may not be. And for what may be, how it must be.

Beyond this music may be more than a way of describing ourselves and the world. Through the resonances it creates with other cycles, vibrations and harmonies, it may actually be reshaping ourselves and the world. Thus music must not only face the terrifying responsibility that faces all of art and human creativity. But even more, if what we create alters what already exists, then our responsibilities are those of gods and not those of children, which we persist in being.

ON TIME AND FREQUENCY

Whenever I look at a piece of sheet music, I am intrigued by how the symbolism of music shows us that we invariably discriminate and separate time from frequency (or pitch as musicians prefer to call it).



In written music, time moves from left to right horizontally, while pitch goes vertically from bottom to top as frequency increases. We understand that pitch or frequency is the reciprocal of time, $f = 1/t$. So pitch and duration are just two different ways of looking at time. Why do we view time in these two distinct ways and how do we decide where to stop viewing time as duration and changeover to view time as pitch? Is there more involved than just inverting the $1/t$ equation? The equation tells us that there are as many frequencies between zero and one as there is time from one, or now, to infinity. But what is **one**, what does **one** stand for?

Depending on the loudness, the average human ear can hear sounds from about 20 hertz (cycles/second) to 16,000 hertz. Depending on the tempo there can be up to about M.M.240, that is at extreme prestissimo, about 240 quarter notes per minute. This value is equivalent to a quarter note having a duration of one quarter of a second, an eighth note one eighth of a second, a sixteenth note one sixteenth of a second, etc. Here the time durations of notes are approaching the same values as the frequencies we hear at the lowest levels of pitch. So it appears that somewhere in the range say 8 to 16 hertz we make the switch of preference between time and frequency.

The second is the shortest time unit that humans find useful to measure sensory experience, (nanoseconds and femtoseconds are for computers). We express time periods longer than a second in numbers of seconds, (or in units of multiple seconds, such as minutes, days, years). But we express time periods shorter than a second in frequency units or hertz. (There is, however, an ambiguous region between about 1 second and 1/20th second (or 20 hertz) where both systems are used. Also note here that the number of motion picture frames per second needed to create for us the illusion of continuous motion is from 8 to 16). Evidently then, there is something fundamental in the internal human clock that switches in this zone.

One hypothesis is that humans use the Schuster Electron Time[1] [SET] of 0.121 second as a zeitgeber. Since this value is very close to 1/8 second, we might say that [SET] is the metronome that governs our time sense. We switch to frequency representations at times shorter than [SET] and to duration representations at times longer than [SET]. It is probably not fortuitous that the duration value of the second is near this period, but it does seem fortuitous that this value is related to the rotation period of the earth.

Another matter of interest in the musical utilization of time and sound is that in both the duration and pitch zones there are intervals of silence. In the horizontal zone, there is a brief silence between the sounding of each note. (One classical composer held that the whole purpose of music was to give quality to these intervals of silence). In the vertical zone there are non-pitch intervals between the values of pitch that are set by scales or modes. All of this is present in our music, but somehow musical notation obscures it from us. But then there are no symbols that carry all the reality of that which they symbolize.

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New 0.120498

where r_e is the radius of the electron, m_e is its mass and G is the gravitational constant.

MUSICAL SCALES

The **Pythagorean** scale is based on third harmonics, or "fifths". Starting with a fundamental frequency C by adding successive fifths we get:

C,G,D,A,E,B Then subtracting one fifth gives: F,C,G,D,A,E,B

When arranged in the following order, we obtain the eight note **diatonic** scale:

C,D,E,F,G,A,B,C' Where t is a "full tone" and s is a "semi-tone"

t t s t t t s

If the process of adding successive fifths is continued, we get:

F,C,G,D,A,E,B,F#,C#,G#,Eb,Bb,F The "circle of fifths"

The twelve notes of this set, arranged in the order:

C,C#,D,Eb,E,F,F#,G,G#,A,Bb,B,C'

is called the **chromatic** scale in which all the intervals are semi-tones

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The **Just Intonation** scale employs fifth harmonics ["thirds", t] as well as third harmonics ["fifths", f]. The just intonation diatonic scale is constructed as follows:

C, D=2f, E=t, F= –f, G=f, A=t–f, B=t+f

The **Mean Tone** scale uses an altered fifth which is 22 cents less than the perfect fifth. Four such fifths in succession lead to a perfect third. C, G, D*,A*,E*=E

The **Equal Temperament** scale divides the octave into twelve equal semi-tones. The frequency ratio for the equal temperament semi-tone is the twelfth root of two = 1.05946, e.g., C=1, C# = 1.05946, D = $(1.05946)^2$, etc. The table gives the frequencies of the diatonic scale in the Pythagorean system [P], Just Intonation system [J], and equal temperament system [E]

	C	D	E	F	G	A	B	C'
P	520	585	658	693	780	877	987	1040
J	520	585	650	693	780	867	975	1040
E	520	584	655	694	779	874	982	1040

If the octave is divided into 1200 parts called cents, then an equal temperament semi-tone = 100 cents. Using cents, comparing the just intonation and equal temperament scales:

	C	D	E	F	G	A	B	C'
J	0	204	385	498	702	885	1088	1200
E	0	200	400	500	700	900	1100	1200

2002 #19

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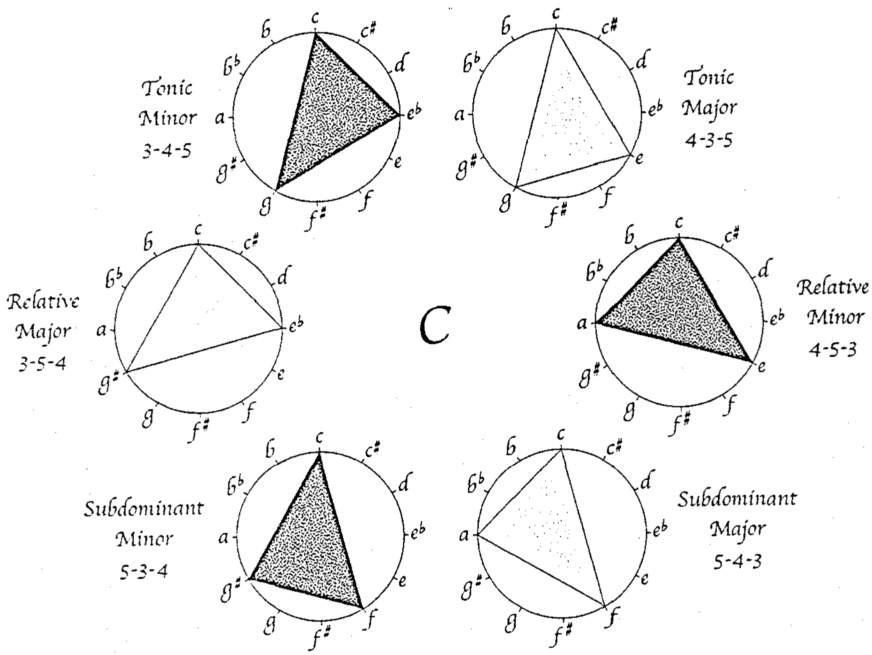
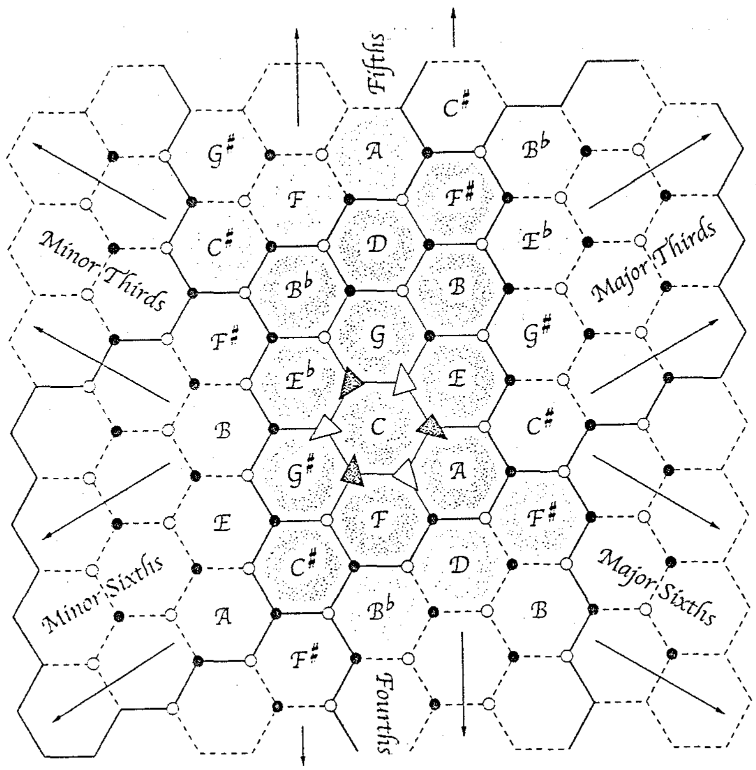

Tonic Minor 3-4-5
Tonic Major 4-3-5
Relative Major 3-5-4
C
Relative Minor 4-5-3
Subdominant Minor 5-3-4
Subdominant Major 5-4-3


Fifths
Minor Thirds
Major Thirds
Minor Sixths
Major Sixths
Fourths

QUADRIVIUM

See 1994 #5
1997 #32

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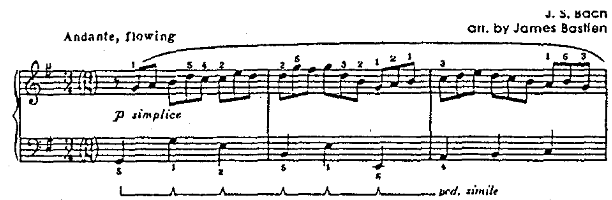

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